LAHORE UNIVERSITY OF MANAGEMENT SCIENCES
Department of Electrical Engineering

AI 501 Mathematics for Artificial Intelligence
Quiz 04 Solutions

Name: ____________________

Campus ID: ____________________

Total Marks: 10
Time Duration: 15 minutes

Question 1 (6 marks)

Select ALL (upto three) correct choices.

- Which of the following is/are true about kNN algorithm?
  (a) It can be used for both supervised and unsupervised learning tasks.
  (b) A small k value makes the model sensitive to noise.
  (c) Computational complexity to carry out prediction does not depend on the size of the training data.
  (d) It requires the data to be scaled or normalized.

  Solution: (b), (d)

- For kNN classifier, what tends to be correct about decreasing the k in kNN algorithm?
  (a) The boundary become smoother
  (b) The model becomes more computationally expensive during training.
  (c) The model is prone to overfit to the data.
  (d) The bias of the model increases.

  Solution: (c)

- What is the time complexity of finding the k nearest neighbors for a given test point in kNN?
  (a) $O(1)$
  (b) $O(n)$
  (c) $O(n^2)$
  (d) $O(nlog(n))$

  Solution: (b)

- What is/are the drawback of using a very large value for k in kNN?
  (a) It can struggle to accurately classify minority classes in imbalanced data.
  (b) The decision boundary becomes overly simplistic.
  (c) A large k guarantees high accuracy on unseen data.
  (d) The risk of overfitting increases.

  Solution: (a), (b)

- For a wide matrix A, the inverse (either left or right) exists. Which of the following statements hold true for matrix A.
  (a) Left inverse exists.
  (b) All rows are linearly independent.
  (c) The matrix is under-determined.
  (d) The matrix is over-determined.

  Solution: (b), (c)

- Which of the following matrices is guaranteed to have a pseudo-inverse but may not have a left or right inverse?
  (a) An orthogonal matrix with full rank.
  (b) A rectangular matrix with linearly dependent rows.

(c) A rectangular matrix with linearly dependent columns.

(d) A square non-singular matrix.

Question 2 (4 marks)

For an $m \times n$ matrix A and its pseudo-inverse $A^\dagger$, show that $A = AA^\dagger A$ and $A^\dagger = A^\dagger A A^\dagger$ in each of the following cases.

(a) A is tall with linearly independent columns.

Solution: $A^\dagger$ is the left inverse of A, so $A^\dagger A = I$. Therefore,

1. $AA^\dagger A = A(A^\dagger A) = AI = A$,
2. $A^\dagger A A^\dagger = (A^\dagger A)A^\dagger = IA^\dagger = A^\dagger$.

(b) A is wide with linearly independent rows.

Solution: $A^\dagger$ is the right inverse of A, so $AA^\dagger = I$. Therefore,

1. $AA^\dagger A = (AA^\dagger)A = IA = A$,
2. $A^\dagger A A^\dagger = A^\dagger(AA^\dagger) = A^\dagger I = A^\dagger$.