LUMS
A Not-for-Profit University

Department of Electrical Engineering
School of Science and Engineering

EE212 Mathematical Foundations for Machine Learning and Data Science

ASSIGNMENT 3

Due Date: 13:30, Thursday, December 1, 2022.
Format: 7 problems, for a total of 100 marks
Instructions:

- You are allowed to collaborate with your peers but copying your colleague's solution is strictly prohibited. This is not a group assignment. Each student must submit his/her own assignment.
- Solve the assignment on blank A4 sheets and staple them before submitting.
- Submit in-class or in the dropbox labeled EE-212 outside instructor's office.
- Write your name and roll no. on the first page.
- Feel free to contact the instructor or the teaching assistants if you have any concerns.

- You represent the most competent individuals in the country, do not let plagiarism come in between your learning. In case any instance of plagiarism is detected, the disciplinary case will be dealt with according to the university's rules and regulations.

Problem 1 (12 marks)

Given a function $f(x, y) = arctan(x/y)$, where $f : R^2 \mapsto R$.

(a) [**6 marks**] Find the partial derivatives of the above function.

(b) [**4 marks**] Derive the gradient vector in part(a).

(c) [**2 marks**] Discuss the dimensions of the gradient vector.

Problem 2 (28 marks)

Consider the function $f : R^2 \mapsto R^3$ with

$$f_1(x) = -x_1^5 + 1/x_1 x_2^3$$

$$f_2(x) = arctan(x_2/x_1^2)$$

$$f_3(x) = -x_2^6$$

where $x \in R^2$, $f \in R^3$

(a) **[12 marks]** Compute the Jacobian matrix, **J**, for the given functions.

(b) **[2 marks]** Discuss the dimensions of **J**.

(c) **[3 marks]** If possible, find $|\mathbf{J}|$.

(d) **[9 marks]** Calculate the Hessian matrix, **H**, of f_2 and f_3.

(e) **[2 marks]** Discuss the dimensions of **H**.

Problem 3 (10 marks)

Suppose 32% of students own a dog, 30% of students own a cat, and 22% of the students that have a dog also own a cat. An student is chosen at random and found to have a cat. What is the probability the student also owns a dog?

Problem 4 (10 marks)

Consider an exam that contains 10 multiple-choice questions with 4 possible choices for each question, only one of which is correct.
Suppose a student is to select the answer for every question randomly. Let X be the number of questions the student answers correctly. Then, X has a binomial distribution with parameters n = 10 and p = 0.25.

(a) **[5 marks]** What is the probability for the student to get no answer correct?

(b) **[5 marks]** What is the probability for the student to get two answers correct?

Problem 5 (10 marks)

The number of injuries per week in a particular factory is know to follow a Poisson distribution with a mean of 0.5.
Find the probability that in a particular week:

(a) **[5 marks]** There will be less than two accidents.

(b) **[5 marks]** There will be more than two accidents.

Problem 6 (15 marks)

Consider a cancer diagnostic test. We use the following notations;
A person has cancer / does not have cancer $\rightarrow X = 1/X = 0$
Test is positive / Test is negative $\rightarrow Y = 1/Y = 0$
We are given the following information.

- $P(Y = 0|X = 1) = 0.2$
  Missed detection
- $P(Y = 1|X = 0) = 0.1$
  False alarm
- $P(X = 1) = 0.004$
  Chance of someone having cancer in the population.

(a) **[6 marks]** If someone tests positive, what is the probability that they have cancer?
i.e., $P(X = 1|Y = 1)$

(b) **[5 marks]** A lot of people took the test, and those who tested positive want to be tested again. Probably due to your answer to part (a). In the reduced sample space of people who tested positive the first time, find the probability that they have cancer given that they test positive again.
i.e., $P(X = 1|Y = 1)$

(c) **[4 marks]** Building on part (b), after how many consecutive positive test results will the probability that the person has cancer given they tested positive be greater than 0.99. You can simulate this by writing a simple code in Python/Matlab.

Problem 7 (15 marks)

Consider the density function:

$$f_X(x) = \begin{cases} (p+1)x^p & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

(a) **[8 marks]** Find $E[X]$.

(b) **[7 marks]** Find $Var(X)$.

— End of Assignment —