LUMS
A Not-for-Profit University

Department of Electrical Engineering
School of Science and Engineering

EE240 Circuits I - Fall 2020

ASSIGNMENT 1 – SOLUTIONS

Q 1. (a) To create a voltage divider, you need two resisters, R_1 and R_2. Connect the lights in parallel to R_2 as specified by the question. Start by finding the equivalent resistance of the lights R_L and R_2.

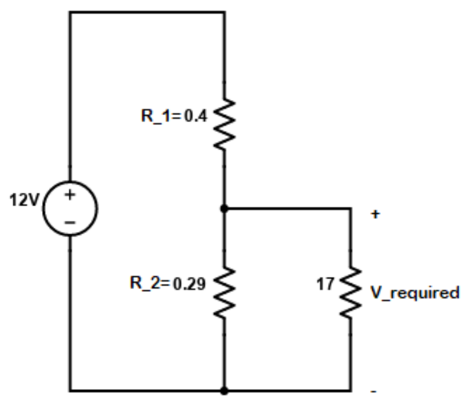
$$R_{eq} = \frac{17 \cdot R_2}{17 + R_2}$$

The required voltage is 5V, so using the relation,

$$V_{required} = \frac{R_{eq}}{R_1 + R_{eq}} \cdot V_{total}$$
$$5 = \frac{\frac{17R_2}{17+R_2}}{\frac{17R_2}{17+R_2} + R_1} \cdot 12$$
$$\frac{5}{12} = \frac{17R_2}{17R_2 + R_2R_1 + 17R_1}$$

Notice that there can be multiple solutions to this equation,

$$5 = 17R_2$$
$$R_2 = 0.29\Omega$$
$$12 = 17(0.29) + (0.29)(R_1) + 17R_1$$
$$R1 = 0.40\Omega$$



(b)

$$P = \frac{V^2}{R} = \frac{5^2}{17k} = \frac{25}{17k} = 1.47mW$$

(c)

$$V_{new} = \frac{0.29}{0.29 + 0.40} \cdot 12$$
$$V_{new} = 5.04V$$

Removing the lights parallel to R_2 increases the R_{eq} that results in a greater voltage across R_2, here the change in voltage was 0.04V.

(d)

$$I_{total} = 13A$$
$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{3} + \frac{1}{5} + \frac{1}{11}$$
$$R_{total} = 1.60\Omega$$
$$i_1 = \frac{1.60}{3} \cdot 13 = 6.9A$$
$$i_2 = \frac{1.60}{5} \cdot 13 = 4.2A$$
$$i_3 = \frac{1.60}{11} \cdot 13 = 1.9A$$

Q 2. (a) Use dimensional analysis to find what each expression represents,

$$\frac{dw}{dq} \to \frac{joules}{charge} \to V(t)$$
$$\frac{dq}{dt} \to \frac{charge}{time} \to I(t)$$
$$p(t) = V(t) \cdot I(t)$$
$$p(t) = 0.25 \cdot e^{3200t} - 0.5 \cdot e^{-2000t} + 0.25 \cdot e^{-800t}\ kW$$

(b)

$$p(625\mu s) = 42.2W$$

(c)

$$w(t) = \int_0^\infty p(t)dt$$
$$w(t) = \int_0^\infty (0.25e^{-3200t} - 0.5e^{-2000t} + 0.25e^{-800t})dt$$
$$= \frac{0.25 \cdot e^{-3200t}}{-3200}\Big|_0^\infty - \frac{0.5 \cdot e^{-2000t}}{-2000}\Big|_0^\infty + \frac{0.5 \cdot e^{-800t}}{-800}\Big|_0^\infty$$
$$= \frac{0.25}{3200} - \frac{0.5}{2000} + \frac{0.25}{800}$$
$$= 140mJ$$

Q 3. (a) (i)

$$\frac{1}{2} \cdot C \cdot V^2 = 32$$
$$C = 7.11 \cdot 10^{-4} = 710\mu F$$

(ii)

$$C = \frac{\epsilon \cdot A}{d}$$

By making A the subject,

$$A = \frac{(710 \cdot 10^{-6}) \cdot (0.5 \cdot 10^{-2})}{\epsilon}$$

(iii) Lets call the new capacitance due to halving the distance as,

$$C_{d/2} = \frac{\epsilon \cdot A}{0.25 \cdot 10^{-2}}$$

Using area from the previous part,

$$C_{d/2} = 1.42 \cdot 10^{-3} F$$

(the capacitance is doubled)
Lets call the capacitace due to half the area as,

$$C_{A/2} = 355\mu F$$

(the capacitance is halved)

(iv) For half the distance:

$$q_{discharge} = (C_{d/2} \cdot V) - (C \cdot V)$$

$$q_{discharge} = (1.42 \cdot 10^{-3})(300) - (710 \cdot 10^{-6})(300) = 0.213 Coulombs$$

$$Current discharged = 0.213/t$$

For half the area:

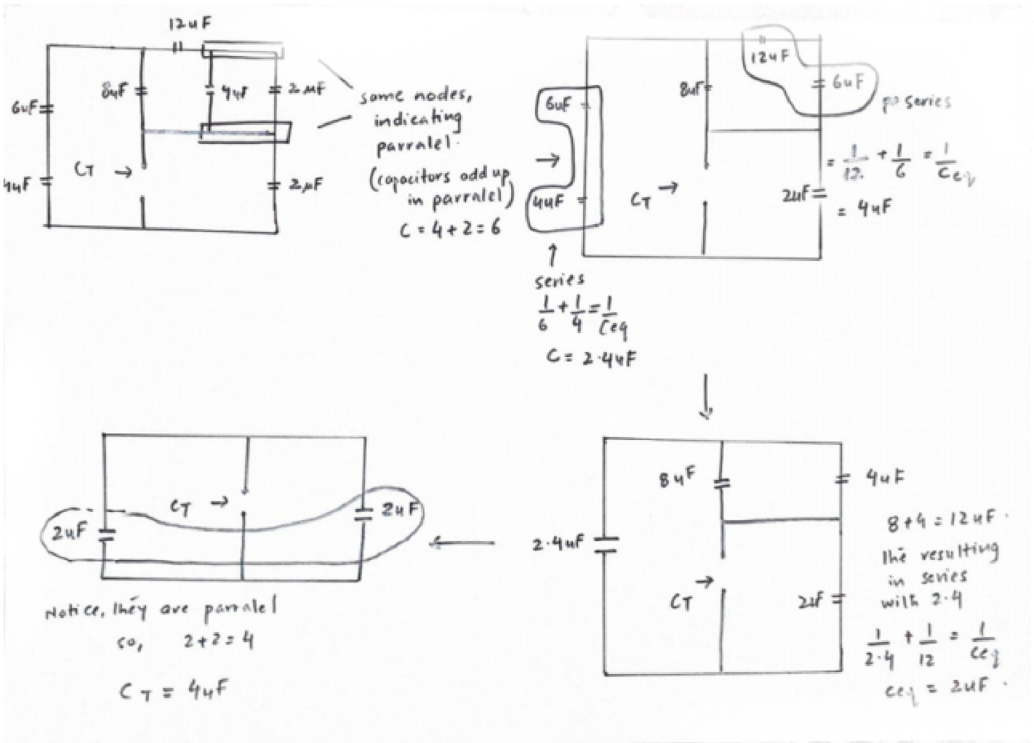
$$q_{discharge} = (C \cdot V) - (C_{A/2} \cdot V)$$

$$q_{discharge} = (710 \cdot 10^{-3})(300) - (1.42 \cdot 10^-3)(300) = 0.1065 Coulombs$$

$$Current discharged = 0.1065/t$$

(b)

$$C_T = 4\mu F$$



Q 4. (a) Using passive sign convention,for car A, the power is computed as follows

$$P = vi = 10 \cdot 12 = 120W$$

Since the power is positive,car A was initially dead.

(b)

$$\text{total time} = 60 \cdot 6 = 360sec$$

$$w(t) = \int_{t=0}^{360} 120dt = 120(360 - 0) = 43200J = 43.2kJ$$

(c) (i)

$$v = L\frac{\mathrm{d}i}{\mathrm{d}t}$$

for $t < 0$:

$$v = (25 \cdot 10^{-3}) \cdot (0) = 0$$

for $t > 0$:

$$v = (25 \cdot 10^{-3})\frac{\mathrm{d}i}{\mathrm{d}t}$$

where,

$$\frac{\mathrm{d}i}{\mathrm{d}t} = 10e^{-t}$$

So,

$$v = (25 \cdot 10^{-2}) \cdot e^{-t} = 0.25 \cdot e^{-t}mV$$

Thus,

$$v(t) = \begin{cases} 0V, & t < 0 \\ 0.25 \cdot e^{-t}V, & t > 0 \end{cases}$$

(ii)

$$w(t) = \frac{1}{2} \cdot L \cdot i(t)^2$$

For $t < 0$,

$$w(t) = \frac{1}{2} \cdot L \cdot (0)^2$$
$$w(t) = 0$$

For $t > 0$,

$$w(t) = \frac{1}{2} \cdot (25 \cdot 10^{-3}) \cdot (10 \cdot 10^{-3}(1 - e^{-t}))^2$$
$$w(t) = (1.25 \cdot 10^{-3}) \cdot (1 - e^{-t})^2$$
$$w(t) = 1.25(1 - e^{-t})^2 \mu J$$

Thus,

$$w(t) = \begin{cases} 0, & t < 0 \\ 1.25(1 - e^{-t})^2 \mu J, & t > 0 \end{cases}$$

(d) (i) Step 1: Write $i(t)$ as a piece-wise function using the waveform

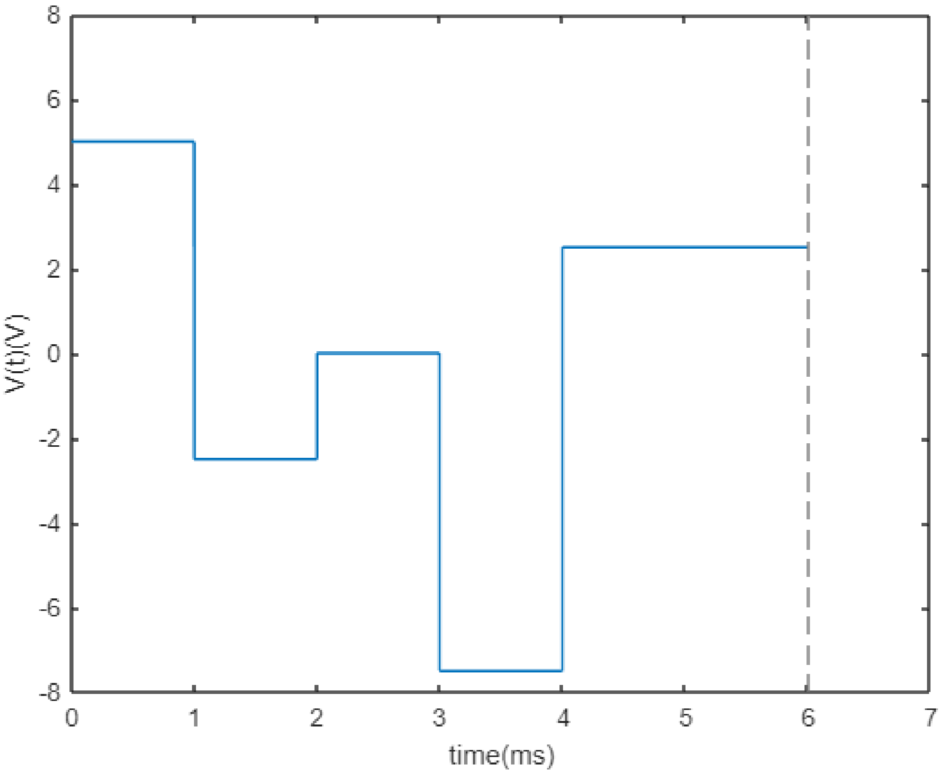
$$i(t) = \begin{cases} 10tA, & 0 < t \leq 1\, ms \\ -5t + 0.015A, & 1 < t \leq 2\, ms \\ 5A, & 2 < t \leq 3\, ms \\ -15t + 0.05A & 3 < t \leq 4\, ms \\ 5t - 0.03A & 4 < t \leq 6\, ms \end{cases}$$

Step 2: Now use the relation for inductor to find the piece-wise function for $v(t)$

$$v = L \cdot \frac{\mathrm{d}i}{\mathrm{d}t}$$

$$v(t) = \begin{cases} 0.5 \cdot 10 = 5V, & 0 < t \leq 1\, ms \\ 0.5 \cdot 0.5 = -2.5V, & 1 < t \leq 2\, ms \\ 0V, & 2 < t \leq 3\, ms \\ 0.5 \cdot -15 = -7.5V, & 3 < t \leq 4\, ms \\ 0.5 \cdot 5 = 2.5V, & 4 < t \leq 6\, ms \end{cases}$$

Step 3: Plot the wavefunction for $v(t)$



Matlab code:

```
syms t
y = piecewise(0 <= t <= 1, 5, 1 < t <= 2, -2.5,2 < t <= 3, 0, 3 < t <= 4, -7.5,
4 < t <= 6, 2.5)
fplot(y)
axis([0 7 -8 8])
xlabel('time(ms)')
ylabel('V(t)(V)')
```

(ii)

$$w(t) = \frac{1}{2} \cdot L \cdot i^2$$

$t = 1.7 \, lies \, in \, the \, range \, 1 < t < 2 :$

$$\text{i(t)} = -5t + 0.015A$$

$$w(1.7ms) = \frac{1}{2} \cdot 0.5 \cdot (-5(1.7 \cdot 10^{-3}) + 0.015)^2$$

$$= 1.056 \cdot 10^{-5} J$$

$t = 4.2 \, lies \, in \, the \, range \, 4 < t < 6 :$

$$i(t) = 5 \cdot t - 0.03$$

$$w(4.2ms) = \frac{1}{2} \cdot 0.5 \cdot (5(4.2 \cdot 10^{-3}) - 0.03)^2$$

$$= 2.025 \cdot 10^{-5} J$$

(iii)

$$p(t) = v(t) \cdot i(t)$$

$$t = 1.2 \, lies \, in \, the \, range \, 1 < t < 2:$$

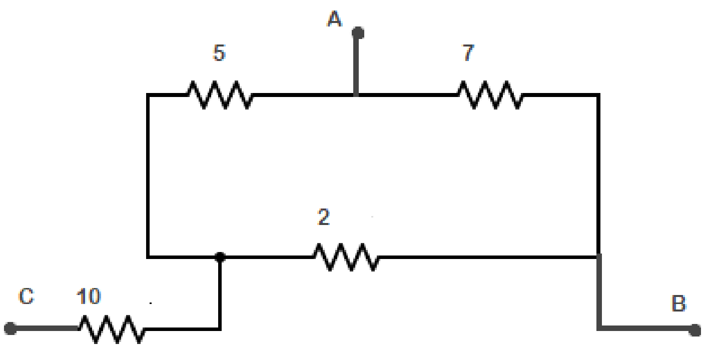
$$\text{i(t)} = -5 \cdot t + 0.015A$$
$$v(t) = -2.5V$$
$$p(t) = -2.5 \cdot (0.015 - 5(1.2)) = -0.0225W$$

$$t = 2.8 \, lies \, in \, the \, range \, 2 < t < 3:$$

$$i(t) = 5$$
$$v(t) = 0$$
$$p(t) = 0$$

Notice the sign of power at $t = 1.2$ and also that power absorbed is zero at $t = 2.8$, thus the model is an over-simplification.

Q 5. (a) The best approach to this question will be to play around with the resistors under different combinations.



(b) The circuit is just a combination of parallel and series resistors

$$R_{eq} = 10.3\Omega$$

Q 6. (a) (i) Using the relation,

$$L = \mu \cdot \frac{N^2 \cdot A}{l}$$
$$L = \mu \cdot \frac{N^2 \pi r^2}{l}$$

By making N the subject,

$$N = 149 \, turns$$

This is only possible for $50\mu m$ and $100\mu m$ wires.

(ii) Resistance of the inductor increases.

(iii) Using the relation in (i),

$$L = 27.5\mu H$$

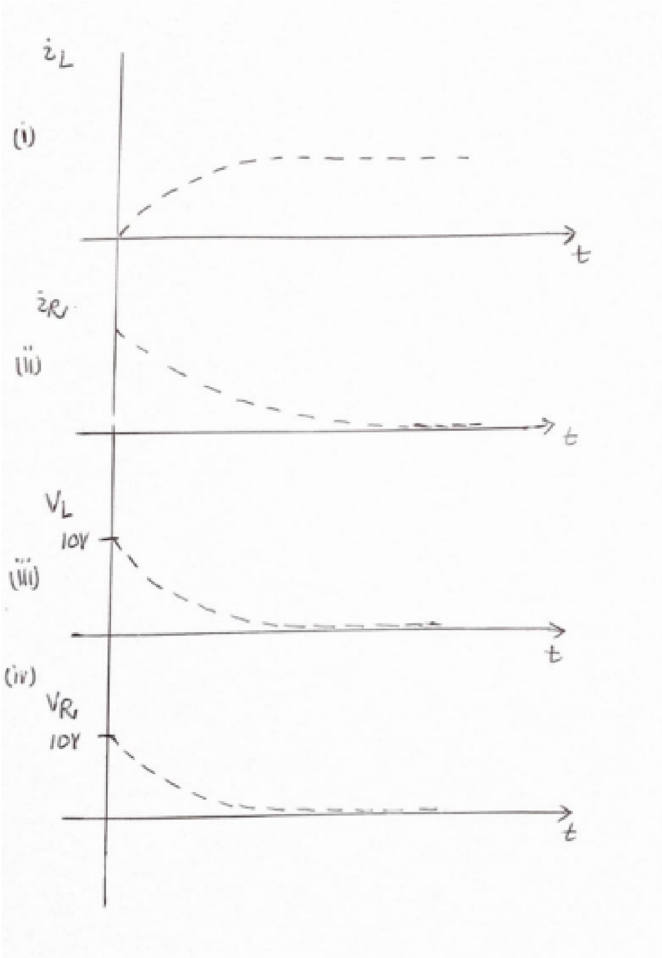
(b) For inductors, the total magnetic flux,

$$\Psi = N \cdot \Phi \quad (where \, \Phi \, is \, the \, flux, \; N \, is \, the \, number \, of \, turns)$$
$$V = \frac{\mathrm{d}\Psi}{\mathrm{d}t} = N \cdot \frac{\mathrm{d}\Phi}{\mathrm{d}t}$$
$$\frac{\mathrm{d}\Phi}{\mathrm{d}t} = \frac{V}{N} = \frac{24}{1250} = 19.2Wb/s$$

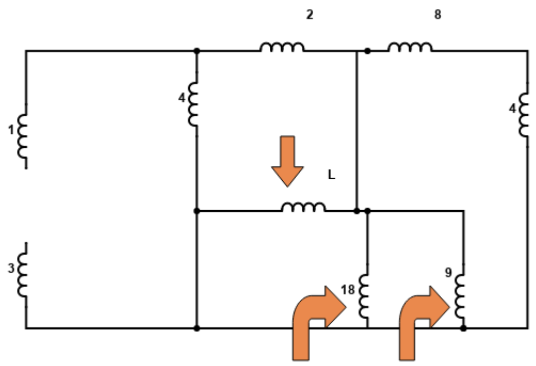
(c) Inductor does not allow instantaneous change in current so in the beginning most of the current passes through the resistor. Inductor gradually allows more current to pass through. When 1A current passes through 10ohm resistor mainting 10V at its terminals. Later when all current is passing through inductor, there isn't any change in current so according to

$$V = L \cdot \frac{\mathrm{d}i}{\mathrm{d}t}$$

voltage approaches 0.



Q 7. (a) Redraw the circuit into a more familiar form.



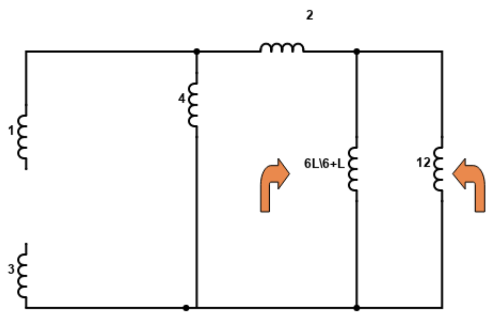
The arrowed inductors are in parallel since they share common nodes,

$$\frac{1}{18} + \frac{1}{9} = \frac{1}{L_1}$$
$$L_1 = 6H$$

Now, combine this in parallel with L,

$$\frac{1}{L_1} + \frac{1}{6} = \frac{1}{L_2}$$
$$L_2 = \frac{6L}{6+L}$$

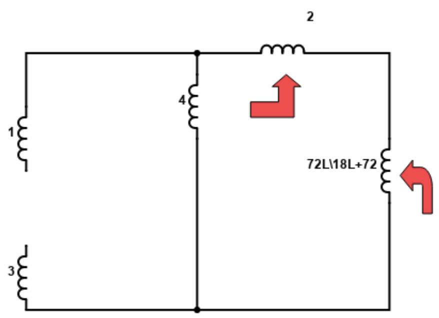
The resulting circuit looks like this (notice that the inductors $4H$ and $8H$ are in series):



The two arrowed inductors are in parallel,

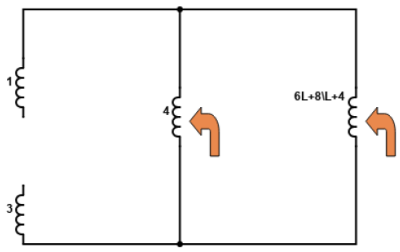
$$\frac{1}{12} + \frac{6L}{6+L} = \frac{1}{L_3}$$
$$\frac{1}{L_3} = \frac{12(6+L)+6L}{72L}$$
$$L_3 = \frac{72L}{18L+72}$$

The resulting circuit looks like this:



The arrowed inductors are in series

$$L_4 = L_3 + 2 = \frac{6L+8}{L+4}$$



The arrowed inductors are parallel to eachother,

$$\frac{1}{4} + \frac{L+4}{6L+8} = \frac{1}{L_5}$$
$$\frac{1}{L_5} = \frac{10L+32}{24L+32}$$
$$L_5 = \frac{24L+32}{10L+24}$$

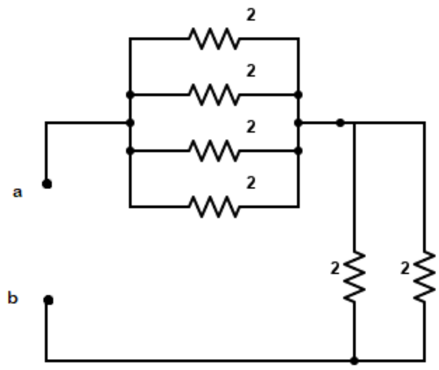
Notice, that L_5 is in series with the remaining two inductors and the resultant inductance equals 5.5H.

$$L_5 + 4 = 5.5$$
$$L_5 = 1.5$$

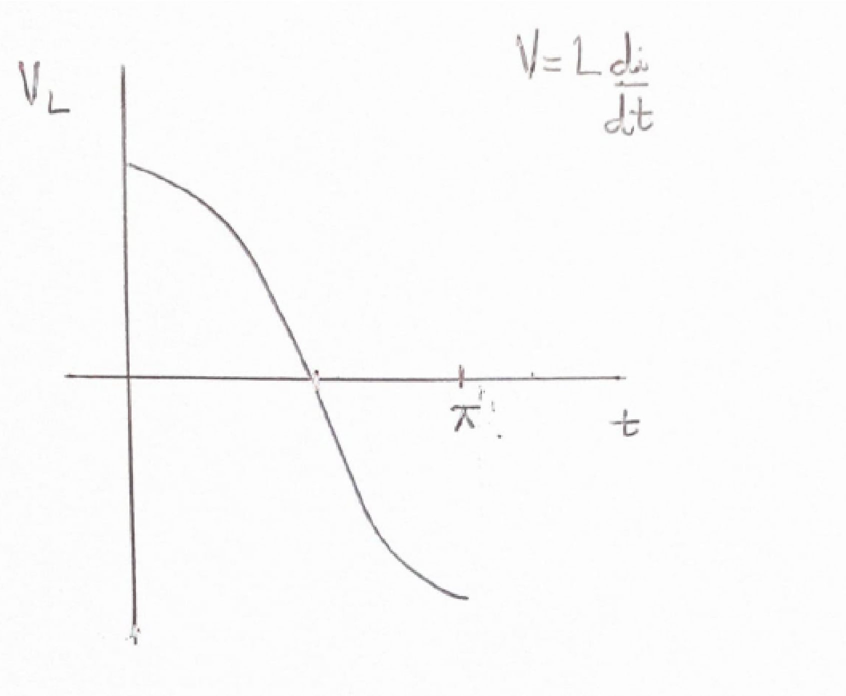
Using the expessionfor L_5,

$$L = \frac{4}{9}H$$

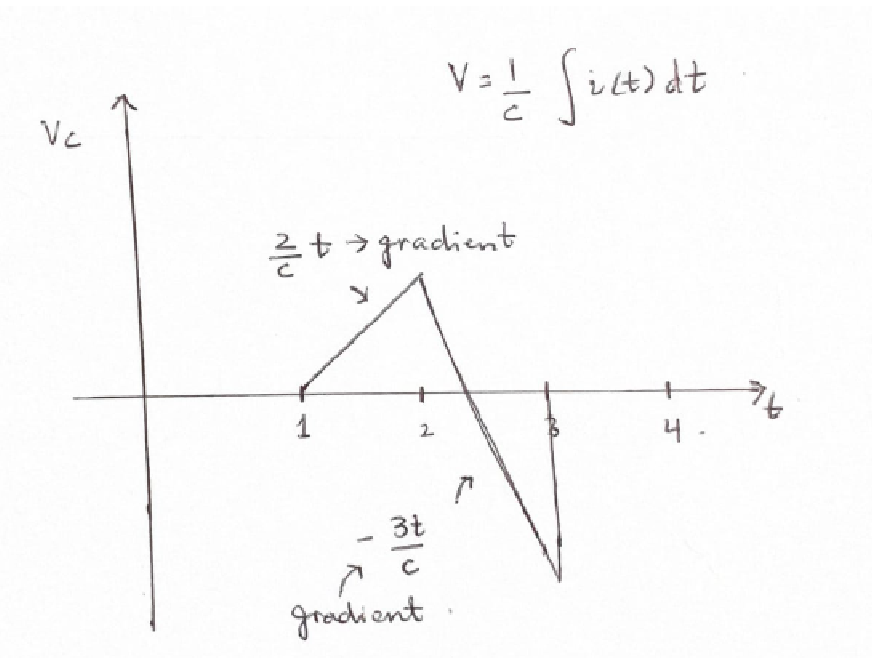
(b) Recall the effect of adding inductors in series and in parallel.



Q 8. (a) For the inductor,(sine turns into cosine)



For the capacitor,



(b) (i)

$$P = VI$$
$$P = 0,\ t < 1$$

Using the graph for V_c in (a) for $1 < t \leq 2$,

$$I(t) = 2$$
$$V(t) = \frac{2t}{C} - \frac{2}{C}$$

(C is the capacitance)

$$
\begin{aligned}
P &= I \cdot (\frac{2t}{C} - \frac{2}{C}) \\
w(t) &= \int_1^2 2 \cdot (\frac{2t}{C} - \frac{2}{C})\, dt \\
3 \cdot 10^6 &= \frac{4}{C} \\
C &= \frac{4}{3} \mu F
\end{aligned}
$$

(ii)

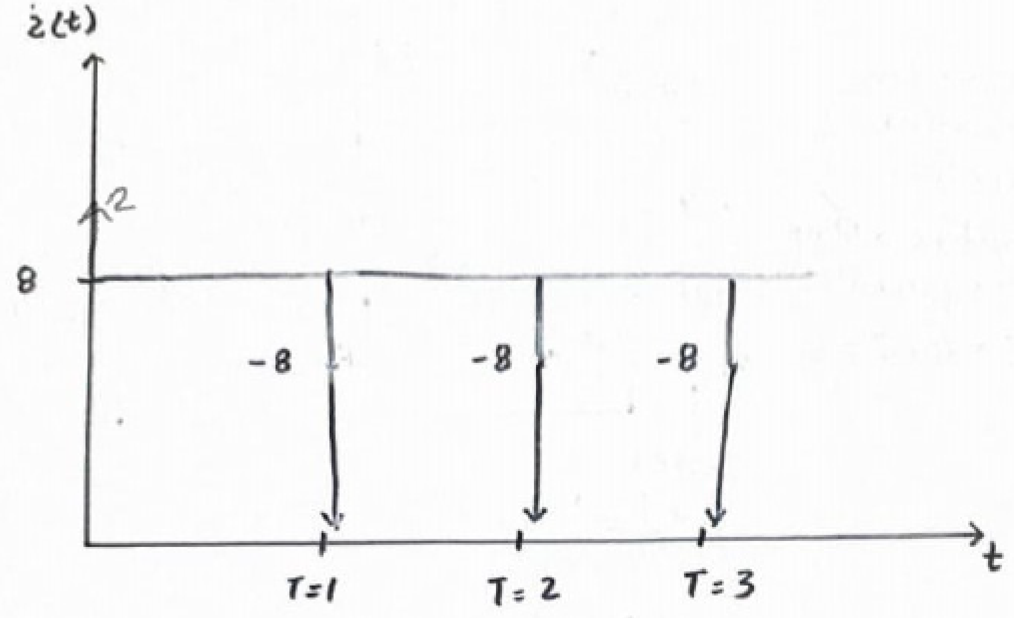
$$
\begin{aligned}
P &= VI \\
P &= \sin(t) \cdot L\cos(t),\ 0 < t \leq \pi/2 \\
w(t) &= \int_0^{\pi/2} L\sin(t)\cos(t)\, dt \\
w &= 3.5 \cdot 10^{-3} = 0.5L \\
L &= \frac{3.5 \cdot 10^{-3}}{0.5} = 7mH
\end{aligned}
$$

(c) For instantaneous change in voltage or current,

$$\frac{\mathrm{d}i}{\mathrm{d}t} \text{ or } \frac{\mathrm{d}v}{\mathrm{d}t}$$

becomes very large (as a change is needed in a very small amount of time), hence according to the relation infinite current voltage would be required. Providing infinite current or voltage is not possible in electrical circuits, hence a capacitor or inductor does not allow an instantaneous change in voltage or current, respectively.
Capacitor for voltage spikes and Inductor for current surges.

Q 9. (a) Take derivative of the sawtooth signal,



Between $0 < t < T$

$= \dfrac{10 - 2}{1} = 8 \rightarrow$ gradient.

At $t = nT$ $n = 1, 2, 3 \ldots$, the derivative is undefined.

However, we have a step of -8 (8 downwards)

(b)

$$i(t) = \begin{cases} 8 \cdot 10^{-3} tA, & 0 < t \leq 2\,ms \\ -8 \cdot 10^{-6} A & 2 < t \leq 4\,ms \\ 0A & t > 4\,ms \end{cases}$$

Use the relation between the current and voltage for capacitors to obtain piece-wise functions for $v(t)$,

$$v(t) = \frac{1}{C} \cdot \int i(t)\,dt$$

For time between 0ms and 2ms, the expression for $v(t)$ is

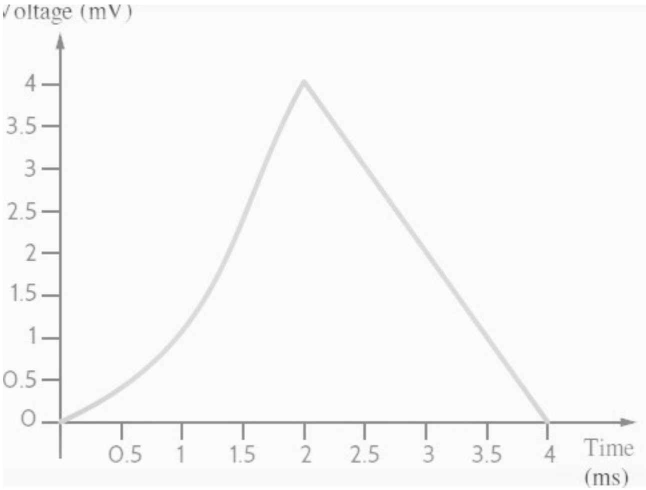
$$\begin{aligned} v(t) &= \frac{1}{4 \cdot 10^{-6}} \cdot \int_0^t 8 \cdot 10^{-3} t\,dt \\ &= 1000t^2 mV \end{aligned}$$

So, at $t = 2ms$

$$v(t) = 4mV$$

For time between 2ms and 4ms, the expression for $v(t)$ is,

$$v(t) = 4mV + \frac{1}{4 \cdot 10^{-3}} \int_{2 \cdot 10^{-3}}^{t} -8 \cdot 10^{-6}\,dt$$
$$v(t) = -2t + 8 \cdot 10^{-3} mV$$



Voltage (mV)
4
3.5
3
2.5
2
1.5
1
0.5
0
0.5
1
1.5
2
2.5
3
3.5
4
Time
(ms)