Lahore University of Management Sciences

AI 501 – Mathematics for Artificial Intelligence

Fall 2024

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Course URL (if any)	https://www.zubairkhalid.org/ai501_2024.html

Course Teaching Methodology (Please mention following details in plain text)
• Teaching Methodology: In-person classes (2.5 hours meeting once a week) • Attendance is mandatory as maintaining a good attendance will help students in many ways. Students not frequently attending the lecture will find it difficult to cope with the course. We will also have assessments during the class.

Course Basics				
Credit Hours	3			
Lecture(s)	Nbr of Lec(s) Per Week	1	Duration	2 hours and 30 minutes
Recitation/Lab (per week)	Nbr of Lec(s) Per Week	0	Duration	
Tutorial (per week)	Nbr of Lec(s) Per Week	1	Duration	1 hour (if needed)

Course Distribution	
Core	MS AI
Elective	None
Open for Student Category	MS AI
Close for Student Category	All other students (Open for Auditing)

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COURSE DESCRIPTION
This course offers an in-depth exploration of the mathematical principles that form the foundations of machine learning (ML) and artificial intelligence (AI). Aimed at graduate students and industry professionals, this course is designed to provide a rigorous understanding of the mathematical concepts crucial for developing, implementing, and evaluating ML and AI algorithms. In broad brush terms, we will be covering the following topics in the course: • **Vector and Matrix Operations**: Understanding vectors, matrices, and their operations is fundamental to data representation and transformations in ML. The course covers vector spaces, linear transformations, eigenvalues, and eigenvectors, focusing on their practical applications in data analysis and feature extraction. • **Linear and Logistic Regression**: Students will explore regression techniques, crucial for prediction and classification tasks. The course delves into the least squares method, regularization techniques, and logistic regression, linking theoretical concepts to practical applications in supervised learning. • **Matrix Decompositions**: The course includes detailed discussions on eigenvalue decomposition (EVD) and singular value decomposition (SVD), highlighting their importance in dimensionality reduction, data compression, and noise reduction. • **Dimensionality Reduction**: Techniques such as Principal Component Analysis (PCA) and Linear Discriminant Analysis (LDA) are explored, providing tools to manage and visualize high-dimensional data. • **Optimization Techniques**: Essential for training ML models, optimization topics cover gradient descent, convex optimization, and advanced methods like Newton's method. Students will learn to implement these techniques to minimize cost functions effectively. • **Probability and Statistics**: Fundamental probabilistic concepts are covered, including random variables, distributions, Bayes' theorem, and inference methods. This foundation is crucial for understanding probabilistic models and Bayesian inference. • **Machine Learning Algorithms**: The course includes a comprehensive overview of key ML algorithms, including Support Vector Machines (SVM), decision trees, and neural networks. Advanced topics such as deep learning and convolutional neural networks are also introduced. • **Real-World Applications and Case Studies**: Throughout the course, theoretical concepts are linked with practical applications through case studies and real-world examples, providing students with insights into how ML and AI are applied across various domains.

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COURSE PREREQUISITE(S)	
•	Pre-requisites: None
•	Co-requisites: None

COURSE OBJECTIVES
• Foundational Knowledge: Equip students with a robust understanding of linear algebra, calculus, probability theory, and optimization techniques, which are essential for ML and AI. • Practical Applications: Demonstrate the application of mathematical concepts in real-world ML and AI scenarios, including data processing, model training, and performance evaluation. • Analytical Thinking: Foster analytical thinking by encouraging students to critically assess different algorithms and their suitability for various tasks, considering both theoretical foundations and practical applications.

Grading Distribution: Component Details and weightages
Assignments, 25 %
Quizzes, 25 %
Mid-Exam and Mid-Viva, 20 %
Final Exam and Final Viva, 30 %

Textbook(s)/Supplementary Readings

Books:

- S.Boyd and L. Vandenberghe. **Introduction to Applied Linear Algebra – Vectors, Matrices, and Least Squares.** Cambridge University Press, 2019
- M. P. Deisenroth, A. A. Faisal and Cheng Soon Ong. **Mathematics for Machine Learning**. Cambridge University Press, 2019
- Ali H. Sayed, **Inference and Learning from Data,** Foundations (Volume 1). Cambridge University Press, 2022
- G. Strang. **Introduction to Linear Algebra.** 2016
- J. A. Gubner, **Probability and Random Processes for Electrical and Computer Engineers**, Cambridge University Press, 2006.
- S. L. Miller and D. Childers, **Probability and Random Processes: With Applications to Signal Processing and Communications.**
- A. Papoulis and S.U. Pillai, **Probability, Random Variables, and Stochastic Processes.**
- Class notes will be provided to supplement these readings

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Support Services
LUMS offers a range of academic and other services to support students. These are mentioned below, and you are encouraged to use these in addition to in-class assistance from course staff. For a complete list of campus support services available for you click here (https://advising.lums.edu.pk/#supportservices).

Campus supports & Key university policies
Campus Supports Students are strongly encouraged to meet course instructors and TA's during office hours for assistance in course-content, understand the course's expectations from enrolled students, etc. Beyond the course, students are also encouraged to use a variety of other resources. (Instructors are also encouraged to refer students to these resources when needed.) These resources include Counseling and Psychological Services/CAPS (for mental health), LUMS Medical Center/LMC (for physical health), Office of Accessibility & Inclusion/ OAI (for long-term disabilities), advising staff dedicated to supporting and guiding students in each school, online resources (https://advising.lums.edu.pk/advising-resources), etc. To view all support services, their specific role as well as contact information click here (https://advising.lums.edu.pk/#supportservices). **Academic Honesty/Plagiarism** LUMS has zero tolerance for academic dishonesty. Students are responsible for upholding academic integrity. If unsure, refer to the student handbook and consult with instructors/teaching assistants. To check for plagiarism before essay submission, use similarity@lums.edu.pk. Consult the following resources: 1) Academic and Intellectual Integrity (http://surl.li/gpvwb), and 2) Understanding and Avoiding Plagiarism (http://surl.li/gpvwo). **LUMS Academic Accommodations/ Petitions policy** Long-term medical conditions are accommodated through the Office of Accessibility & Inclusion (OAI). Short-term emergencies that impact studies are either handled by the course instructor or Student Support Services (SSS). For more information, please see Missed Instrument or 'Petition' FAQs for students and faculty (https://rb.gy/8sj1h) **LUMS Sexual Harassment Policy** LUMS and this class are a harassment-free zone. No behavior that makes someone uncomfortable or negatively impacts the class or individual's potential will be tolerated. To report sexual harassment experienced or observed in class, please contact me. For further support or to file a complaint, contact OAI at oai@lums.edu.pk or harassment@lums.edu.pk. You may choose to file an informal or formal complaint to put an end to the offending behavior. You can also call their Anti-Harassment helpline at 042-35608877 for advice or concerns. *For more information: Harassment, Bullying & Other Interpersonal Misconduct: Presentation (http://surl.li/gpvwt)*

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Course Topics and Schedule

Week 1: Introduction and Vector Operations

- Course Overview: Objectives, structure, and relevance to ML/AI
- Vectors in Machine Learning: Importance, representation, and operations
- Vector Space, Subspace: Definitions, examples, importance in ML
- Basis and Dimension: Conceptual understanding, applications in data representation
- Angle and Distance: Cosine similarity, Euclidean distance
- Correlation Coefficient: Measuring linear relationships

Week 2: Advanced Vector Concepts, Matrix Operations and Linear Transformations

- Vector Spaces: Span, basis, linear independence
- Orthonormal Vectors: Concept and importance in ML
- Matrix Notation and Operations: Addition, multiplication, transposition
- Matrix-Vector and Matrix-Matrix Products: Interpretations and applications
- Block Matrices: Structure and use cases
- Practical Applications: Transformations, data projections

Week 3: Solving Systems of Linear Equations

- System Formulation: Representation and solutions
- Matrix Inverses: Computation and application
- Pseudo-Inverses: Moore-Penrose inverse, applications in least squares

Week 4: Linear Regression and Logistic Regression in Machine Learning – Formulation

- Supervised Learning Basics: Nomenclature, problem setup
- Train-Test Split: Cross-validation, model evaluation
- Regression and Classification Overview
- Linear and Multiple Regression: Model formulation, loss functions
- Least Squares Formulation: Regularization, ridge regression
- Logistic Regression: Basics, link to classification, practical applications
- Classification Metrics: Accuracy, precision, recall, F1 score, ROC curves

Week 5: Eigenvalue and Singular Value Decompositions

- Eigenvalue Decomposition (EVD): Theory and computation
- Diagonalization: Importance in simplifying matrix operations
- Hands-On Examples: Computing eigenvalues and eigenvectors
- Singular Value Decomposition (SVD): Theory, computation, applications
- Low rank approximations

Week 6: Advanced Matrix Concepts

- Gram-Schmidt Process: Orthogonalization, QR decomposition
- Polar Decomposition: Theoretical understanding, applications
- Minimal Polynomial and Jordan Canonical Form: Theory and applications
- Advanced Matrix Applications in ML: Case studies

Week 7: Curse of Dimensionality and Dimensionality Reduction Methods

- Curse of Dimensionality: Challenges in high-dimensional data
- Principal Component Analysis (PCA): Dimensionality reduction, data visualization
- Linear Discriminant Analysis (LDA): Concepts, differences from PCA, applications

Week 8: Multivariate Calculus

- Functions and Limits: Basics and relevance to optimization
- Derivatives and Gradients: Partial derivatives, gradient vectors
- Jacobian and Hessian Matrices: Computation and interpretation
- Application: Least-squares solutions

Week 9: Optimization Techniques

- Optimization Basics: Objective functions, local and global minima
- Convex Sets and Functions: Definitions, properties, and significance
- Gradient Descent: Algorithm, variants (stochastic, mini-batch)
- Advanced Methods: Newton's method, conjugate gradient
- Optimization in machine learning algorithms: Linear and logistic regression

Week 10: Support Vector Machine

- Formulation: Hard SVM and Soft SVM
- Optimization problem formulation (including the concept of duality in optimization)
- Applications and kernel trick

Week 12: Probability Theory Fundamentals

- Probability Basics: Axioms, conditional probability
- Random Variables: Discrete and continuous, expectation, variance
- Important Distributions: Bernoulli, Binomial, Gaussian
- Bayes Theorem
- Expectation and Variance

Week 13: MLE and MAP Estimation

- Bayes' Theorem and Bayesian inference
- Naive Bayes Classifier, Naïve Bayes Classifier for Text Classification
- MLE and MAP: Estimation techniques, comparison
- Regression: a Bayesian Perspective: ML and MAP Estimators, Bayesian Approach

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Week 14: Probabilistic Machine Learning

- Bayesian Inference
- Predictive posterior distribution – challenges
- Sampling: Rejection Sampling, Monte Carlo, Specific Sampling Techniques (like Box Muller)
- Approximate Inference: Variational Inference, Markov Chain Monte Carlo, Laplace Approximation