Department of Electrical Engineering
School of Science and Engineering

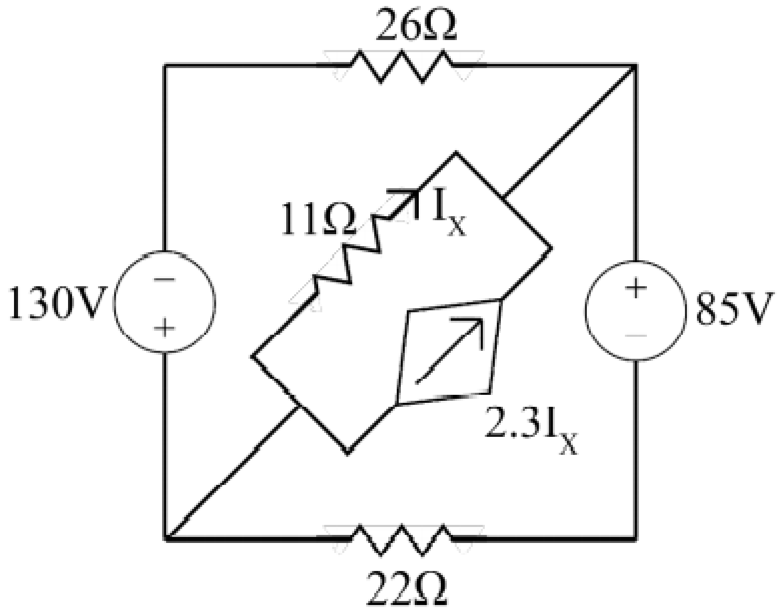
EE240 Circuits I - Fall 2020

ASSIGNMENT 2 – SOLUTIONS

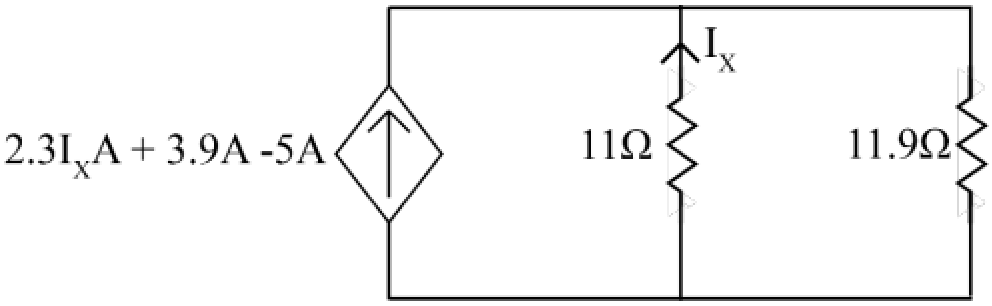
Q 1. (a) (i) 4 equations

(ii) 3 loops

(b) First approach: Apply Source Transformation on all parallel current sources and the corresponding resistors i.e. (5A-3Ω),(5A-23Ω), (5A-17Ω).
Add resistance and voltage sources in series to produce the following circuit.



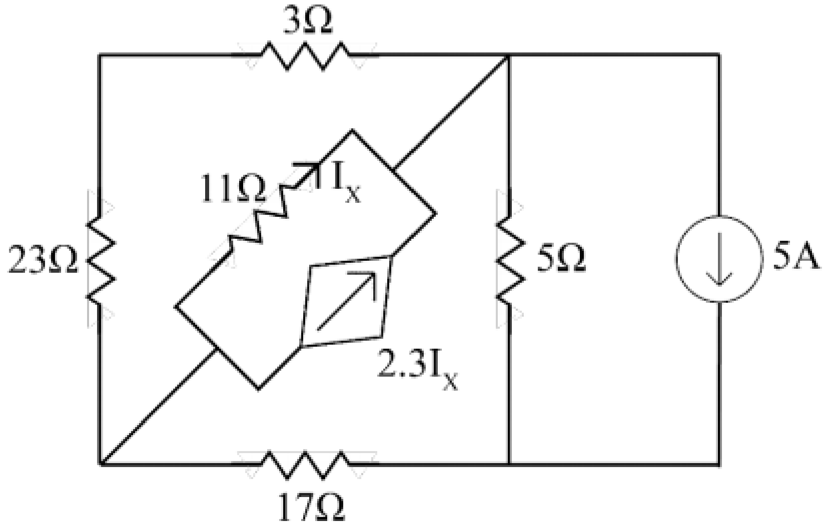
Apply Source Transformation again on all series voltage sources and resistors i.e. (130V-26Ω), (85V-22Ω).
Add resistance and current sources in parallel to produce the following circuit



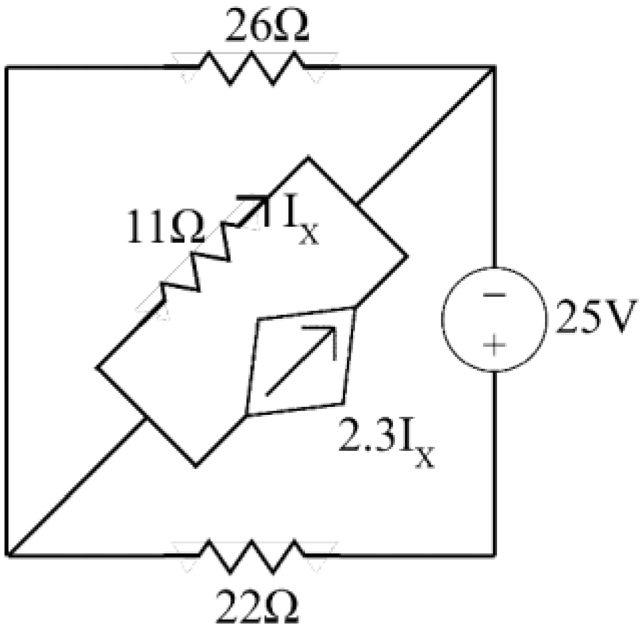
Note that the current controlled current source depends upon the current flowing through the 11Ω resistor, that is why it cannot be combined withthe 11.9 Ω resistor, also because we need to find I_x.
Use the current divider rule to find I_x,

$$-I_x = \frac{11.9\Omega}{11\Omega + 11.9\Omega} \cdot (2.3I_x + 3.9A - 5A)$$
$$I_x = 0.26A = 0.3A$$

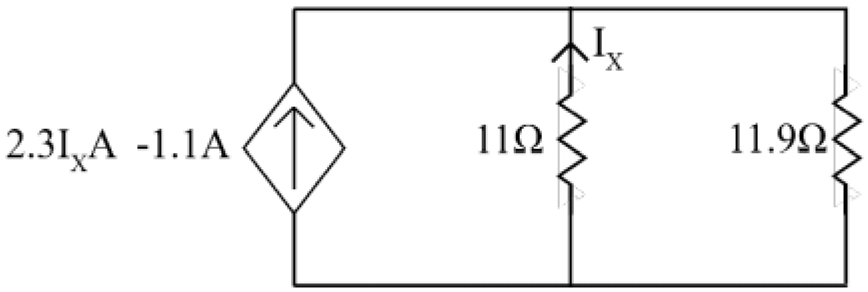
Alternative approach:Instead of applying source transformation on 3 pairs, we can combine the 3 current sources to produce the following circuit.



Now apply Source Transformation on the parallel current source and resistor. i.e.(5A-5Ω). Also, combine the 2 resistors in series i.e. 23Ω +3Ω to produce the following circuit.



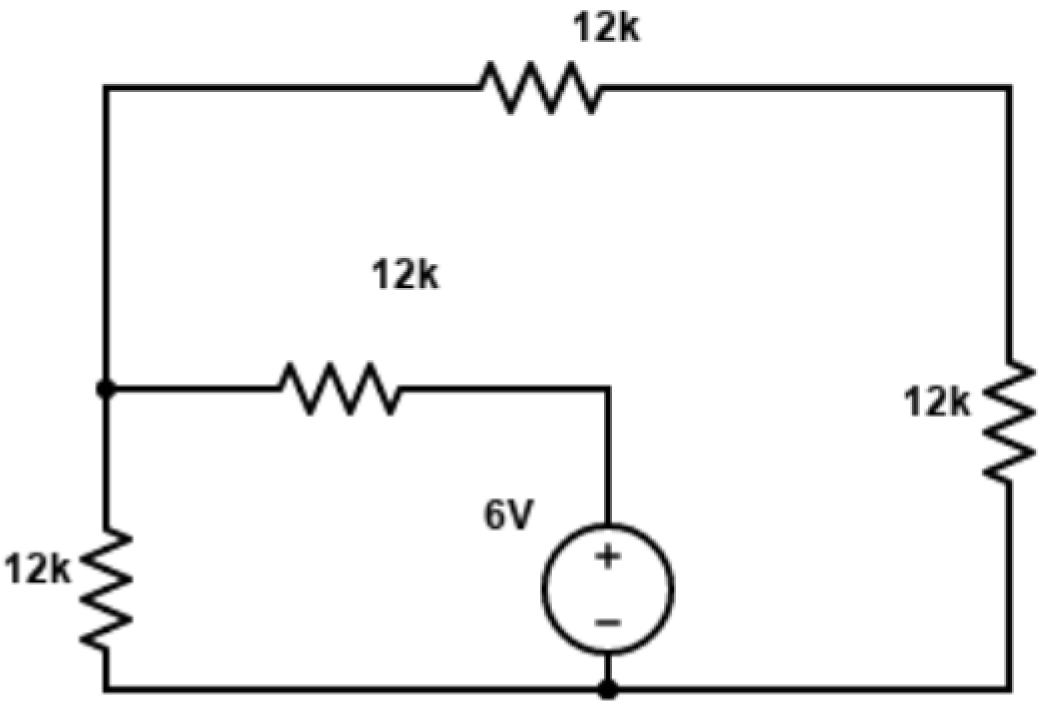
Apply Source Transformation on the series voltage source and resistor. i.e. (25V-22Ω).
Add resistance and current sources in parallel to produce the following circuit.
Note: The Current Controlled Current Source depends upon the current flowing through the 11Ω resistor, that is why we can not combine it with the 11.9Ω resistor, also because we need to find the current I_x.



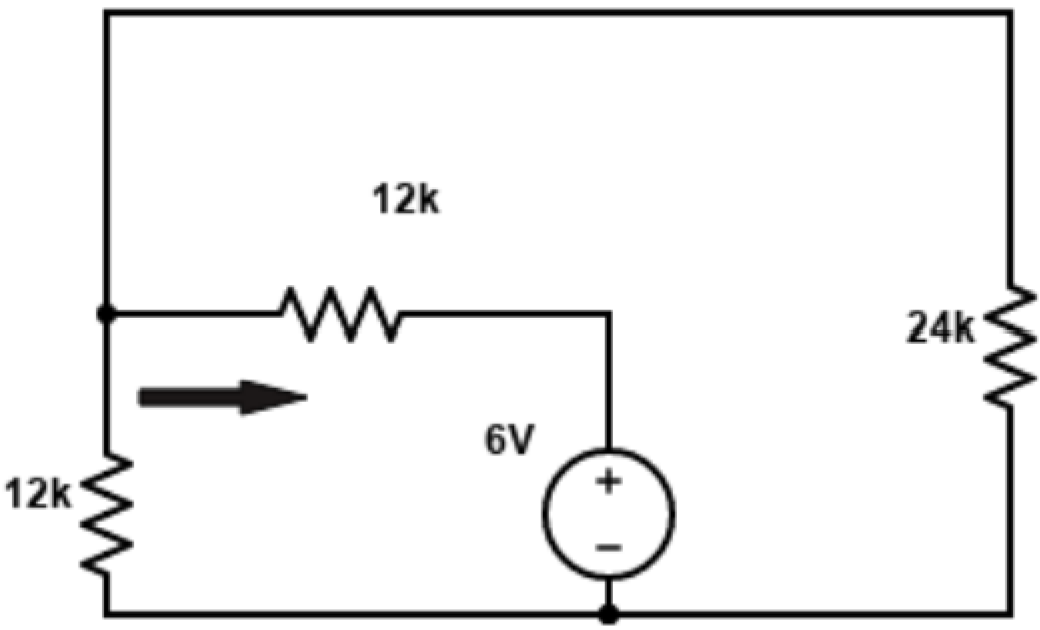
Use the current divider rule to find I_x,

$$-I_x = \frac{11.9\Omega}{11\Omega + 11.9\Omega} \cdot (2.3I_x - 1.1A)$$
$$I_x = 0.3A$$

Q 2. (a) Step 1: Keep the voltage source, open-circuit the current source(find current due to voltage source I_V)

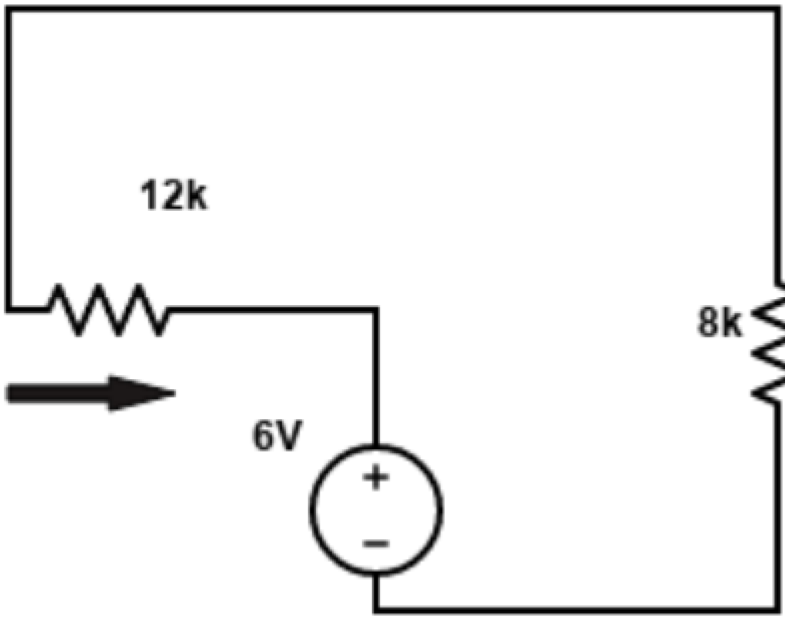


Simplify the resitors in series,



Simplify the resistors in parallel,

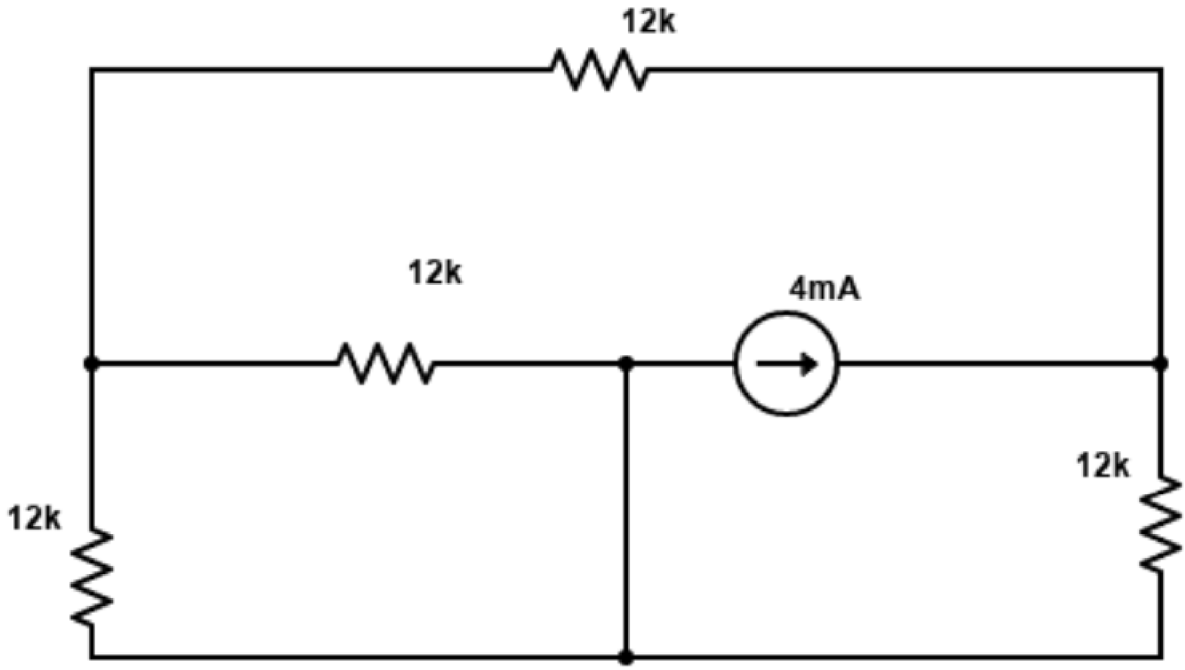
$$12k\Omega || 24k\Omega = 8k\Omega$$



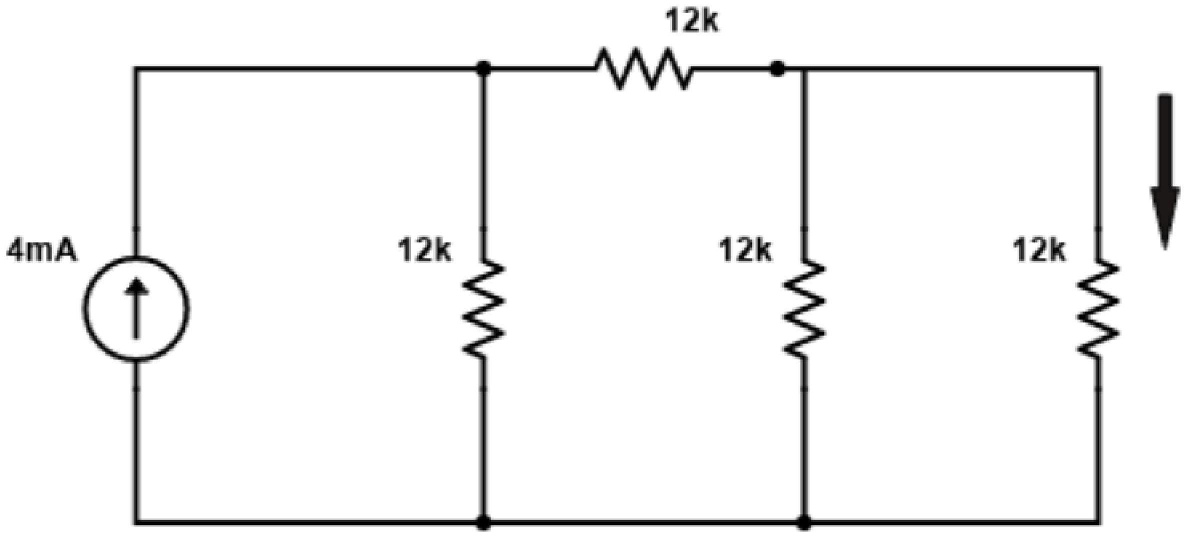
Notice the polarity of the voltage source and the direction of the current

$$I_V = -\frac{6}{20000} = -3 \cdot 10^{-4}A = -0.3mA$$

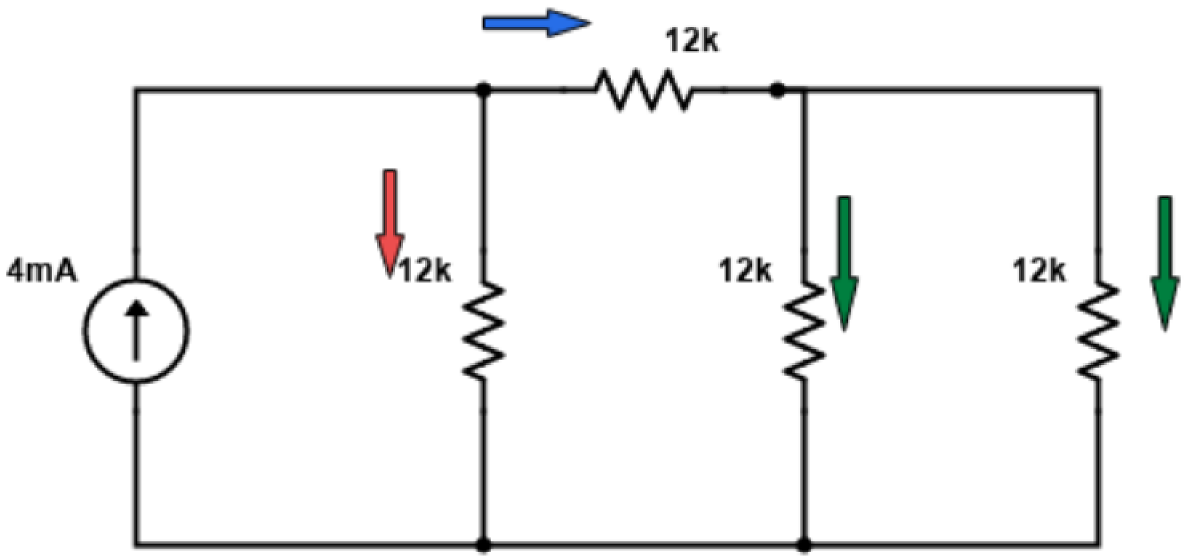
Step 2: Keep the current source, short-circuit the voltage source(find the current due to the current source I_I)



Redraw the circuit,



Consider the first two divisions(blue and pink) of the current in the figure below, the current for the blue arrow I_{blue} is (using current divider rule),



$$I_{blue} = \frac{12}{30} \cdot 4mA = 1.6 \cdot 10^{-3}A = 1.6mA$$

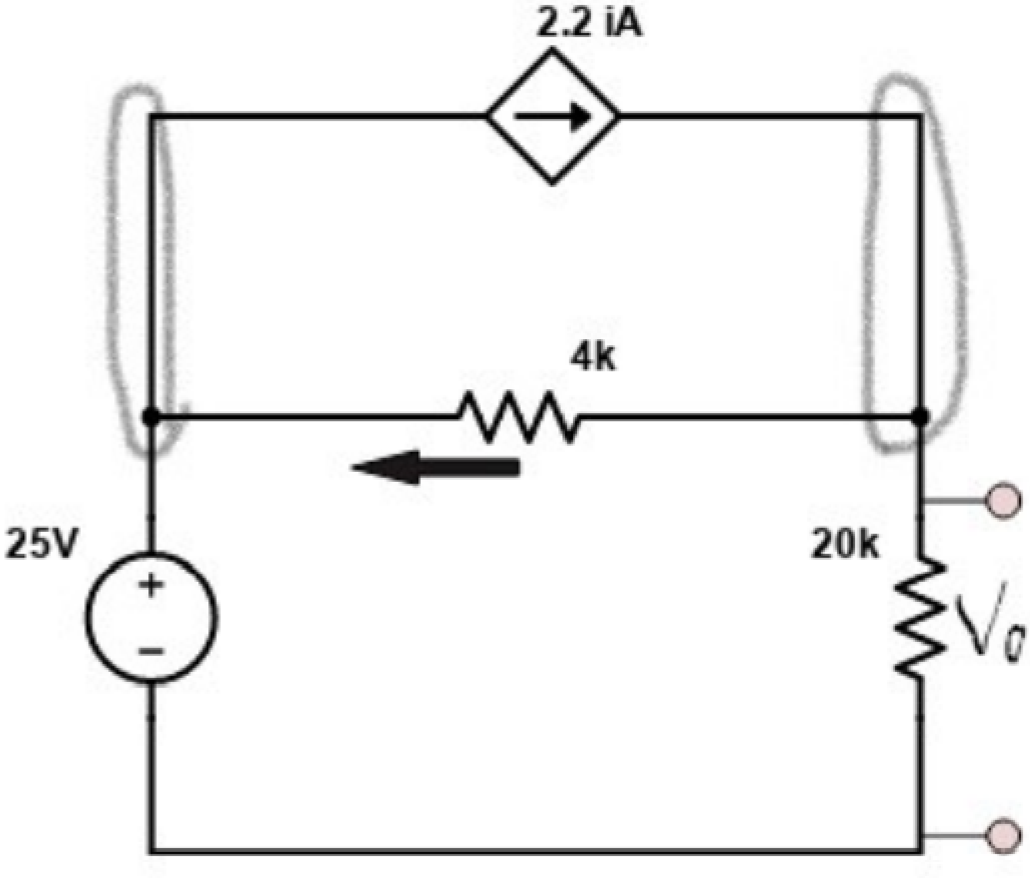
This current will divide equally into the two branches represented by two green arrows,

$$I_I = \frac{1.6mA}{2} = 0.8mA$$

Therefore,

$$I_0 = I_V + I_I = 0.8mA + (-0.3mA)$$
$$I_0 = 0.5mA$$

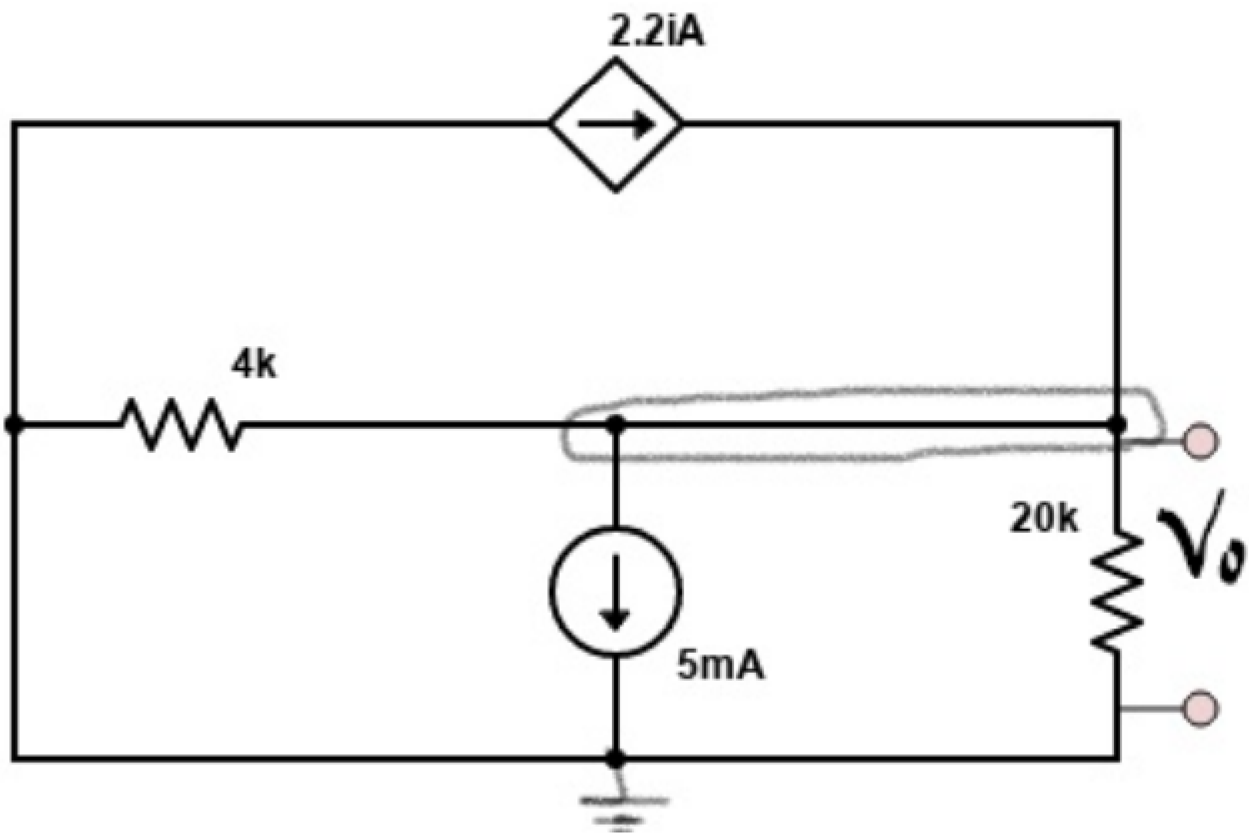
(b) Step 1: Keep the voltage source, short circuit the current source (find the voltage due to voltage source V_{0V}).



Apply nodal analysis on the nodes circled,

$$\frac{V_{0V}-25}{4000}+\frac{V_{0V}}{20000}=2.2i_A$$
$$\frac{V_{0V}}{4000}+\frac{V_{0V}}{20000}-\frac{25}{4000}=2.2\left(\frac{V_{0V}-25}{4000}\right)$$
$$\frac{3V_{0V}}{10000}-\frac{2.2V_{0V}}{4000}=\frac{2.2(-25)}{4000}+\frac{25}{4000}$$
$$V_{0V}=30V$$

Step 2: Keep the current source, open-circuit the voltage source (find the voltage due to the current source V_{0I})



Apply nodal analysis,

$$\frac{V_{0I}}{20000} + (5 \cdot 10^{-3}) + \frac{V_{0I}}{4000} = 2.2 \cdot \frac{V_{0I}}{4000}$$
$$V_{0I} = 20V$$

So,

$$V_0 = V_{0V} + V_{0I} = 30V + 20V = 50V$$

(c) (i) Resultant resistor combinations for every voltage source will cause a scaled version of the actual voltage to appear across the terminals a and b. The effect of one of the voltage sources $(V_1, V_2, V_3 ... V_M)$,

$$v_m = a_m V_m$$

(ii) Resultant resistor combinations for every current source will cause a scaled version of the actual voltage to appear accross the terminals a and b. The effect of one of the current sources $(I_1, I_2 ... I_{N-1})$,

$$v_n = I_n R_n$$

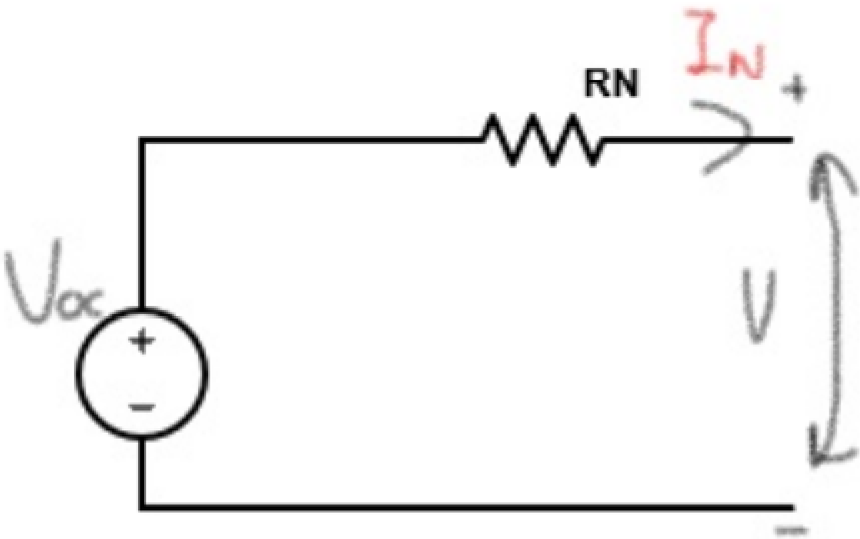
(iii) The effect of the N^{th} current source,

$$v_I = -I_N R_N$$

(iv) Superpose the effect of all the sources,

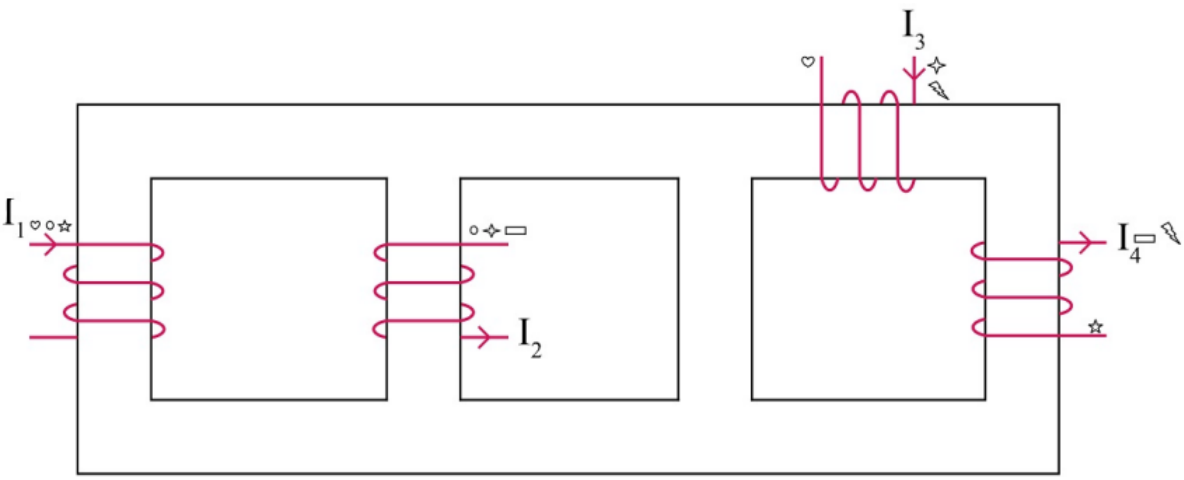
$$V = \sum_{m=1}^{M} v_m + \sum_{n=1}^{N-1} v_n - I_N R_N$$
$$V = v_{oc} - I_N R_N$$

Note: This expression is equivalent to the following circuit (essential the Thevenin theorem),



Q 3. Output is shorted.
V_2 and I_3 should be replaced by V_2.
I_1 and I_2 can not be connected in series.
V_1 and V_3 can not be connected in parallel.
V_4 and I_4 should be replaced by I_4.

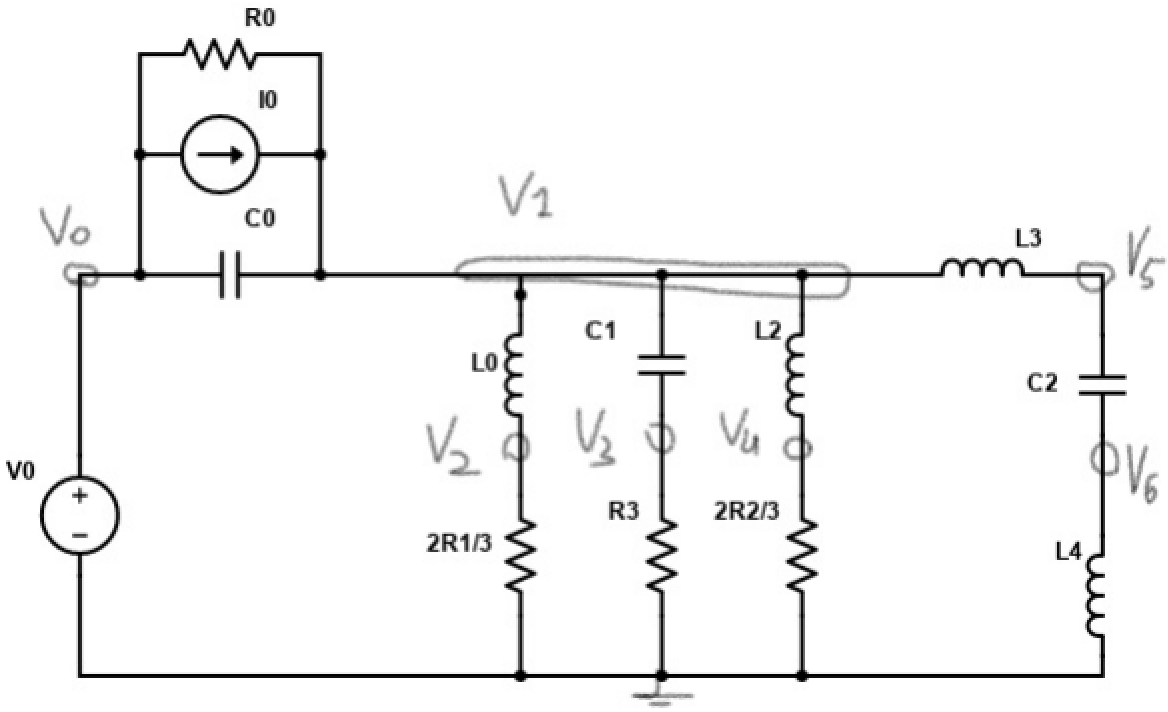
Q 4. Follow the dot convention:



1–2	○
1–3	♡
1–4	☆
2–3	✧
2–4	▭
3–4	⚡

Q 5. (a) (i) 7 nodes, 6 equations.

(ii) Redraw the circuit,



Equation 1 (Node 1):

$$\frac{V_1 - V_0}{R_0} + C_0 \frac{\mathrm{d}V_1 - V_0}{\mathrm{d}t} + \frac{1}{L_0}\int (V_1 - V_2)dt + C_1 \frac{\mathrm{d}V_1 - V_3}{\mathrm{d}t} + \frac{1}{L_2}\int (V_1 - V_4)dt + \frac{1}{L_3}\int (V_1 - V_5)dt = I_0$$

Equation 2 (Node 2):

$$\frac{1}{L_0}\int (V_2 - V_1)dt + \frac{V_2}{\frac{2}{3}R_1} = 0$$

Equation 3 (Node 3):

$$C_1 \frac{\mathrm{d}V_3 - V_1}{\mathrm{d}t} + \frac{V_3}{R_3} = 0$$

Equation 4 (Node 4):

$$\frac{1}{L_2}\int (V_4 - V_1)dt + \frac{V_4}{\frac{2}{3}R_2} = 0$$

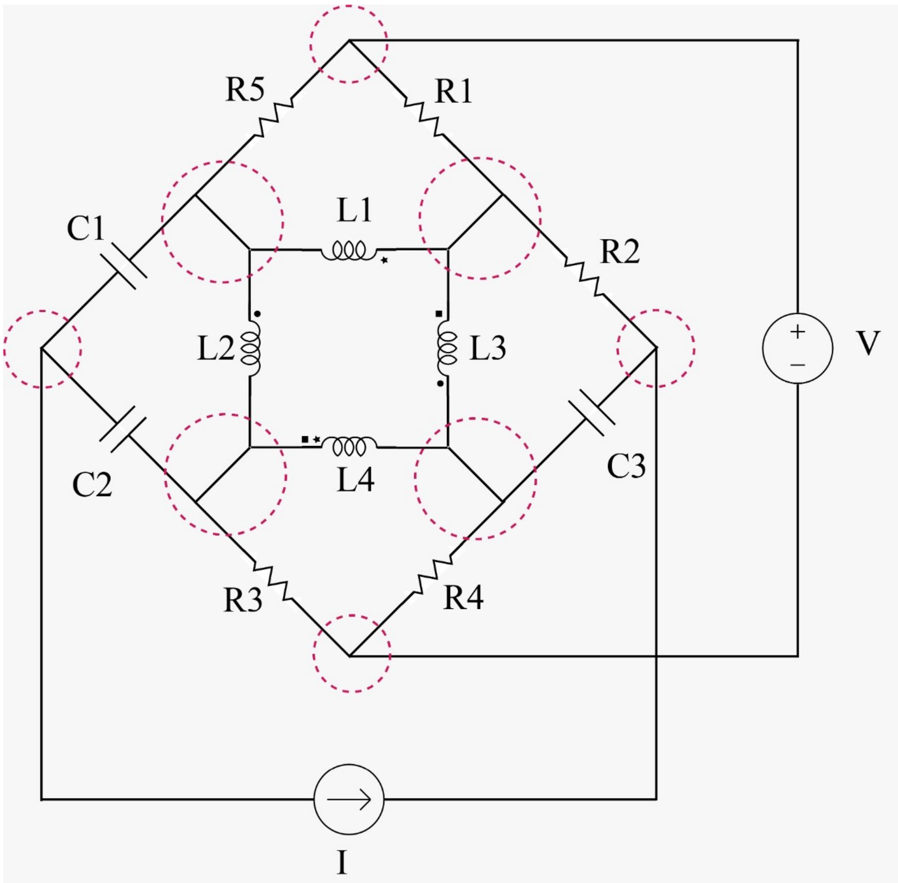
Equation 5 (Node 5):

$$\frac{1}{L_3}\int (V_5 - V_1)dt + C_2 \frac{\mathrm{d}V_5 - V_6}{\mathrm{d}t} = 0$$

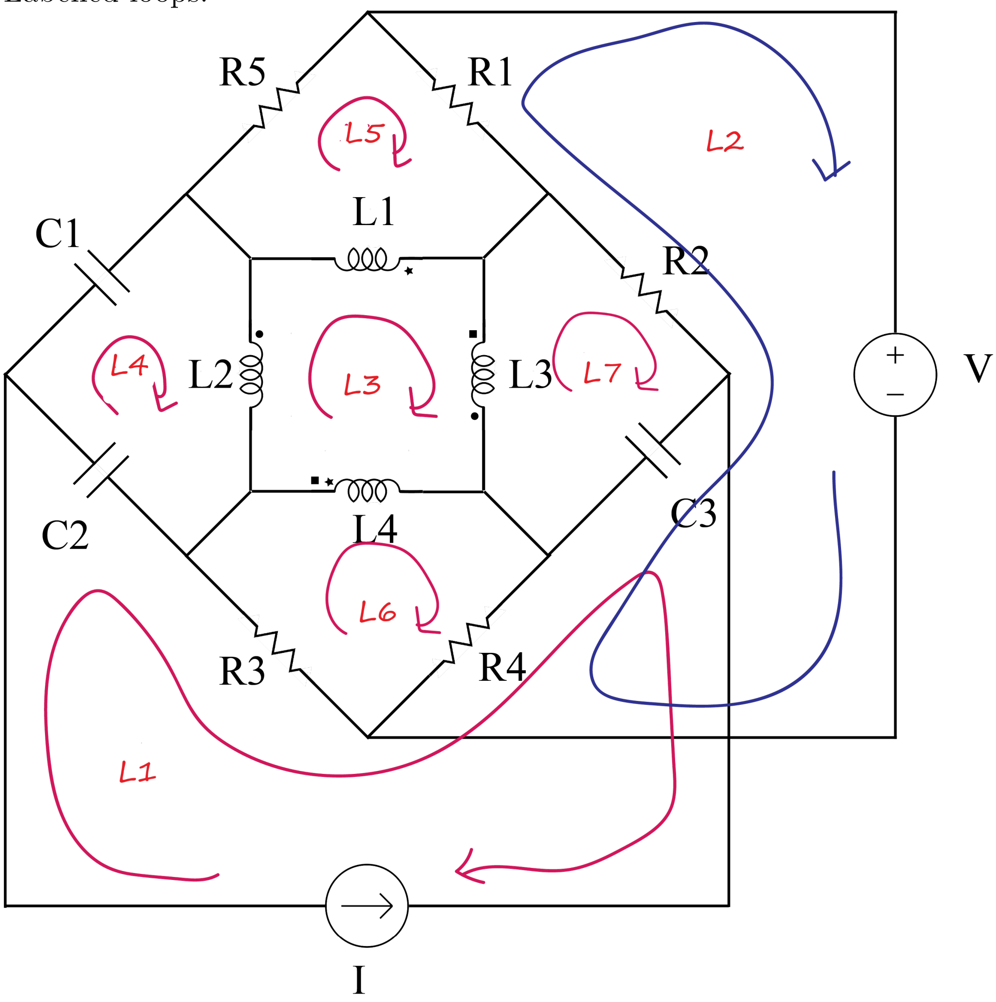
Equation 6 (Node 6):

$$C_2 \frac{\mathrm{d}V_6 - V_5}{\mathrm{d}t} + \frac{1}{L_4}\int (V_6)dt = 0$$

Q 6. (i) Label all the nodes on the circuit.



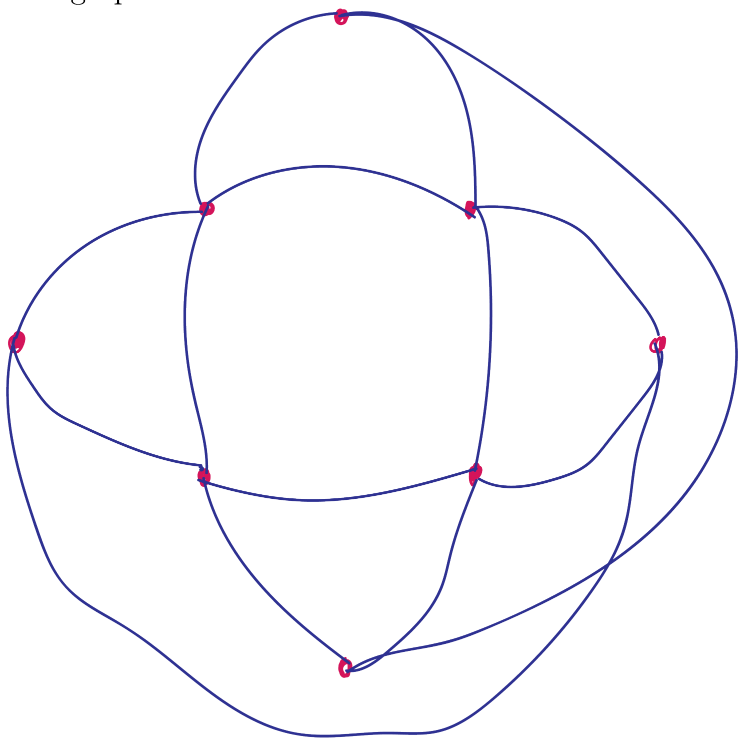
Labelled loops:



(ii) Use loop analysis. There are 8 nodes and 5 complete loops, so loop analysis will yield less equations making it.

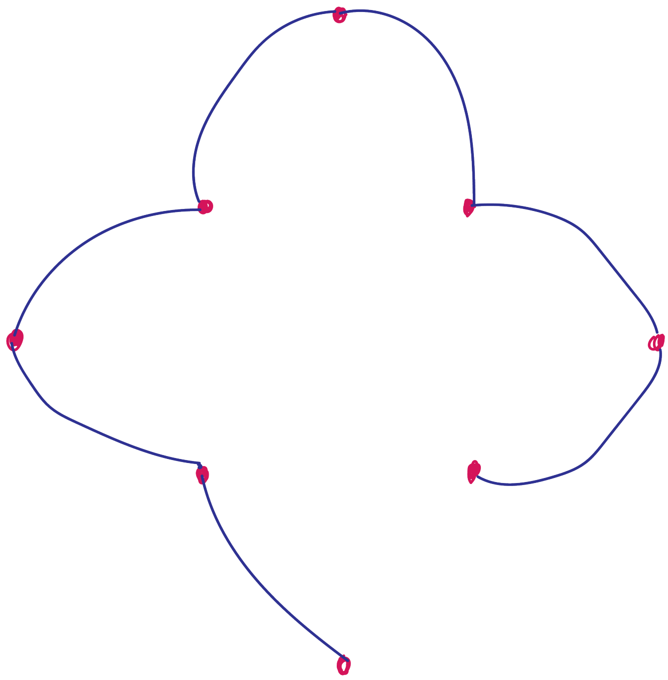
(iii) 14 branches

(iv) The graph of the circuit:



(v) Calculate the number of chords in the tree.
number of chords = number of nodes − 1 = 7 Chords

(vi) The tree of the circuit:



(vii) Loop 1 (L1):

$$\frac{1}{C_2}\int(I_1 - I4)dt + (I_1 - I_6)R_3 + (I_1 + I_2 - I_6)R_4 + \frac{1}{C_3}\int(I_1 + I_2 - I_7)dt = 0$$

Loop 2 (L2):

$$(I_1 + I_2 - I_6)R_4 + \frac{1}{C_3}\int(I_1 + I_2 - I_7)dt + (I_2 - I_7)R_2 + (I_2 - I_5)R_1 = -V$$

Loop 3 (L3):

$$L_1\frac{\mathrm{d}(I_3-I_5)}{\mathrm{d}t}+L_3\frac{\mathrm{d}(I_3-I_7)}{\mathrm{d}t}+L_4\frac{\mathrm{d}(I_3-I_6)}{\mathrm{d}t}+L_2\frac{\mathrm{d}(I_3-I_4)}{\mathrm{d}t}$$
$$+M_{14}\frac{\mathrm{d}(I_3-I_6)}{\mathrm{d}t}+M_{23}\frac{\mathrm{d}(I_3-I_7)}{\mathrm{d}t}+M_{32}\frac{\mathrm{d}(I_3-I_4)}{\mathrm{d}t}$$
$$-M_{34}\frac{\mathrm{d}(I_3-I_6)}{\mathrm{d}t}+M_{41}\frac{\mathrm{d}I_3-I_5}{\mathrm{d}t}-M_{43}\frac{\mathrm{d}(I_3-I_7)}{\mathrm{d}t}=0$$

Loop 4 (L4):

$$\frac{1}{C_2}\int(I_4-I_1)dt+\frac{1}{C_1}\int I_4dt+L_2\frac{\mathrm{d}I_4-I_3}{\mathrm{d}t}-M_{23}\frac{\mathrm{d}(I_7-I_3)}{\mathrm{d}t}=0$$

Loop 5 (L5):

$$(I_5-I_2)R_1+L_1\frac{\mathrm{d}(I_5-I_3)}{\mathrm{d}t}-M_{14}\frac{\mathrm{d}(I_6-I_3)}{\mathrm{d}t}+I_5R_5=0$$

Loop 6 (L6):

$$(I_6-I_1)R_3+(I_6-I_1-I_2)R_4+L_4\frac{\mathrm{d}(I_6-I_3)}{\mathrm{d}t}-M_{34}\frac{\mathrm{d}(I_7-I_3)}{\mathrm{d}t}+M_{41}\frac{\mathrm{d}(I_5-I_3)}{\mathrm{d}t}=0$$

Loop 7 (L7):

$$(I_7-I_2)R_2+\frac{1}{C_3}\int(I_7-I_1-I_2)dt+L_3\frac{\mathrm{d}(I_7-I_3)}{\mathrm{d}t}+M_{32}\frac{\mathrm{d}(I_4-I_3)}{\mathrm{d}t}-M_{34}\frac{\mathrm{d}(I_6-I_3)}{\mathrm{d}t}=0$$

(viii)

$$\begin{bmatrix}
R_3+R_4+\frac{1}{C_2}\int dt+\frac{1}{C_3}\int dt & R_4+\frac{1}{C_3}\int dt & 0 & -\frac{1}{C_2}\int dt & 0 & (-R_3-R_4) & \frac{-1}{C_3}\int dt \\
R_4+\frac{1}{C_3}\int dt & R_1+R_2+R_3+\frac{1}{C_3}\int dt & 0 & 0 & -R_1 & -R_4 & -R_2-\frac{1}{C_3}\int dt \\
0 & 0 & (L_1+L_2+L_3+L_4+M_{14}+M_{23}-M_{32}+M_{34}+M_{41}+M_{43})\frac{\mathrm{d}}{\mathrm{d}t} & (-L_2-M_{32})\frac{\mathrm{d}}{\mathrm{d}t} & (-L_1-M_{14})\frac{\mathrm{d}}{\mathrm{d}t} & (-L_4-M_{14}-M_{34})\frac{\mathrm{d}}{\mathrm{d}t} & (-L_3-M_{32}-M_{43})\frac{\mathrm{d}}{\mathrm{d}t} \\
\frac{1}{C_2}\int dt & 0 & (-L_2-M_{23})\frac{\mathrm{d}}{\mathrm{d}t} & \frac{1}{C_2}\int dt+\frac{1}{C_1}\int dt+L_2\frac{\mathrm{d}}{\mathrm{d}t} & 0 & 0 & 0 \\
0 & -R_1 & (-L_1-M_{41})\frac{\mathrm{d}}{\mathrm{d}t} & 0 & R_1+R_5+L_1\frac{\mathrm{d}}{\mathrm{d}t} & -M_{14}\frac{\mathrm{d}}{\mathrm{d}t} & 0 \\
(-R_3-R_4) & -R_4 & (-L_4-M_{14}-M_{34})\frac{\mathrm{d}}{\mathrm{d}t} & 0 & -M_{14}\frac{\mathrm{d}}{\mathrm{d}t} & R_3+R_4+L_4\frac{\mathrm{d}}{\mathrm{d}t} & -M43\frac{\mathrm{d}}{\mathrm{d}t} \\
\frac{-1}{C_3}\int dt & -R_2-\frac{1}{C_3}\int dt & (-L_3-M_{32}-M_{43})\frac{\mathrm{d}}{\mathrm{d}t} & 0 & 0 & -M_{43}\frac{\mathrm{d}}{\mathrm{d}t} & R_2+\frac{1}{C_2}\frac{\mathrm{d}}{\mathrm{d}t}+L_3\frac{\mathrm{d}}{\mathrm{d}t}
\end{bmatrix}
\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \end{bmatrix}$$

$$=\begin{bmatrix} 0 \\ -V_0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$