EE240 – Circuits I

Mid Examination (Fall 2018)

SOLUTIONS

November 6, 2018 06:00 pm–08:30 pm

Student ID ____________________________ Name _________________________________

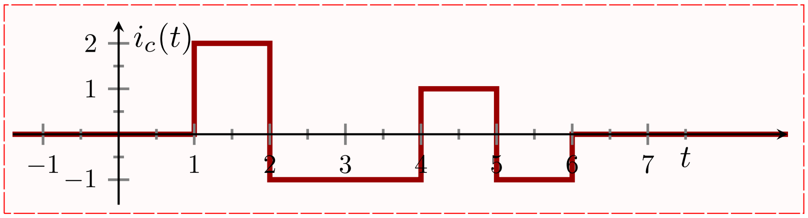
Signature ____________________

INSTRUCTIONS:

- Do not flip this page over until told to do so.
- **The exam needs to be solved on this book and not on blue book.**
- If you need the blue book for rough work, please ask the exam staff.
- The exam is closed book and notes. You are allowed to bring calculator and one A4 sheet with you with *hand-written* notes on both sides.
- Read all the questions before you start working on the exam.
- You cannot keep your mobile phone(s) with you (even on silent mode or switched off).

Part 1: Sources and I-V Characteristics of R, L, C

Problem 1. (**10 pts**) The current $i_c(t)$ through the capacitor of capacitance $\frac{1}{2}F$ is shown in Figure 1 below.



(a) (**1 pts**) Express $i_c(t)$ as piecewise function of time.

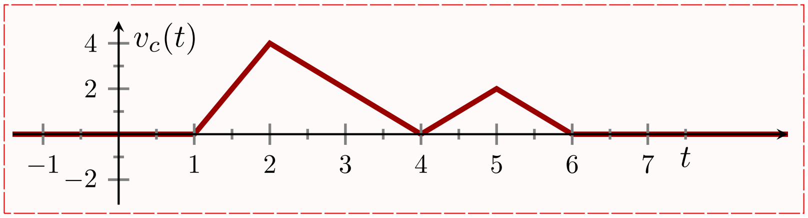
$$i_c(t) = \begin{cases} 0 & t < 1 \\ 2 & 1 \leq t < 2 \\ -1 & 2 \leq t < 4 \\ 1 & 4 \leq t < 5 \\ -1 & 5 \leq t < 6 \\ 0 & 6 \leq \end{cases}$$

(b) (**8 pts**) Assuming that the voltage is zero for times $t \leq -1$ seconds, determine the voltage across the capacitor and **plot** for $0 \leq t \leq 7$ seconds.

Let $v_c(t)$ be the voltage across capacitor.

$$v_c(t) = \frac{1}{C}\int_{-\infty}^{t} i_c(t)dt$$

$$v_c(t) = 2\begin{cases} 0 & t < 1 \\ 2t - 2 & 1 \leq t < 2 \\ -t + 4 & 2 \leq t < 4 \\ t - 4 & 4 \leq t < 5 \\ -t + 6 & 5 \leq t < 6 \\ 0 & 6 \leq \end{cases}$$

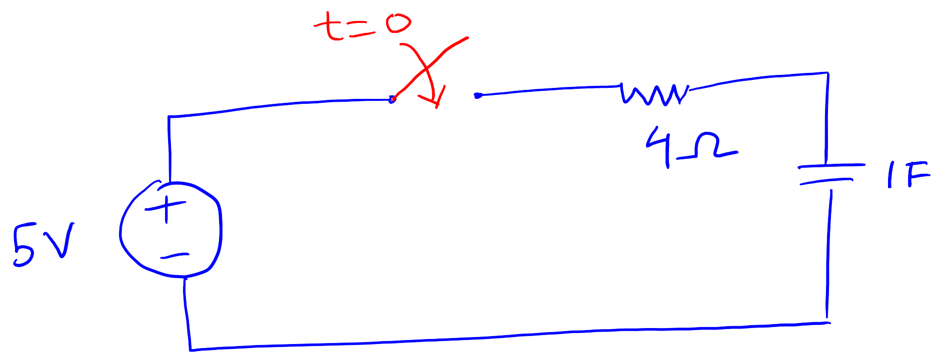


(c) (**1 pts**) Determine the energy stored in the capacitor at $t = 3.5$ seconds.

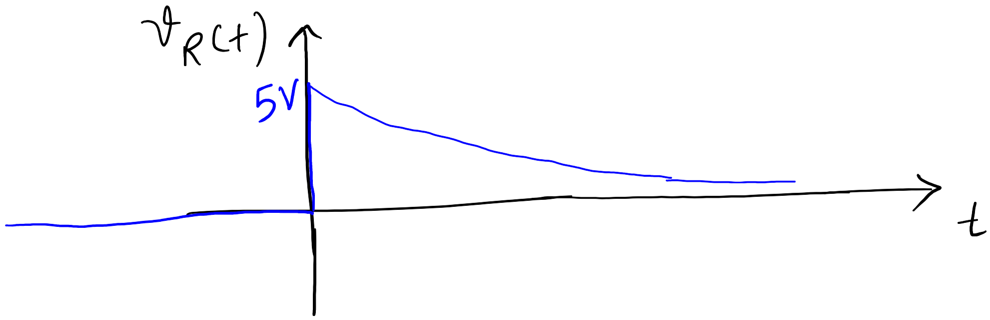
$w_c(t) = \frac{1}{2}C\big(v_c(t)\big)^2 \Rightarrow w_c(3.5) = \frac{1}{4}\big(v_c(3.5)\big)^2 = 0.25J$.

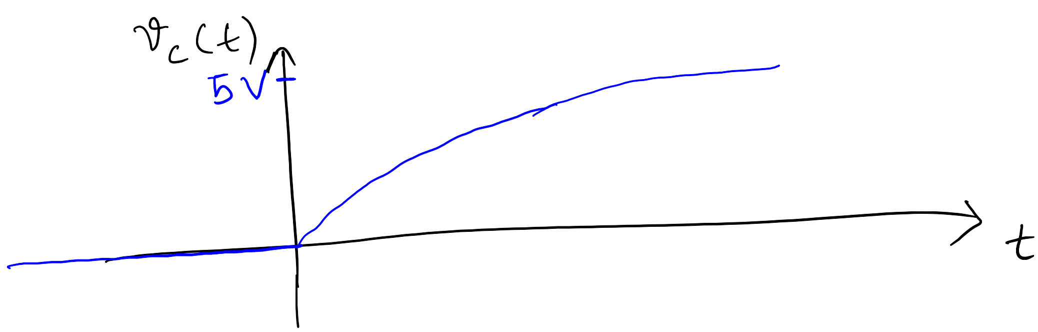
Problem 2. (**6 pts**) Consider a circuit where the DC voltage source of $5V$ is connected to a series combination of $4\ \Omega$ resistor and $1F$ capacitor through the switch. Assume that the switch is initially open and is closed at $t = 0$ and the capacitor is uncharged before the switch is closed, that is, the capacitor voltage $v_c(t) = 0$ for all $t < 0$.

(a) (**1 pts**) Draw the circuit and indicate the current $i(t)$ through the circuit and the voltages $v_R(t)$ and $v_C(t)$ across the resistor and the capacitor respectively.

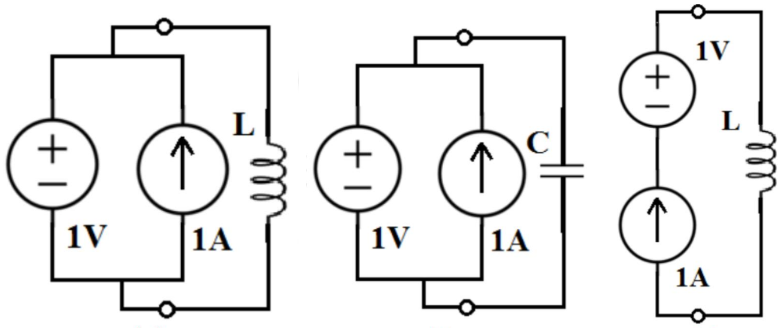


(b) (**5 pts**) Plot the waveforms (not to the scale) of the voltages $v_R(t)$ and $v_C(t)$.

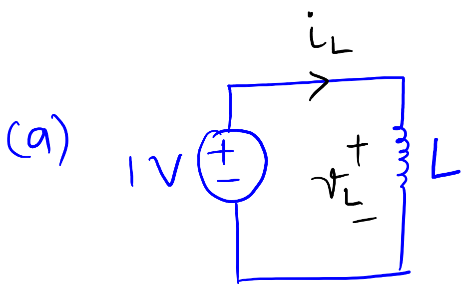


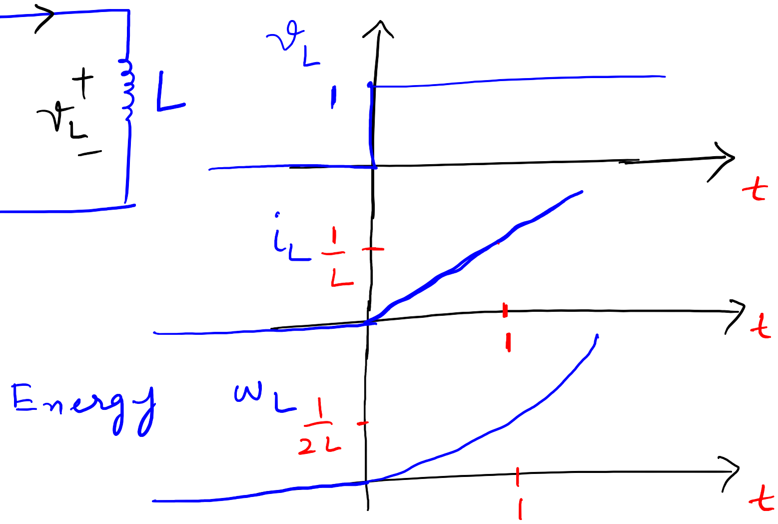


Problem 3. (**9 pts**) Consider the following four circuits. Assuming that the sources are switched on at $t = 0$ and the elements do not carry any current or voltage before the sources are turned-on, draw the voltage, current and energy waveform for $0 \le t \le 1$ seconds for each circuit (element). Total of 9 waveforms.



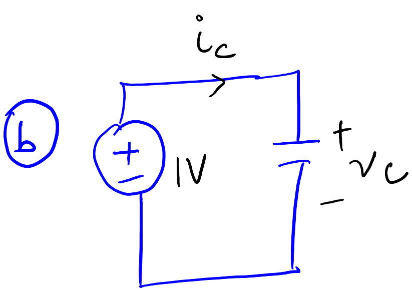
(a) (b) (c)

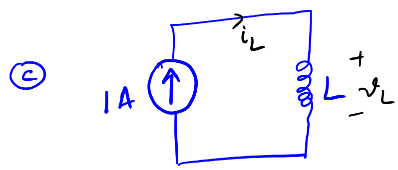


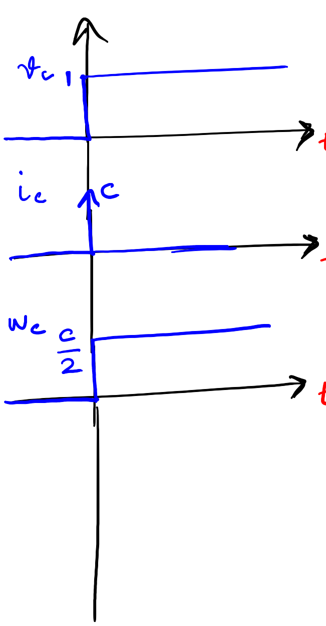


$$i_L = \frac{1}{L}\int_{-\infty}^{t} v\,dt = \begin{cases} \frac{t}{L} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$w_L = \frac{1}{2} L i^2 = \begin{cases} \frac{1}{2}\frac{t^2}{L} & t \geq 0 \\ 0 & t < 0 \end{cases}$$





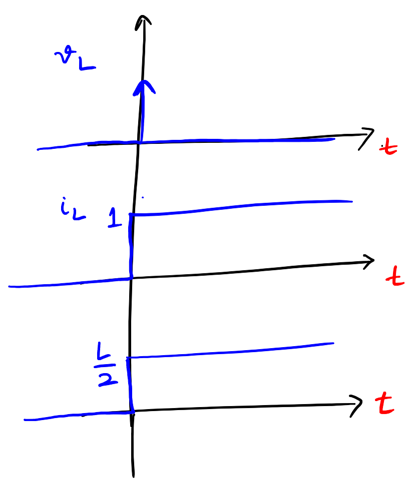


$i_c = C\frac{dv_c}{dt}$

impulse

$i_c = C\,\delta(t)$

$w_c = \frac{1}{2} C v_c^2$



$v_L = L\frac{di_L}{dt}$

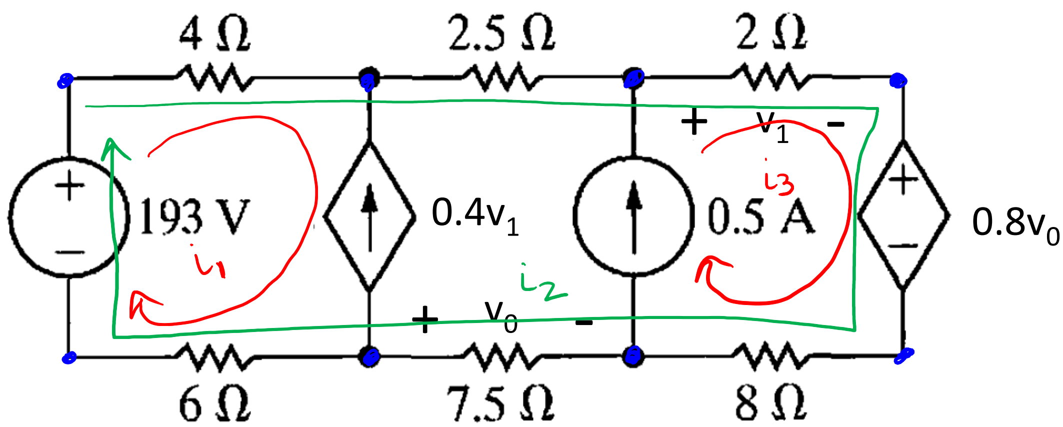
$= L\,\delta(t)$

$w_L = \frac{1}{2} L i_L^2$

Part 2: Network Topology, Network Equations and Equivalent Circuits

Problem 4. (**20 pts**)

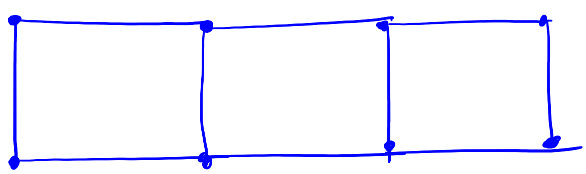
Consider the circuit given below.



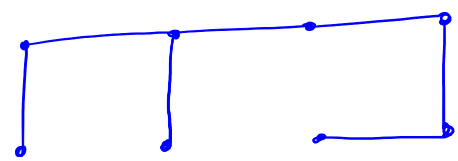
Nodes : Blue dots

(a) (**5 pts**) Draw the graph and one tree of the circuit. Determine the number of nodes and number of branches in a circuit.

Graph



Tree (one possibility)



no. of nodes = n = 8

no. of branches = b = 10

(b) (**2 pts**) Determine the number of network equations required for carrying out i) nodal analysis and ii) loop analysis.

Nodal Analysis : No. of equations = $n-1 = 7$

Loop Analysis : No. of equations = $b-(n-1) = 3$

(c) (**9 pts**) Carry out the loop analysis, that is, identify and determine the loop currents.

Controlled Sources

$v_o = -7.5 i_2$

$v_1 = (i_2 + i_3)(2)$

$v_1 = 2 i_2 + 1$

Currents indicated on circuit

$i_1 = -0.4 v_1 = -0.8 i_2 - 0.4$

$i_3 = 0.5A$

Green loop : $10 i_1 + 30 i_2 + 10 i_3 + 0.8 v_o = 193$

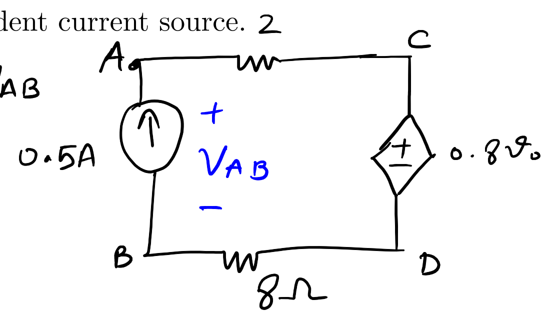
$10(-0.8 i_2 - 0.4) + 30 i_2 + 5 + (0.8)(-7.5 i_2) = 193$

$16 i_2 = 192 \Rightarrow i_2 = 12A$

$i_1 = -0.8 i_2 - 0.4$

$i_1 = -10A$

(d) (**4 pts**) Determine the power delivered by the independent current source.



We find the voltage across current source as V_{AB}

$v_{AB} = v_{AC} + v_{CD} + v_{DA}$

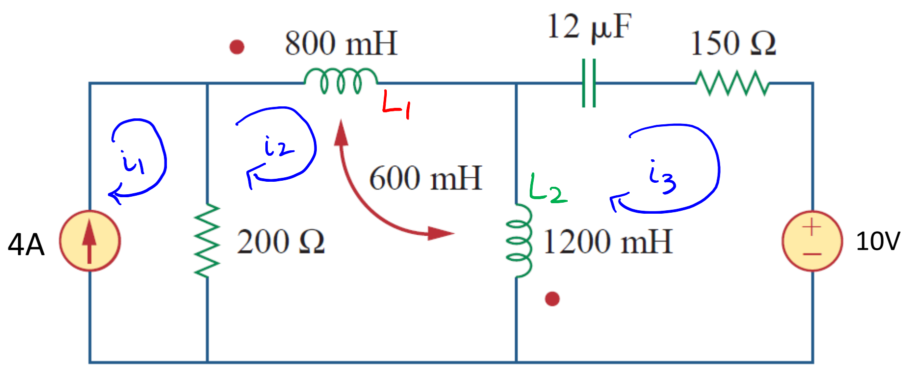
$= 2(i_2 + i_3) + (0.8)(-7.5 i_2) + 8(i_2 + i_3)$

$= 25 - 72 + 100 = 53V$

Power $= V_{AB}\,(0.5) = 26.5$ Watts

↑ i_3

Problem 5. (**10 pts**) Consider the circuit given below.



(a) (**5 pts**) Formulate the network equations using loop analysis.

Loop 1:

$$\boxed{i_1 = 4A}$$

Loop 2:

$$0.8\frac{di_2}{dt} + 1.2\frac{d}{dt}(i_2 - i_3) + 200(i_2 - i_1)$$

$$+ 0.6\frac{d}{dt}(i_3 - i_2) - 0.6\frac{di_2}{dt}$$

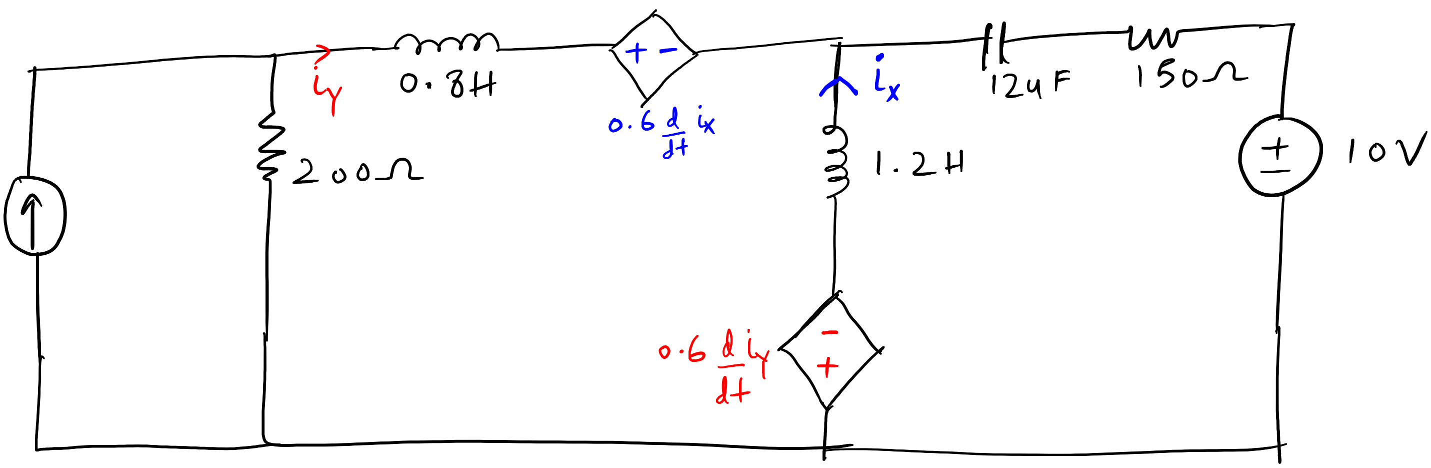
Effect on L_1 due to current $(i_3 - i_2)$ entering dot of L_2.

Effect on L_2 due to i_2 current entering dot of L_1

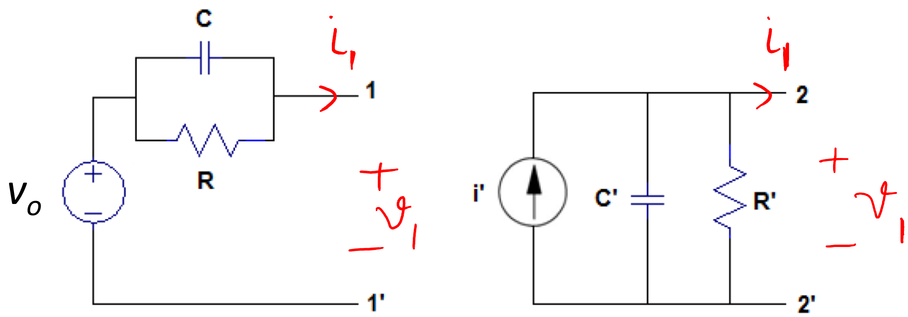
Loop 3:

$$\frac{1}{12u}\int i_3\,dt + 150 i_3 + 1.2\frac{d(i_3 - i_2)}{dt} + 0.6\frac{di_2}{dt} = -10$$

(b) (**4 pts**) Find an equivalent circuit where each coupled inductor is replaced with an uncoupled inductor and a controlled source. Draw the equivalent circuit.



Problem 6. (**6 pts**) Find the values of i', R' and C' in terms of R, C and v_o such that the networks shown below are equivalent at terminals 11 and 22.



$$i_1 = \frac{v_0 - v_1}{R} + C\frac{d}{dt}(v_0 - v_1)$$

$$i_1 = \left(\frac{1}{R} + C\frac{d}{dt}\right)v_0 - \frac{v_1}{R}$$

$$- C\frac{d}{dt}v_1 \quad — (1)$$

$$i_1 = i' - \frac{v_1}{R'} - C'\frac{dv_1}{dt} \quad — (2)$$

By comparing (1) and (2)

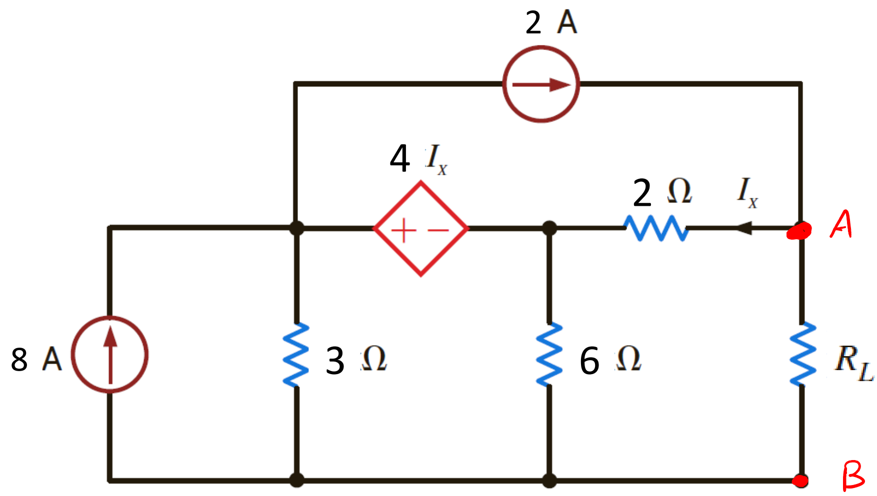
$$i' = \frac{v_0}{R} + C\frac{dv_0}{dt}$$

$$R' = R$$

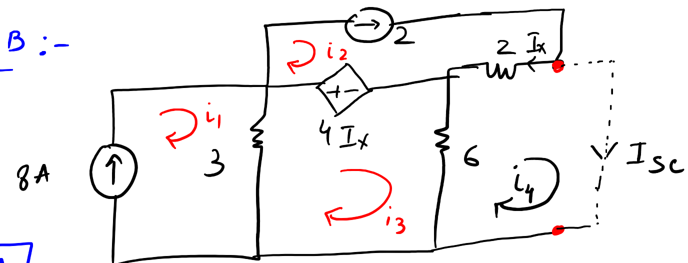
$$C' = C$$

Part 3: Additional Analysis Techniques

Problem 7. (**12 pts**) For the circuit given below, determine the value of R_L for maximum power transfer to R_L using Thevenin's Theorem.



Find Thevenin Equivalent across AB :-



V_{th} : $i_1 = 8A$, $i_2 = 2A = i_x$

Loop 3: $9i_3 - 3i_1 + 4i_x = 0$

$\Rightarrow 9i_3 - 24 + 8 = 0 \Rightarrow \boxed{i_3 = \frac{16}{9} A}$

$V_{th} = V_{AB} = 2i_2 + 6i_3 = 4 + \frac{32}{3} = \boxed{\frac{44}{3} V}$

I_{sc} : $i_1 = 8A$, $i_2 = 2A$, $i_x = i_2 - i_4$, $i_4 = I_{sc}$

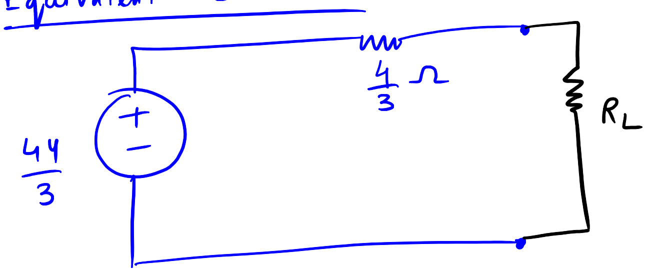
Loop 3: $9i_3 - 3i_1 + 4i_x - 6i_4 = 0 \Rightarrow 9i_3 - 24 + 8 - 10i_4 = 0 \Rightarrow \boxed{9i_3 - 10i_4 = 16}$

Loop 4: $2(i_4 - i_2) + 6(i_4 - i_3) = 0 \Rightarrow 8i_4 - 6i_3 = 4 \Rightarrow \boxed{4i_4 - 3i_3 = 2}$

$\boxed{i_4 = 11A = I_{sc}}$

$R_{th} = \frac{V_{th}}{I_{sc}} = \frac{4}{3}\,\Omega$

Equivalent Circuit

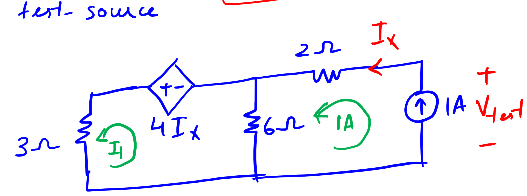


For max. power transfer:

$R_L = R_{th} = \frac{4}{3}\,\Omega$

Alternatively, we can use test source approach method.

* Switch off sources
* Apply 1A current source



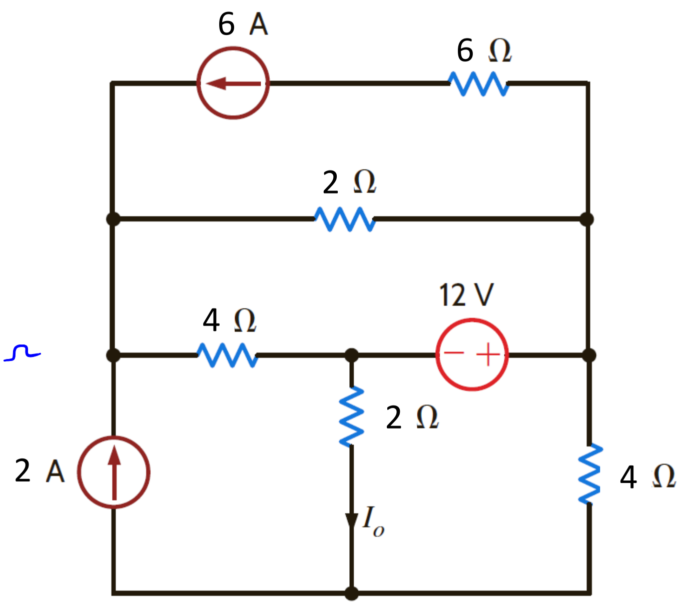
- $I_x = 1A$
- Loop 1: $9i_1 - 4I_x - 6(1) = 0$

  $\Rightarrow \boxed{i_1 = \frac{10}{9} A}$

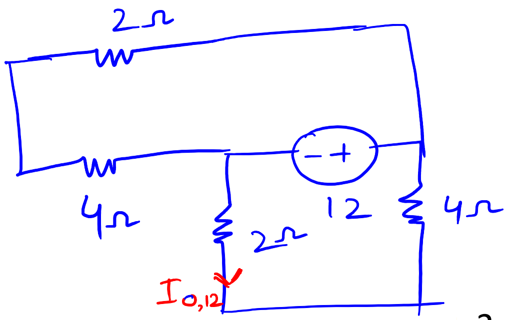
- $V_{test} = (2)(1) + 6\left(1 - \frac{10}{9}\right) = 2 - \frac{2}{3} = \frac{4}{3} V$

  $\Rightarrow R_{th} = \frac{V_{test}}{1} = \boxed{\frac{4}{3}\,\Omega}$

Problem 8. (**9 pts**) Determine I_o using the superposition theorem (principle) for the circuit given below.

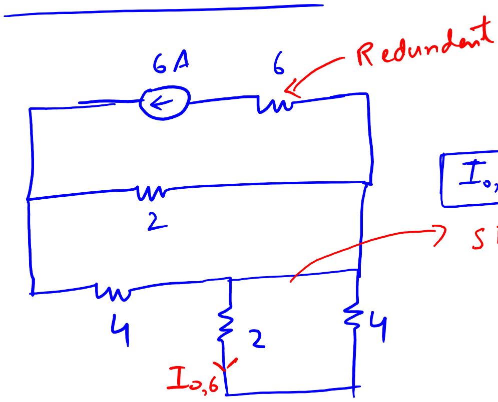


Only 12V ON



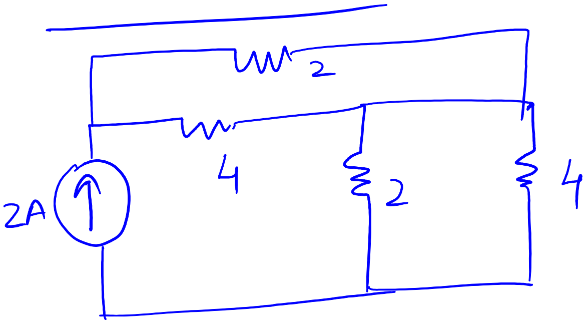
$I_{o,12} = \frac{-12}{6} = -2A$

ONLY 6A ON :

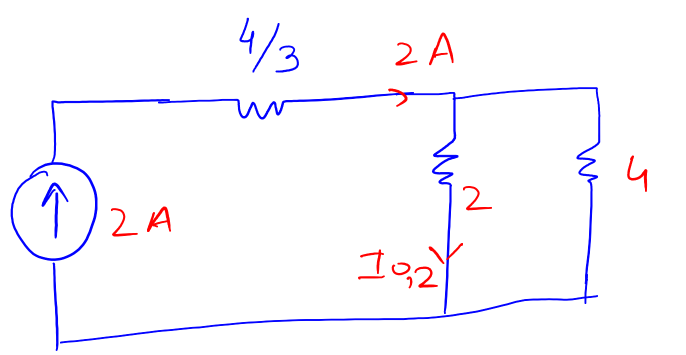


$I_{o,6} = 0$

ONLY 2A ON



$\equiv$

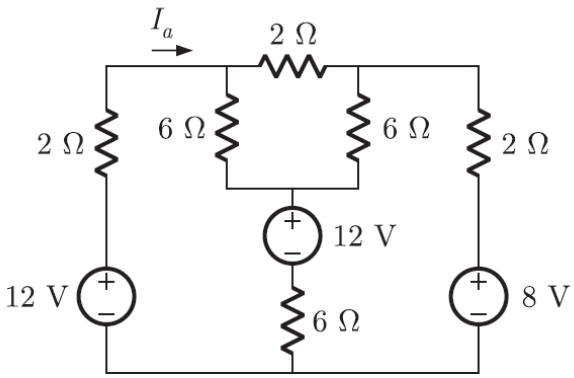


$\Rightarrow \quad I_o = \frac{4}{6} \times 2 = \frac{4}{3} A$

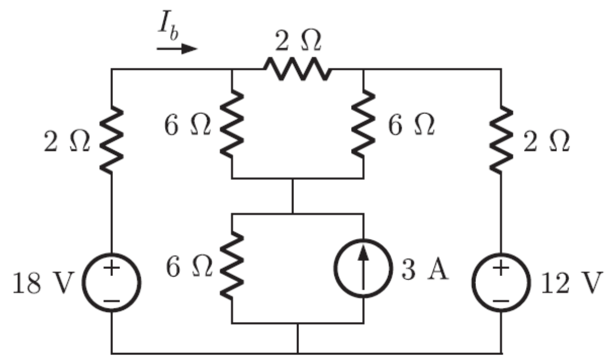
$I_o = I_{o,12} + I_{o,6} + I_{o,2}$

$= -2 + \frac{4}{3} = -\frac{2}{3} A$

Problem 9. (**4 pts**) Given the following two circuits, determine the relation between I_a and I_b. You must provide justification to support your answer.
(Hint: Use the concept of linearity.)



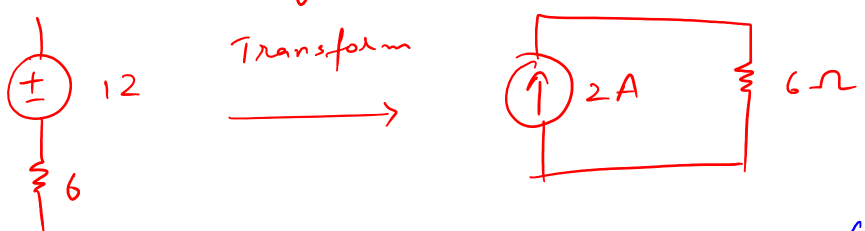
(a)



(b)

This was just a piece of cake!

In (a)

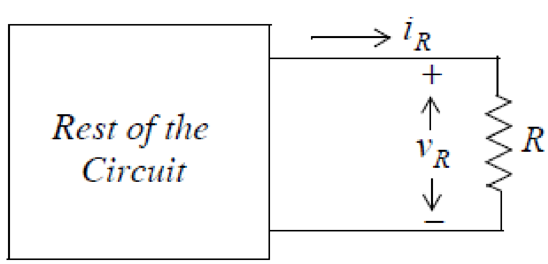


Since the sources in (b) have values (1.5) times the sources in (a); linearity implies $I_b = 1.5 I_a$
(Scaling + Superposition)

Problem 10. (**5 pts**) Consider the network shown below, where the rest of the circuit is pure resistive in nature. The voltage $V_R = 6V$ for $R = 4\Omega$ and the current $i_R = 2.4A$ for $R = 0\Omega$. Determine the voltage V_R in volts when $R = \infty$.

$R = \infty$

$V_R = V_{th}$



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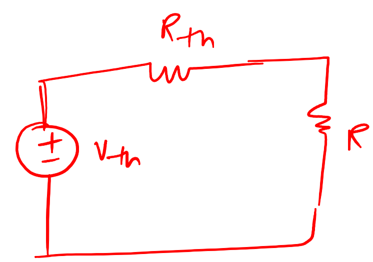

$R = 4$ $\quad V_R = \dfrac{4}{R_{th}+4} V_{th} = 6 \Rightarrow 6R_{th} + 24 = 4V_{th}$

$R = 0$ $\quad i_R = \dfrac{V_{th}}{R_{th}} = 2.4 \Rightarrow V_{th} = 2.4 R_{th} \Rightarrow R_{th} = \dfrac{V_{th}}{2.4}$

$\Rightarrow 2.5 V_{th} + 24 = 4 V_{th} \Rightarrow V_{th} = 16V$