Name: _____

Campus ID: _____

Total Marks: 10
Time Duration: 15 minutes

Question 1 (2 marks)

We use the Euclidean and Manhattan norms to define the regularization term in Ridge and Lasso regression techniques respectively. Consider $\boldsymbol{\theta} \in \mathbb{R}^2$. Plot all the points in (θ_1, θ_2) plane for which $||\boldsymbol{\theta}||_2 = 2$ and $||\boldsymbol{\theta}||_1 = 2$.

Question 2 (8 marks)

In class, we looked at ridge regression where we minimize the loss function

$$\mathcal{L}_{reg}(\boldsymbol{\theta}) = \frac{1}{2}||(\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\theta})||_2^2 + \frac{\lambda}{2}||\boldsymbol{\theta}||_2^2$$

(a) [**6 marks**] Derive the closed-form solution for the ridge regression problem, that is, find $\boldsymbol{\theta}$ that minimizes the loss function. We require you to show all the steps. The following information can be useful: $\nabla \mathbf{a}^T \mathbf{x} = \mathbf{a}$, $\nabla \mathbf{x}^T \mathbf{x} = 2\mathbf{x}$, and $\nabla \mathbf{x}^T \mathbf{P} \mathbf{x} = 2\mathbf{P}\mathbf{x}$ for a symmetric matrix $\mathbf{P}$.

(b) [**2 marks**] How does the solution change as $|\lambda| \to 0$ and $|\lambda| \to \infty$? Also identify in which case the model over-fits and under-fits to the data.