LAHORE UNIVERSITY OF MANAGEMENT SCIENCES
Department of Electrical Engineering

EE212 Mathematical Foundations for Machine Learning and Data Science
Quiz 01 Solutions

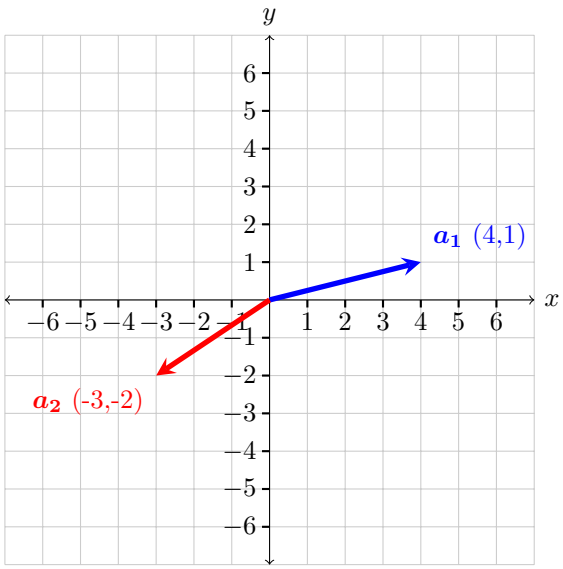
Name: _____

Campus ID: _____

Total Marks: 10
Time Duration: 15 minutes

Question 1 (4 marks)

Let a_1 and a_2 be two vectors in $\mathbb{R}^2$ as shown below:



The linear combination of the given vectors is of form $\beta_1 a_1 + \beta_2 a_2$ where β_1 and β_2 are scalar coefficients of the linear combination.
For the given vectors, express the following combinations on the plot and give ranges for β values:

(a) [2 marks] An affine combination.

Solution: For affine combination, the sum of the coefficients is 1.

(b) [2 marks] A convex combination.

Solution: For convex combination, the sum of the coefficients is 1 and all coefficients are non-negative. i.e., all coefficients are between 0 and 1.

Question 2 (4 marks)

Suppose the 100-vector x represents the distribution of ages in some population of people, with x_i being the number of i-1 year olds, for $i = 1, ...100$. (You can assume that x is not 0 and there is no one in the population over age 99.) Find expressions, using vector notation, for the following quantities.

(a) [2 marks] The total number of people in the population.

Solution: The total population is $1^T x$.

(b) [2 marks] The average age of the population. (You can use ordinary division of numbers in your expression.)

Solution: The sum of the ages across the population is $(0, 1, 2, ..., 99)^T x$. And so the average age is given by

$$\frac{(0, 1, 2, .., 99)^T x}{1^T x}$$

Question 3 (2 marks)

Standardize the following vector:

$$x = \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}$$

Solution:

$$z = \frac{x - avg(x)}{std(x)}$$

$$avg(x) = \frac{4 + 3 + 5}{3} = 4$$

$$x_{de-meaned} = \begin{bmatrix} 4 - 4 \\ 3 - 4 \\ 5 - 4 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

$$std(x) = \sqrt{\frac{(0)^2 + (-1)^2 + (1)^2}{3}} = \sqrt{\frac{2}{3}}$$

$$z = \sqrt{\frac{3}{2}} \begin{bmatrix} 4 - 4 \\ 3 - 4 \\ 5 - 4 \end{bmatrix} = \begin{bmatrix} 0 \\ -\sqrt{\frac{3}{2}} \\ \sqrt{\frac{3}{2}} \end{bmatrix}$$