

LAHORE UNIVERSITY OF MANAGEMENT SCIENCES
Department of Electrical Engineering

EE212 Mathematical Foundations for Machine Learning and Data Science
Quiz 03 Solutions

Name: _____

Campus ID: _____

Total Marks: 10

Time Duration: 15 minutes

Question 1 (6 marks)

Given below is a general representation of the four fundamental subspaces of a matrix $A^{m \times n}$. Here A' denotes the transpose of matrix A .

Redraw and label *exact dimension* for each of the following case.

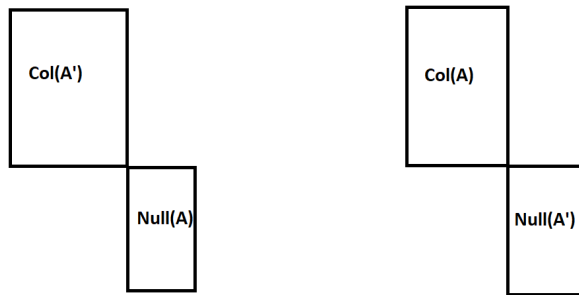


Figure 1: The Four Fundamental Sub-spaces of a Matrix.

(a) $A^{m \times n}$ for $\text{Rank}(A) = r < \min(m, n)$.

Solution:

(b) $A^{m \times n}$ for $m < n$ and $\text{Rank}(A) = m$ (Wide matrix with full row rank).

Solution:

(c) $A^{m \times n}$ for $m > n$ and $\text{Rank}(A) = n$ (Tall matrix with full column rank).

Solution:

Question 2 (4 marks)

This problem is regarding least squares in \mathbf{R}^2 .

Given four points in \mathbf{R}^2 :

$$P_1 = (2, 1), P_2 = (5, 2), P_3 = (7, 3), P_4 = (8, 3),$$

find the best fitting line $y = \beta_0 + \beta_1 x$. You can setup your system of equations as $A\beta = y$ where,

$$A = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ x_3 & 1 \\ x_4 & 1 \end{bmatrix}, \beta = \begin{bmatrix} \beta_1 \\ \beta_0 \end{bmatrix}, y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

Here x and y represent the first and second coordinates for each point respectively.

Formulate A and y and write the solution of β explicitly in terms of A and y such that

$$\|A\beta - y\|_2^2$$

is minimized.

Solution:

$$A = \begin{bmatrix} 2 & 1 \\ 5 & 1 \\ 7 & 1 \\ 8 & 1 \end{bmatrix}, \beta = \begin{bmatrix} m \\ c \end{bmatrix}, y = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 3 \end{bmatrix}$$

We can see that A is tall and full column rank. We can use the left inverse of A to get the ordinary least square solution. $\beta = A^\dagger y$ $\beta = (A^T A)^{-1} A^T y$