Name: _____

Campus ID: _____

Total Marks: 10
Time Duration: 15 minutes

Question 1 (4 marks)

You are working on a project to analyze customer purchasing behavior in an online retail store. You are given three customers' purchase history represented as vectors in a three-dimensional space, where each dimension corresponds to the number of purchases in three different product categories: electronics, clothing, and groceries.

The purchase history vectors for three customers are:

$$\mathbf{v}_A = \begin{pmatrix} 15 \\ 7 \\ 3 \end{pmatrix}, \mathbf{v}_B = \begin{pmatrix} 6 \\ 12 \\ 7 \end{pmatrix}, \mathbf{v}_C = \begin{pmatrix} 14 \\ 2 \\ 9 \end{pmatrix}$$

(a) How are the purchasing habits of customer A correlated with that of customer C?

(b) Another task on the project requires you to form an "average" customer profile from the purchase histories of Customers A, B and C. You are told that the combination must be convex, and that the weight of the purchasing history of Customer A $w_a = 0.3$. Choosing appropriate weights w_b and w_c, compute the "average" customer profile. We can have multiple answers to this problem.

Question 2 (6 marks)

Suppose we have the following three vectors in $\mathbb{R}^3$:

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \mathbf{v}_2 = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}, \mathbf{v}_3 = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$$

(a) Are these vectors mutually orthogonal to each other?

(b) Is this set of vectors linearly dependent?

(c) Do these vectors span $\mathbb{R}^3$?

(d) Compute the normalized version of these vectors such that each vector has a unit 2-norm. Express the following vector as a linear combination of the normalized vectors:

$$\mathbf{x} = \begin{pmatrix} 5 \\ 5 \\ 5 \end{pmatrix}$$