EE 240 – Circuits 1

Assignment # 3 solution

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Ans 1:

$$6i_1 - 8i_2 - 10i_3 + 12i_4 = 8$$

$$2i_1 - 4i_2 + 5i_3 + 6i_4 = 33$$

$$-8i_1 + 20i_2 + 14i_3 - 16i_4 = 10$$

$$5i_1 + 7i_2 + 2i_3 - 10i_4 = -15$$

$$\hookrightarrow \left[\begin{array}{cccc|c} 6 & -8 & -10 & 12 & 8 \\ 2 & -4 & 5 & 6 & 33 \\ -8 & 20 & 14 & -16 & 10 \\ 5 & 7 & 2 & -10 & -15 \end{array}\right]$$

$R_1 = \frac{R_1}{6}$

$R_2 = R_2 - \frac{1}{3}R_1$

$R_3 = \frac{4}{3}R_1 + R_3$

$R_4 = -\frac{5}{6}R_1 + R_4$

$$\left[\begin{array}{cccc|c} 1 & -\frac{4}{3} & -\frac{5}{3} & 2 & \frac{4}{3} \\ 0 & -\frac{4}{3} & \frac{25}{3} & 2 & \frac{91}{3} \\ 0 & \frac{28}{3} & \frac{2}{3} & 0 & \frac{62}{3} \\ 0 & \frac{41}{3} & \frac{31}{3} & -20 & -\frac{65}{3} \end{array}\right]$$

$R_2 = -\frac{3}{4}R_2$

$R_3 = R_3 + 7R_2$

$R_4 = \frac{41}{4}R_2 + R_4$

$$\left[\begin{array}{cccc|c} 1 & -\frac{4}{3} & -\frac{5}{3} & 2 & \frac{4}{3} \\ 0 & 1 & -\frac{25}{4} & -\frac{3}{2} & -\frac{91}{4} \\ 0 & 0 & 59 & 14 & 233 \\ 0 & 0 & \frac{383}{4} & \frac{1}{2} & \frac{1157}{4} \end{array}\right]$$

$R_3 = \frac{1}{59}R_3$

$R_4 = R_4 - \frac{383}{236}R_3$

$$\left[\begin{array}{cccc|c} 1 & -\frac{4}{3} & -\frac{5}{3} & 2 & \frac{4}{3} \\ 0 & 1 & -\frac{25}{4} & -\frac{3}{2} & -\frac{91}{4} \\ 0 & 0 & 1 & \frac{14}{59} & \frac{233}{59} \\ 0 & 0 & 0 & -\frac{1311}{59} & -\frac{5244}{59} \end{array}\right]$$

$R_4 = -\frac{59}{1311}R_4$

$$\left[\begin{array}{cccc|c} 1 & -\frac{4}{3} & -\frac{5}{3} & 2 & \frac{4}{3} \\ 0 & 1 & -\frac{25}{4} & -\frac{3}{2} & -\frac{91}{4} \\ 0 & 0 & 1 & \frac{14}{59} & \frac{233}{59} \\ 0 & 0 & 0 & 1 & 4 \end{array}\right]$$

$R_3 = R_3 - \frac{14}{59} R_4$

$R_2 = R_2 + \frac{3}{2} R_4$

$R_1 = R_1 - 2R_4$

we now have a set of equations that can be solved simultaneously to get solution. However, it can be further simplified:

$$\left[\begin{array}{cccc|c} 1 & -\frac{4}{3} & -\frac{5}{3} & 0 & -\frac{20}{3} \\ 0 & 1 & -\frac{25}{4} & 0 & -\frac{67}{4} \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right]$$

$R_2 = R_2 + \frac{25}{4} R_3$

$R_1 = R_1 + \frac{5}{3} R_3$

$$\left[\begin{array}{cccc|c} 1 & -\frac{4}{3} & 0 & 0 & -\frac{5}{3} \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right]$$

$R_1 = \frac{4}{3} R_2 + R_1$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right]$$

Thus,

$$\boxed{\begin{aligned} i_1 &= 1 \\ i_2 &= 2 \\ i_3 &= 3 \\ i_4 &= 4 \end{aligned}}$$

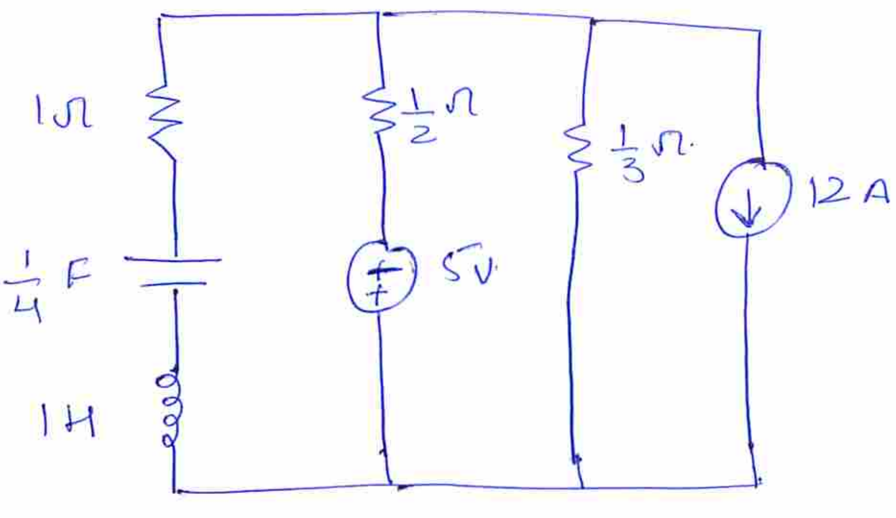
Q2

(a) Networks that have the same voltage. current relationship across the two terminals.

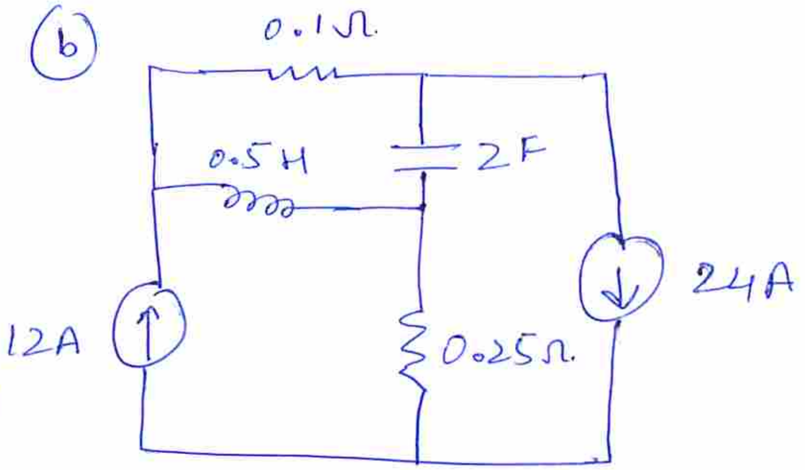
(b) Yes, since the current through the node 1 is same in both circuits and other nodes have the same current flowing through them.

(c) No, since equivalent networks need ~~to~~ not be topologically equivalent.

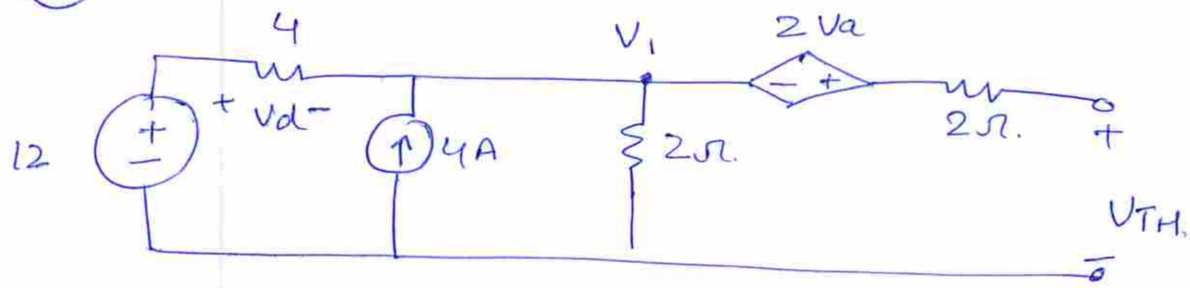
Q3 a



(b)



Q4



Applying Nodal Analysis

$$\frac{V_1 - 12}{4} + \frac{V_1}{2} = 4.$$

$$3V_1 - 12 = 16 \Rightarrow V_1 = \frac{28}{3} V$$

$$Va = 12 - V_1$$

$$= 12 - \frac{28}{3}$$

$$Va = \frac{8}{3} V$$

$$\boxed{V_{TH} = 2Va + V_1 = \frac{44}{3} V.}$$

Isc;

$$12 - V_1 = Va.$$

$$\frac{V_1 - 12}{4} + \frac{V_1}{4} + \frac{V_1 + 2Va}{2} = 4.$$

$$\frac{3V_1 - 12}{4} + \frac{V_1 + 24 - 2V_1}{2} = 4.$$

$$V_1 - 12 + 48 = 16$$

$$\boxed{V_1 = -20 V.}$$

$$\boxed{Va = 32 V.}$$

$$\boxed{Isc = \frac{V_1 + 2Va}{2} = 22 A}$$

$$R_{TH} = \frac{V_{TH}}{I_{SC}} = \frac{2}{3}\Omega$$

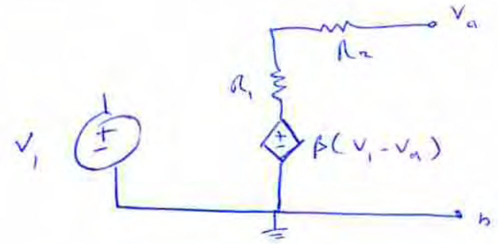
<u>For Max Power</u>

$$R_L = \frac{2}{3}$$

$$i_L = \frac{V_{TH}}{R_L + R_{TH}} = \frac{V_{TH}}{2R_{TH}}.$$

$$= \frac{I_{SC}}{2} = \boxed{11A}$$

Ans 5:



when open circuit, $V_a = V_{Th}$

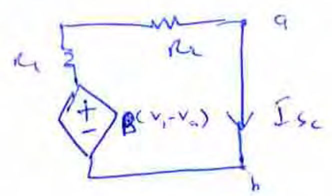
→ since no drop across R_1 & R_2, $V_a = \beta(V_1 - V_a)$

so

$$V_a = \left(\frac{\beta}{\beta+1}\right) V_1$$

$$\boxed{V_{Th} = \left(\frac{\beta}{\beta+1}\right) V_1}$$

For R_{th}, short it:



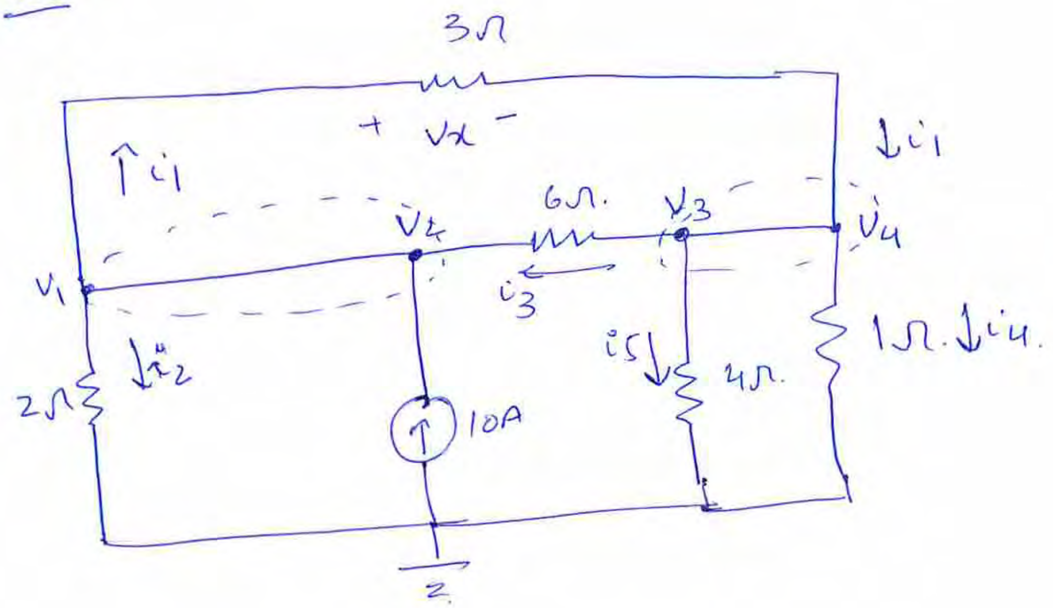
$$I_{sc} = \frac{\beta(V_1 - V_a)}{R_1 + R_2}$$

but $V_a = 0$, so $\boxed{I_{sc} = \frac{\beta V_1}{R_1 + R_2}}$

$$R_{th} = \frac{V_{Th}}{I_{sc}} = \frac{\left(\frac{\beta}{\beta+1}\right) V_1}{\frac{\beta V_1}{R_1 + R_2}} = \boxed{\frac{R_1 + R_2}{\beta+1}}$$

⟵)

Q6



Supernodes :

1 and 2.

3 and 4.

Supernode 1 and 2

$$i_3 + 10 = i_1 + i_2$$

$$\frac{V_3 - V_2}{6} + 10 = \frac{V_1 - V_4}{3} + \frac{V_1}{2}$$

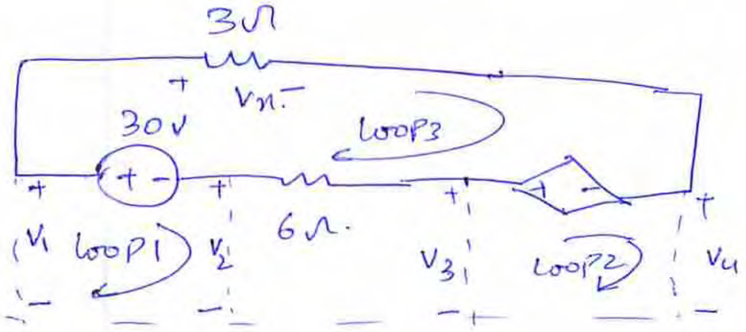
$$5V_1 + V_2 - V_3 - 2V_4 = 60 \quad \text{— (1)}$$

Supernode 3 and 4

$$i_1 = i_3 + i_4 + i_5$$

$$\frac{V_1 - V_4}{3} = \frac{V_3 - V_2}{6} + \frac{V_4}{1} + \frac{V_3}{4}$$

$$= 4V_1 + 2V_2 - 5V_3 - 16V_4 = 0 \quad \text{— (2)}$$



Loop1

$$-V_1 + 20 + V_2 = 0$$

$$V_1 - V_2 = 20. \quad - (3)$$

Loop2

$$-V_3 + 3V_x + V_4 = 0$$

$$V_x = V_1 - V_4$$

$$3V_1 - V_3 - 2V_4 = 0 \quad - (4)$$

Loop3

$$V_x - 3V_x + 6i_3 - 20 = 0$$

we know : $6i_3 = V_3 - V_2$ and $V_x = V_1 - V_4$.

hence :

$$-2V_1 - V_2 + V_3 + 2V_4 = 20 \quad - (5)$$

from (3), $V_2 = V_1 - 20$.

(3) $\longrightarrow$ (1) and (2)

$$6V_1 - V_3 = 2V_4 = 8 \quad - (6)$$

and

$$6V_1 - 5V_3 - 16V_4 = 40 \quad - (7)$$

(4), (6), (7)

$$\begin{bmatrix} 3 & -1 & -2 \\ 6 & -1 & -2 \\ 6 & -5 & -16 \end{bmatrix} \begin{bmatrix} V_1 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 80 \\ 40 \end{bmatrix}$$

Cramer's Rule:

$$\Delta = \begin{bmatrix} 3 & -1 & -2 \\ 6 & -1 & -2 \\ 6 & -5 & -16 \end{bmatrix} = -18 \qquad \Delta_1 = \begin{bmatrix} 0 & -1 & -2 \\ 80 & -1 & -2 \\ 40 & -5 & -16 \end{bmatrix} = -480$$

$$\Delta_3 = \begin{bmatrix} 3 & 0 & -2 \\ 6 & 80 & -2 \\ 6 & 40 & -16 \end{bmatrix} = -3120 \qquad \Delta_4 = \begin{bmatrix} 3 & -1 & 0 \\ 6 & -1 & 80 \\ 6 & -5 & 40 \end{bmatrix} = 840$$

$$\boxed{V_1 = \frac{\Delta_1}{\Delta} = \frac{-480}{-18} = 26.67\,V}$$

$$\boxed{V_3 = \frac{\Delta_3}{\Delta} = \frac{-3120}{-18} = 173.33\,V}$$

$$\boxed{V_4 = \frac{\Delta_4}{\Delta} = \frac{840}{-18} = -46.67\,V}$$

$$\boxed{V_2 = V_1 - 20 = 6.667\,V}$$

Ans 7:

Loop 1

$$40 i_1 - 10 i_2 = -120$$

$$\boxed{4i_1 - i_2 = -12} \quad — (1)$$

loop 2.

$$50 i_2 - 10 i_1 - 10 i_3 = 0$$

$$\boxed{-i_1 + 5i_2 - i_3 = 0} \quad — (2)$$

loop 3:

$$40 i_3 - 10 i_2 = 120$$

$$\boxed{4i_3 - i_2 = +12} \quad — (3)$$

so we get

$$\left[\begin{array}{ccc|c} 4 & -1 & 0 & -12 \\ -1 & 5 & -1 & 0 \\ 0 & -1 & 4 & 12 \end{array}\right] \quad R_2 = R_1 + 4R_2$$

$$\left[\begin{array}{ccc|c} 4 & -1 & 0 & -12 \\ 0 & 19 & -4 & -\cancel{1}2 \\ 0 & -1 & 4 & 12 \end{array}\right] \quad R_3 = R_2 + 19 R_3$$

$$\left[\begin{array}{ccc|c} 4 & -1 & 0 & -12 \\ 0 & 19 & -4 & -12 \\ 0 & 0 & 72 & 216 \end{array}\right]$$

$$72\, i_3 = 216$$

$$\boxed{i_3 = 3A}$$

$$19\, i_2 - 4(3) = -12$$

$$19\, i_2 = 0$$

$$\boxed{i_2 = 0}$$

$$4 i_1 = -12$$

$$\boxed{i_1 = -3A}$$

(—)

Ans 8:

Nodal Equation:

$$\frac{V_1 - 5}{5} + \frac{V_1}{10} + 1 \cdot \frac{dV_1}{dt} = 0$$

$$2V_1 - 10 + V_1 + 10\frac{dV_1}{dt} = 0$$

$$3V_1 + 10\frac{dV_1}{dt} = 10$$

$$\frac{dV_1}{dt} + 0.3V_1 = 1$$

Integration factor $e^{0.3t}$

$$V_1 e^{0.3t} = \int e^{0.3t}\,dt + K$$

$$V_1 = e^{-0.3t}\int e^{0.3t}\,dt + Ke^{-0.3t}$$

$$V_1 = e^{-0.3t} \cdot \frac{e^{0.3t}}{0.3} + Ke^{-0.3t}$$

$$\boxed{V_1 = \frac{10}{3} + Ke^{-0.3t}}$$

at $t = \infty$, $V_1 = \frac{10}{15} \times 5 = \frac{10}{3}$ → so it makes sense

at $t=0$, capacitor s.c. $V_1 = 0$ so $\boxed{K = -\frac{10}{3}}$

$$\boxed{V_1 = \frac{10}{3} - \frac{10}{3}e^{-0.3t}}$$