EE240 – Circuits I

Mid Examination (Fall 2020) - Solutions

November 07, 2020 02:00 pm–04:30 pm

INSTRUCTIONS:

- We require you to solve the exam in a single time-slot of two hours without any external or electronic assistance.
- We encourage you to solve the exam on A4 paper, use new sheet for each question and write sheet number on every sheet.
- Clearly outline all your steps in order to obtain any partial credit.
- The exam is closed book and notes. You are allowed to have one A4 sheet with you with hand-written notes on both sides. Calculators can be used.
- For the sake of completeness, we require you to write the following statement on your first page of submission: *I commit myself to uphold the highest standards of (academic) integrity.*
- If you are ready, please proceed to the next page.

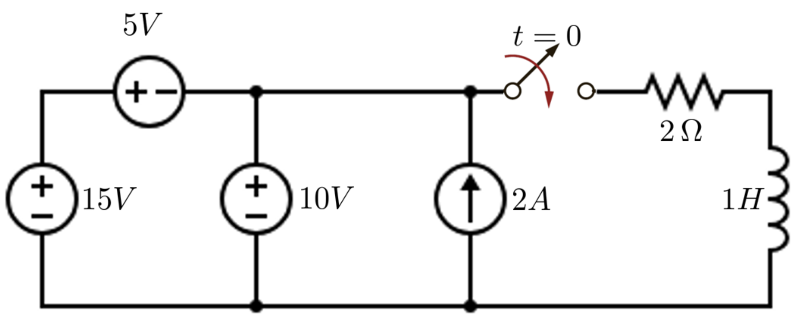
Mapping between exam parts and course learning outcomes (CLOs)

- Part 1: R, L, C Basics, Sources and I-V Characteristics (CLO1)
- Part 2: Network Topology, Network Equations and Kirchhoff's Laws (CLO2)
- Part 3: Additional Analysis Techniques (CLO3)

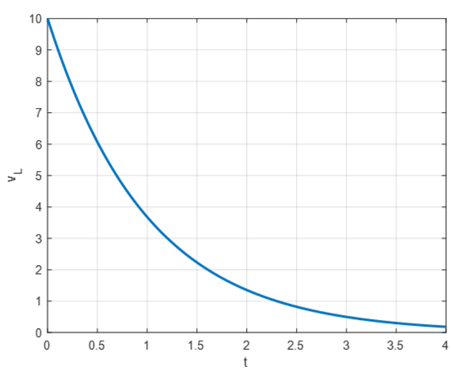
Part 1: R, L, C Basics, Sources and I-V Characteristics

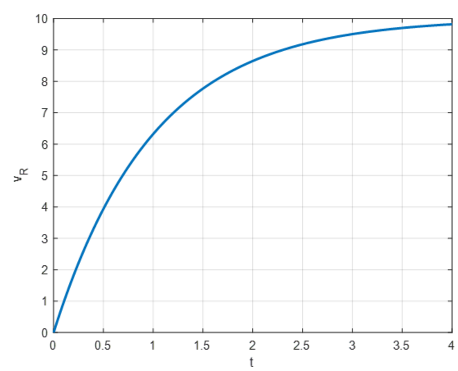
Problem 1. (**10 pts**)

(a) (**5 pts**) For a circuit given below, assume that the switch is initially open and is closed at $t = 0$ and the inductor is not carrying any current before the switch is closed. Label the voltages across resistor and inductor as $v_R(t)$ and $v_L(t)$ respectively and plot the waveforms of the voltages.

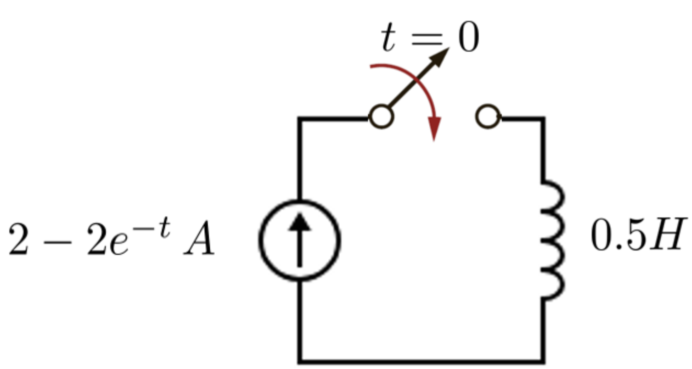


Solutions:
The combination of sources is equivalent to 10V voltage source. Plots given below:





(b) (**5 pts**) For a circuit given below, assume that the switch is initially open and is closed at $t = 0$ and the inductor is not carrying any current before the switch is closed. Determine the voltage across the inductor and the total energy supplied to the inductor.



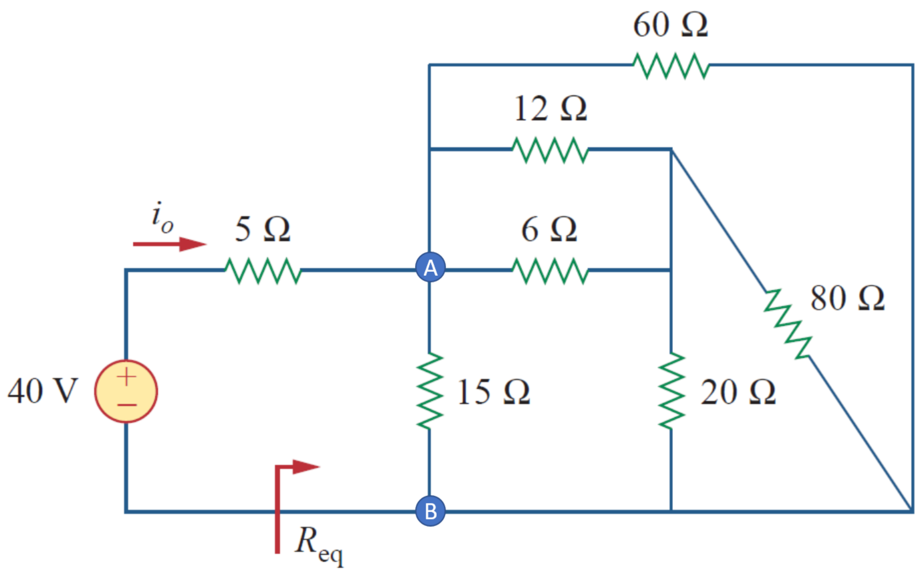
Solutions:

$$v(t) = L\frac{di}{dt} = \frac{1}{2}2e^{-t} = e^{-t}$$

$$(\text{Energy})\, w(t) = \frac{1}{2}Li^2(t) = 1 - 2e^{-t} + e^{-2t}$$

$$(\text{TotalEnergy})\, w(\infty) = 1\, J$$

Problem 2. (**5 pts**) For the circuit given below, determine the equivalent resistance R_{eq} and the current i_o indicated in the circuit. R_{eq} is the equivalent resistance across terminals A and B.

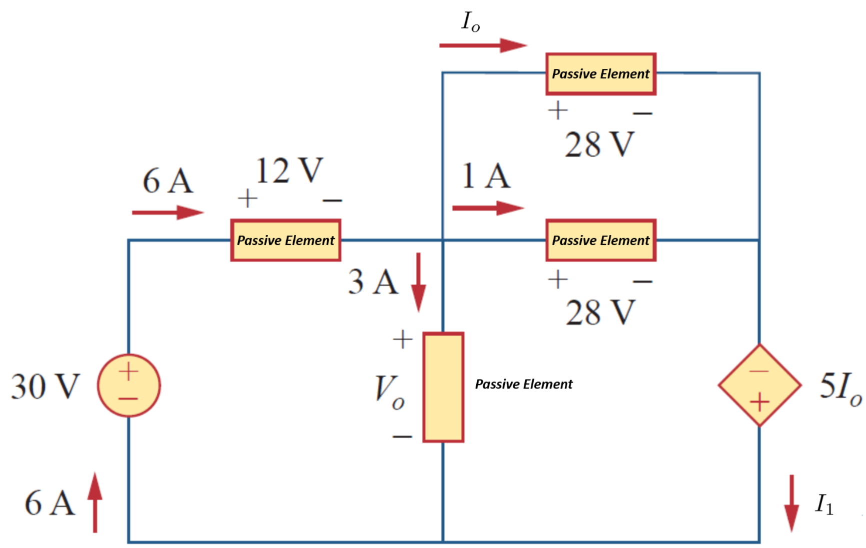


Solutions:

R_{eq} is $(80\|20 + 12\|6)\|60\|15 = (20)\|12 = 7.5\,\Omega$.

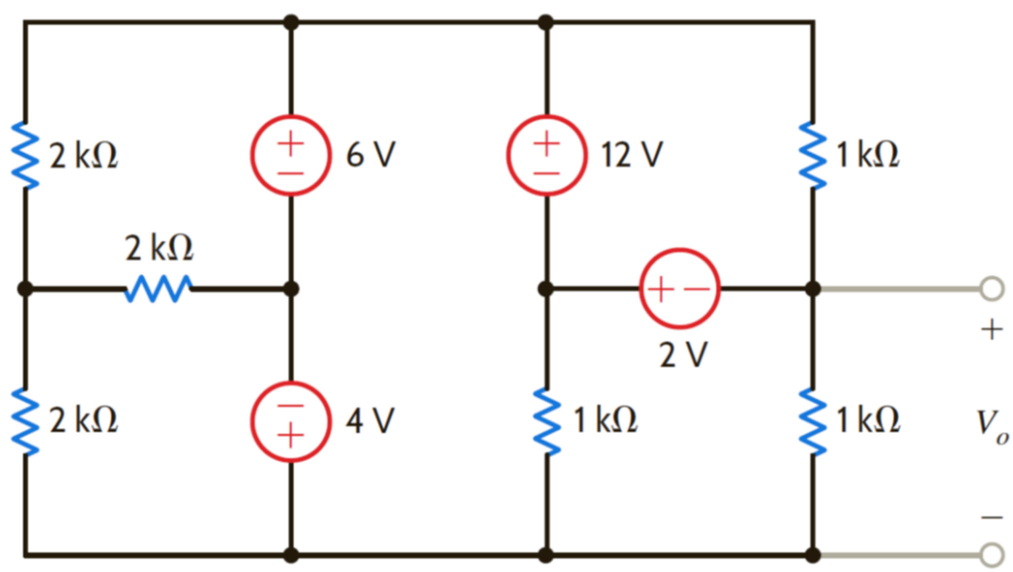
The current $i_o = 40/12.5 = 3.2A$

Problem 3. (**3 pts**) For the circuit given below, determine V_o, I_o and I_1 indicated in the circuit. Provide brief justification or working to support your answer.



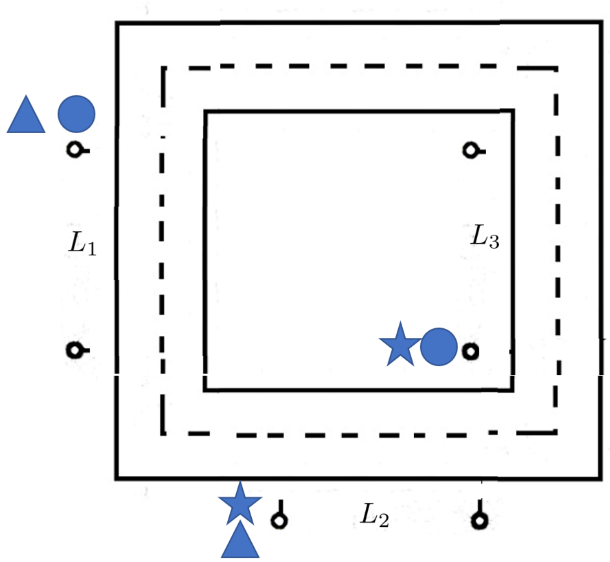
$I_o = 2A, \quad V_o = 18V, \quad I_1 = 3A$

Problem 4. (**2 pts**) For the circuit given below, determine V_o. Provide brief justification or working for your answer.

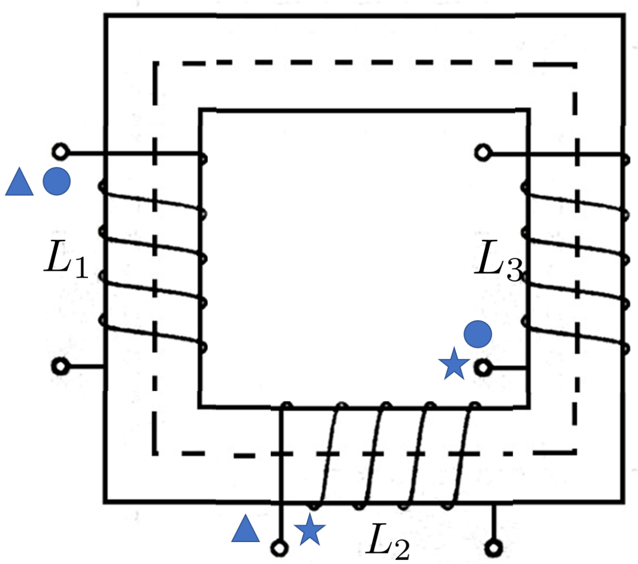


Solutions:
$V_o = -2 - 12 + 6 - 4 = -12V$

Problem 5. (**5 pts**) The figure below shows the dots marked for three windings L_1, L_2 and L_3 on a magnetic flux-conducting core. If the dots are marked using the dot convention, draw the windings on the core with directions consistent with the dots.



Solutions:



Part 2: Network Topology, Network Equations and Kirchhoff's Laws

Problem 6. (**2 pts**)

i) What do we mean by electrically equivalent circuits?

Solutions:
Same $i - v$ characteristics.

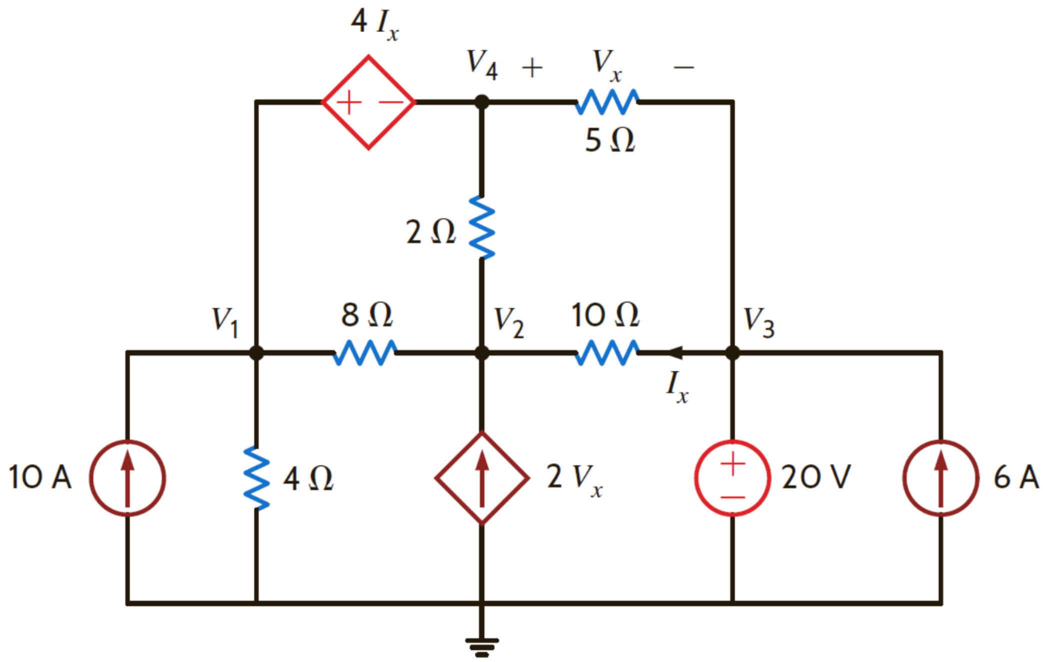
ii) Do you agree with the following statement (support your answer with the justification)?

The two electrically equivalent circuits may not be be topologically equivalent but two 'topologically equivalent' circuits are electrically equivalent.

Solutions:
No! We cannot relate topological equivalence (related to graphical representation) and electrical equivalence (related to $i - v$ characteristics).

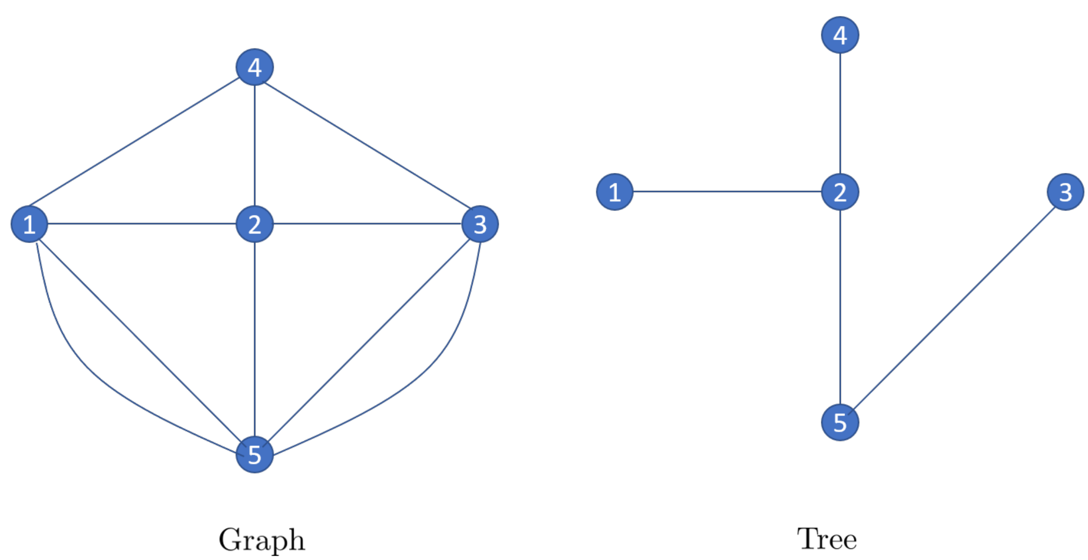
Problem 7. (**18 pts**)

Consider the circuit given below.



(a) (**4 pts**) Draw the graph and one tree of the circuit. Determine the number of nodes and number of branches in a circuit.

Solutions:



Number of nodes $= n = 5$

Number of branches $= b = 10$

(b) (**2 pts**) Determine the number of network equations required for carrying out i) nodal analysis and ii) loop analysis.

Solutions:

Nodal analysis: number of equations $= n - 1 = 4$

Loop analysis: number of equations $= b - n + 1 = 6$

(c) (**9 pts**) Use Kirchhoff current law to determine the nodal voltages indicated on the circuit.

Solutions:

Using KCL to write nodal equations.

Node 1 and Node 4 (Super node):

$$\frac{V_1}{4} + \frac{V_1 - V_2}{8} + \frac{V_4 - V_2}{2} + \frac{V_4 - V_3}{5} - 10 = 0$$

Node 2:

$$\frac{V_2 - V_1}{8} + \frac{V_2 - V_4}{2} + \frac{V_2 - V_3}{10} - 2V_x = 0$$

Node 3:

$$V_3 = 20V$$

Super-node equation:

$$V_1 - V_4 = 4I_x$$

Dependent voltage source:

$$I_x = \frac{V_3 - V_2}{10}$$

Dependent current source:

$$V_x = V_4 - V_3$$

We manipulate these equations to obtain

$$\begin{bmatrix} \frac{1}{8} + \frac{1}{4} & -\frac{1}{8} - \frac{1}{2} & \frac{1}{2} + \frac{1}{5} \\ -\frac{1}{8} & \frac{1}{2} + \frac{1}{8} + \frac{1}{10} & -2 - \frac{1}{2} \\ 5 & 2 & -5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_4 \end{bmatrix} = \begin{bmatrix} 14 \\ -38 \\ 40 \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_4 \end{bmatrix} = \begin{bmatrix} 21.145 \\ 9.074 \\ 16.774 \end{bmatrix} V$$

(d) (**3 pts**) Determine the power supplied by the voltage source.

Solutions:

We use I to denote the current supplied by 20V voltage source. We can apply KCL at node 3 to determine I as

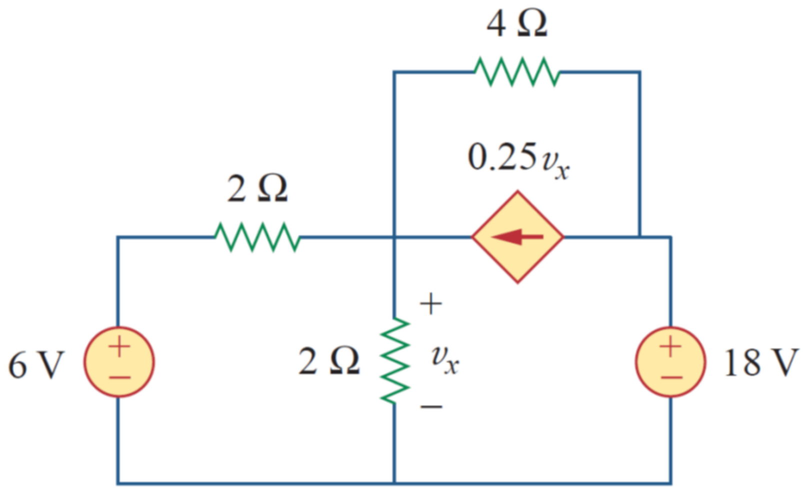
$$I = \frac{V_3 - V_2}{10} + \frac{V_3 - V_4}{5} - 6 = 1.0926 + 0.6452 - 6 = -4.2622A$$

Power is given by

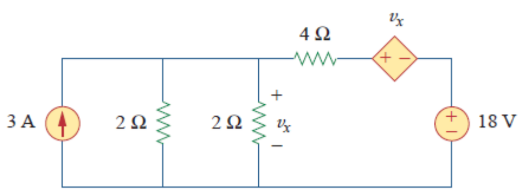
$$P = 20 \times I = -85.2445\, W$$

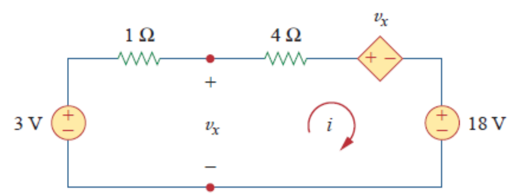
Part 3: Additional Analysis Techniques

Problem 8. (**8 pts**) Determine the voltage v_x in the following circuit using the source transformation technique.



Solutions:
Applying source transformation:





Applying KVL:

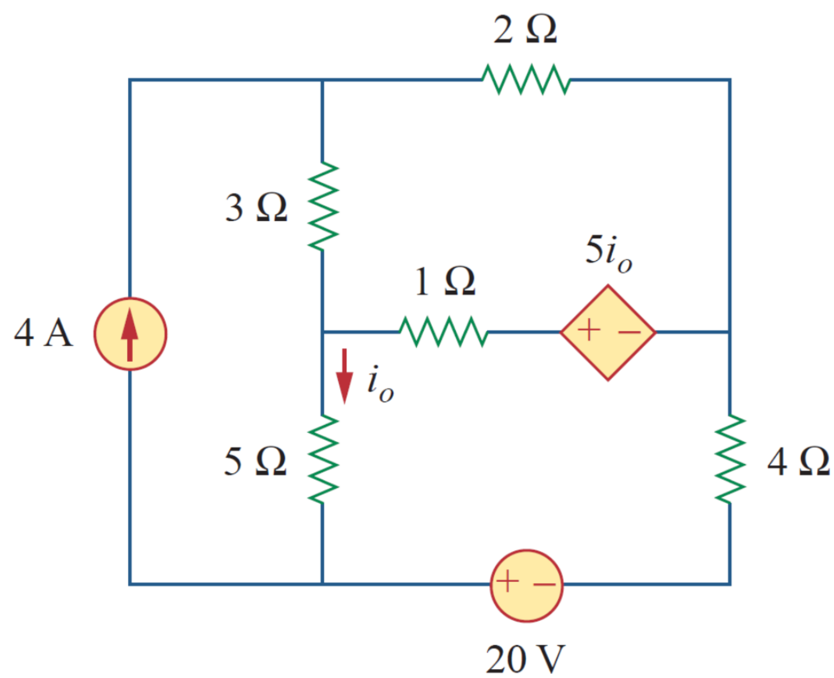
$$-v_x + 4i + v_x + 18 = 0 \quad \Rightarrow \quad i = -4.5 \text{ A}$$

Relating v_x and i:

$$-3 + 1i + v_x = 0 \quad \Rightarrow \quad v_x = 3 - i$$

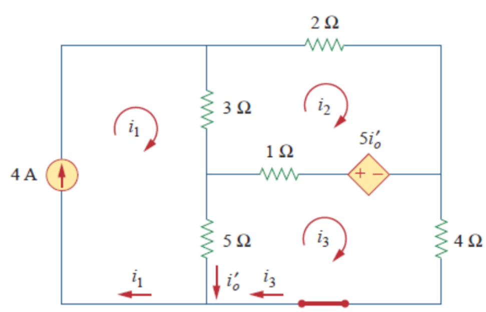
$$v_x = 3 - i = 7.5 \text{ V.}$$

Problem 9. (**12 pts**) For the circuit given below, determine the current i_o using superposition principle. You can use any of the techniques (nodal analysis, loop analysis and source transformation) to carry out analysis when you keep one independent source in the circuit.



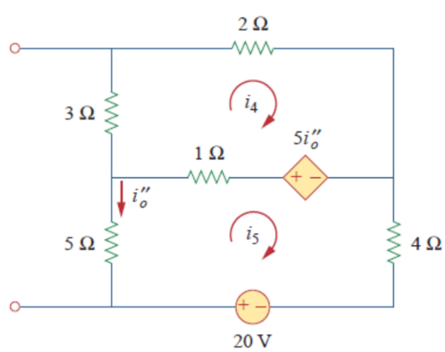
Solutions:

Keeping 4A source:



$$i_o' = \frac{52}{17}\text{ A}$$

Keeping 20V source:



$$i_o'' = -\frac{60}{17}\text{ A}$$

Overall using superpositon principle:

$$i_o = i_o' + i_o'' = -\frac{8}{17}A$$