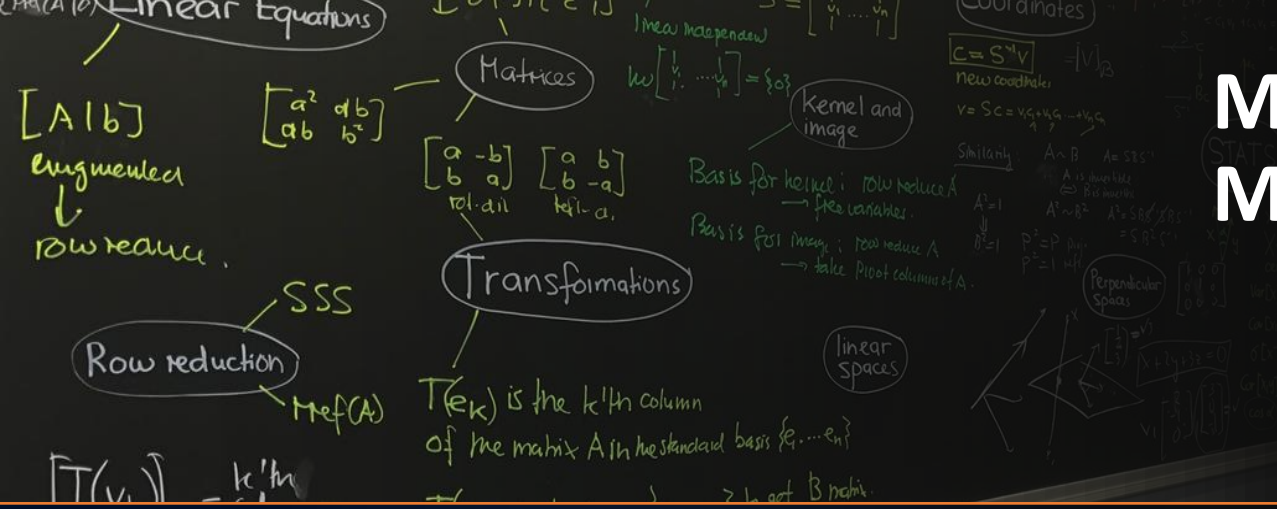


# Mathematical Foundations for Machine Learning and Data Science

## Tutorial 04



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## Problem 01:

You are told that a prize is equally likely to be found behind one out of three closed doors. You point to one of the doors. Then, your friend opens for you one of the remaining two doors, after she makes sure that the prize is not behind it. At this point, you can stick to your initial choice, or switch to the other unopened door. You win the prize if it lies behind your final choice of a door.

As far as the probability of winning is concerned, is there an advantage in switching doors? State precisely any modelling assumptions you are making.

## Problem 02:

For the function  $f(x_1, x_2, x_3) = x_1^2 + 2x_2^3 + 3x_3^4$ , find  $\nabla f$ .

## Problem 03:

Consider the following functions:

$$f_1(x) = \sin(x_1) \cos(x_2), \quad x \in \mathbb{R}^2$$

$$f_2(x, y) = x^T y, \quad x, y \in \mathbb{R}^n$$

- (a) What are the dimensions of  $\frac{\partial f_i}{\partial x}$ ?
- (b) Compute the Jacobians.

## Problem 04:

An experiment consists of tossing two dice.

- (a) [2 marks] Find the sample space  $S$ .
- (b) [3 marks] Find the event  $A$  that the sum of the dots on the dice is greater than 7.
- (c) [3 marks] Find the event  $B$  that the sum of the dots on the dice is greater than 10.
- (d) [2 marks] Find the event  $C$  that the sum of the dots on the dice is greater than 12.

## Problem 05:

A company producing electric relays has three manufacturing plants producing 50, 30, and 20 percent, respectively, of its product. Suppose that the probabilities that a relay manufactured by these plants is defective are 0.02, 0.05, and 0.01, respectively.

- (a) [5 marks] If a relay is selected at random from the output of the company, what is the probability that it is defective?
- (b) [5 marks] If a relay selected at random is found to be defective, what is the probability that it was manufactured by plant 2?