LAHORE UNIVERSITY OF MANAGEMENT SCIENCES Department of Electrical Engineering

AI 501 Mathematics for Artificial Intelligence Quiz 06

Name:			
Campus ID:	_		
Total Marks: 10 Time Duration: 15 minutes			

Question 1 (2 marks)

We use the Euclidean and Manhattan norms to define the regularization term in Ridge and Lasso regression techniques respectively. Consider $\boldsymbol{\theta} \in \mathbb{R}^2$. Plot all the points in (θ_1, θ_2) plane for which $||\boldsymbol{\theta}||_2 = 2$ and $||\boldsymbol{\theta}||_1 = 2$.

Question 2 (8 marks)

In class, we looked at ridge regression where we minimize the loss function

$$\mathcal{L}_{reg}(oldsymbol{ heta}) = rac{1}{2}||(oldsymbol{y} - oldsymbol{X}oldsymbol{ heta})||_2^2 + rac{\lambda}{2}||oldsymbol{ heta}||_2^2$$

- (a) [6 marks] Derive the closed-form solution for the ridge regression problem, that is, find θ that minimizes the loss function. We require you to show all the steps. The following information can be useful: $\nabla \mathbf{a}^T \mathbf{x} = \mathbf{a}$, $\nabla \mathbf{x}^T \mathbf{x} = 2\mathbf{x}$, and $\nabla \mathbf{x}^T \mathbf{P} \mathbf{x} = 2\mathbf{P} \mathbf{x}$ for a symmetric matrix \mathbf{P} .
- (b) [2 marks] How does the solution change as $|\lambda| \to 0$ and $|\lambda| \to \infty$? Also identify in which case the model over-fits and under-fits to the data.