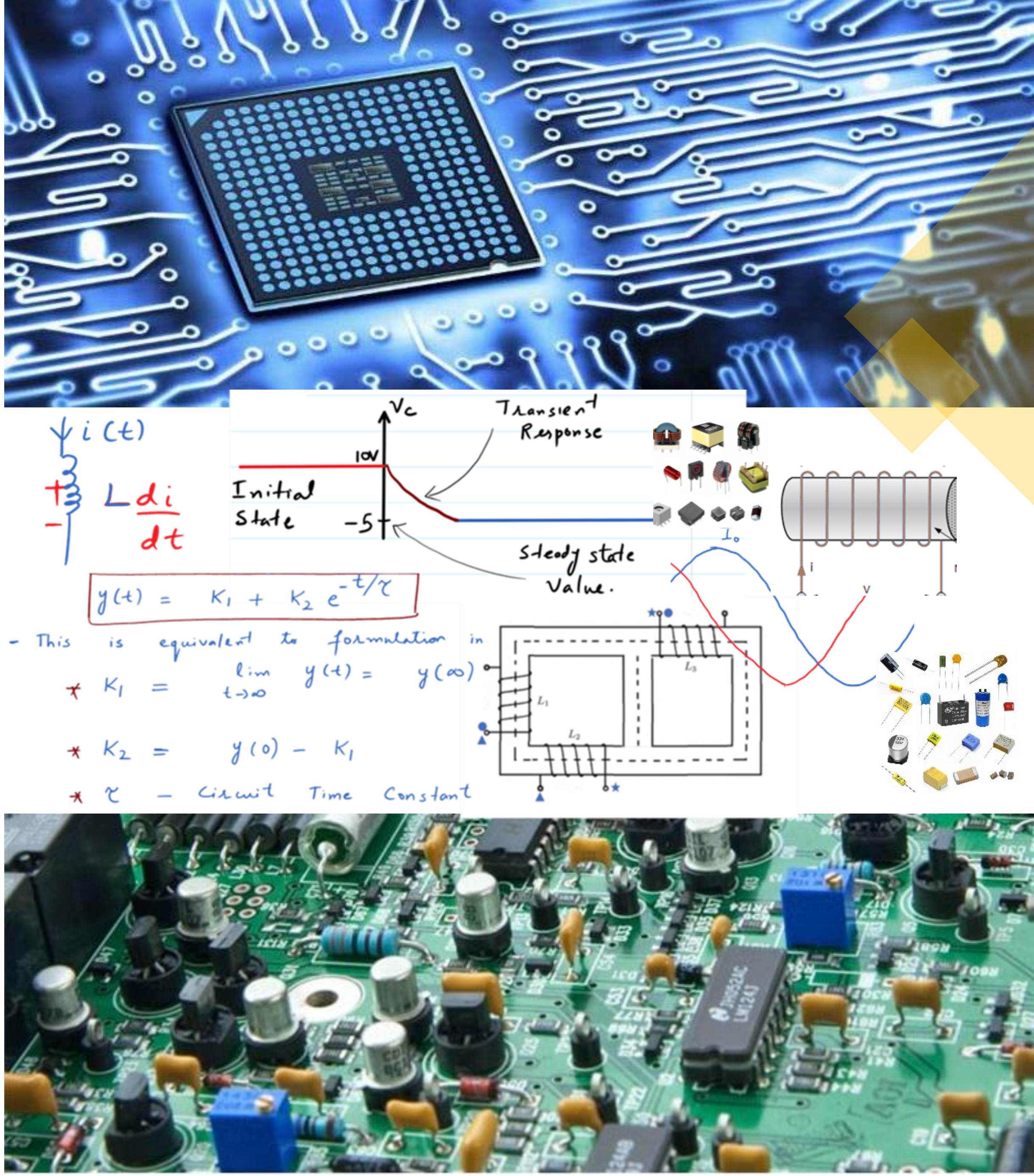
EE 240 Circuits I

Dr. Zubair Khalid

Department of Electrical Engineering
School of Science and Engineering
Lahore University of Management Sciences

https://www.zubairkhalid.org/ee240_2020.html

- Thevenin's Theorem and Norton's Theorem
- Equivalent Circuit



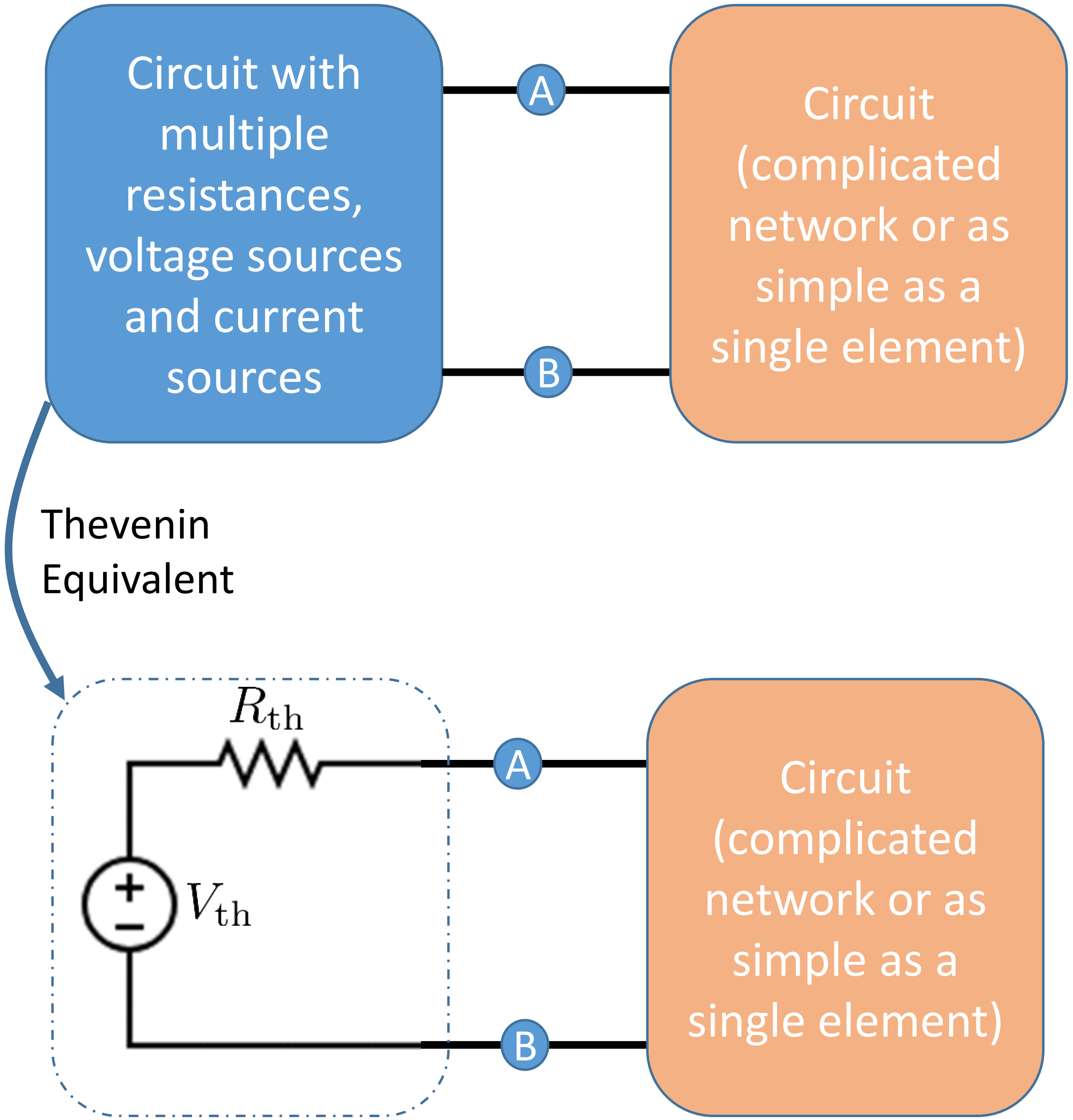
Thevenin's Theorem

Overview

Thevenin theorem is used to change a complicated circuit into a simple equivalent circuit consisting of a single voltage source, referred to as Thevenin voltage V_{th} in series with a single resistance, referred to as Thevenin Resistance R_{th}.

Thevenin Theorem Statement:

Any circuit containing only resistances, voltage sources, and current sources be replaced at the terminals A-B by an equivalent combination of a voltage source V_{th} in a series connection with a resistance R_{th}.



Thevenin's Theorem

How to Obtain Thevenin Equivalent

Key Idea: Use the concept of equivalence: same current and voltage characteristics across terminals A-B.

Only keep the circuit for which we want to find the equivalent circuit; disconnect the rest of the circuit.

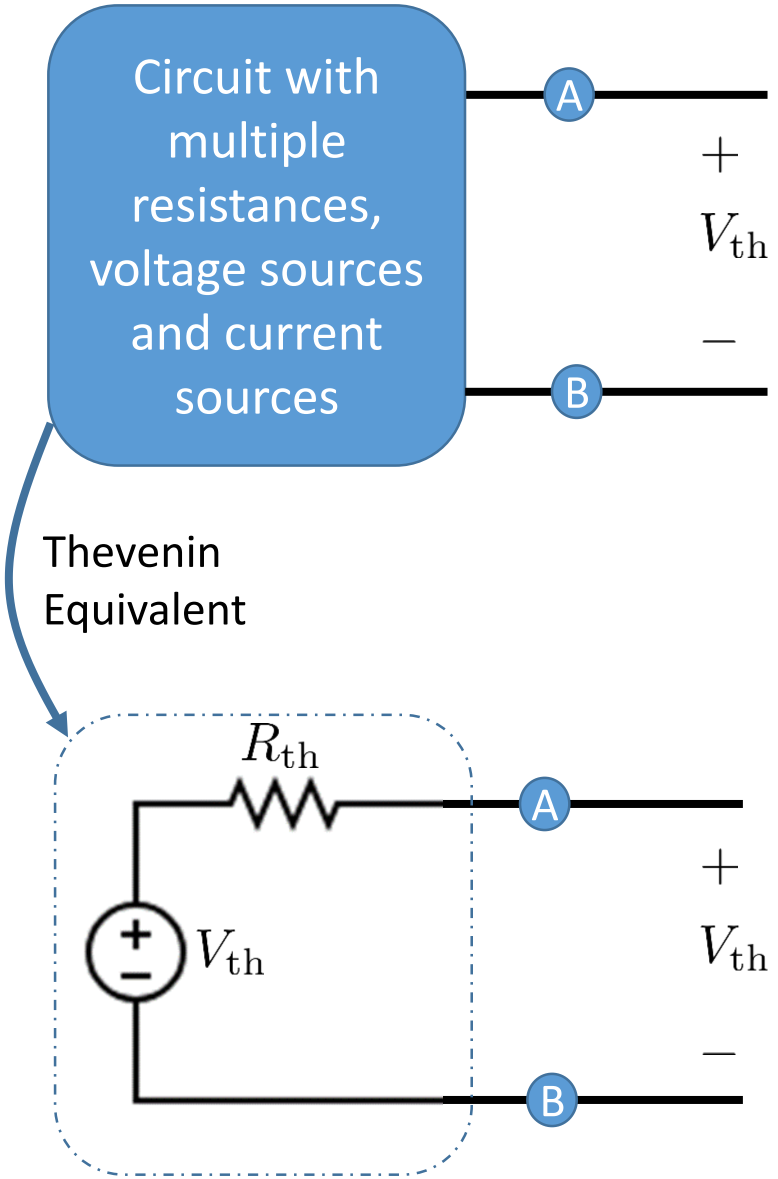
Determine V_{th}:

As no current is flowing from A to B,
V_{th} is simply a voltage across terminals A-B.

We can determine V_{th} by analysisng the circuit inside blue box and determine the voltage across terminals A-B.

We will learn different methods to obtain Thevenin Resistance

Method 1:



Thevenin's Theorem

How to Obtain Thevenin Equivalent

Determine R_{th}:

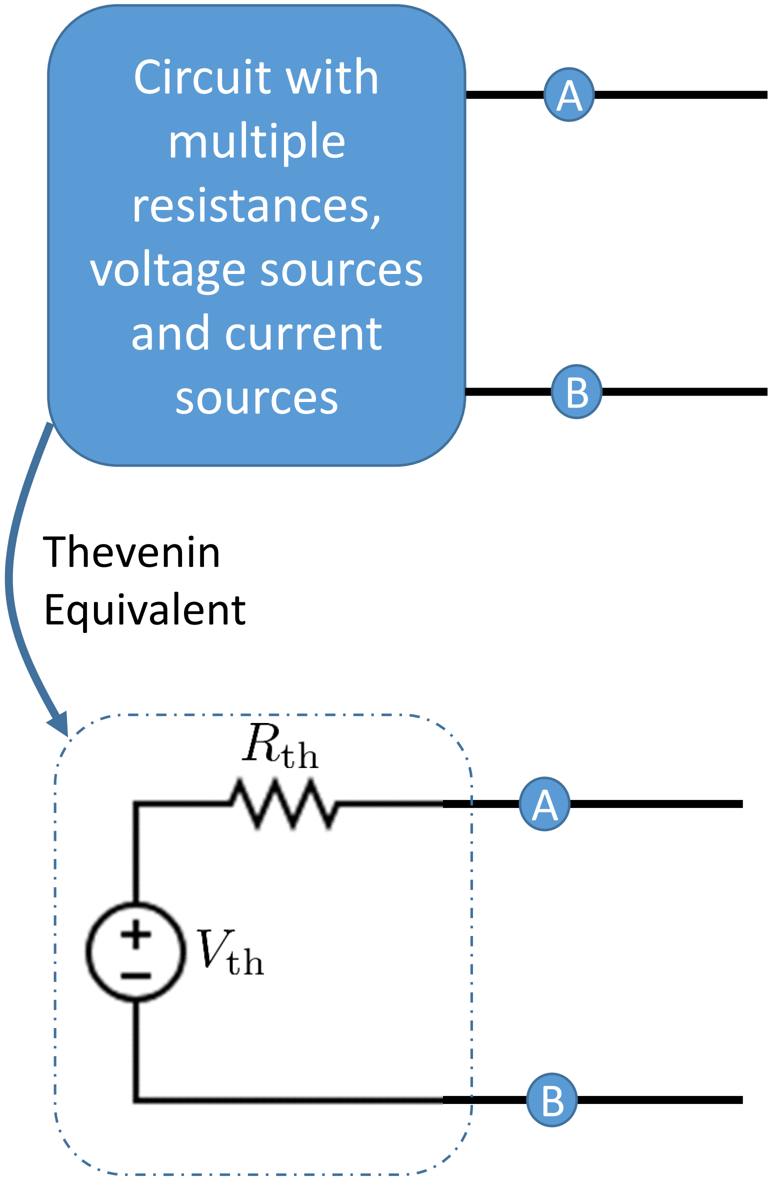
Method 1:

- Switch off the independent sources and determine the equivalent resistance across terminals A and B.

Method 2:

- Short-circuit terminals A and B and determine the current flowing from A to B, referred to as I_{SC}.
- Using this current, we can determine R_{th} as

$$R_{\text{th}} = \frac{V_{\text{th}}}{I_{\text{sc}}}$$



Thevenin's Theorem

How to Obtain Thevenin Equivalent

Determine R_{th}:

Method 3:

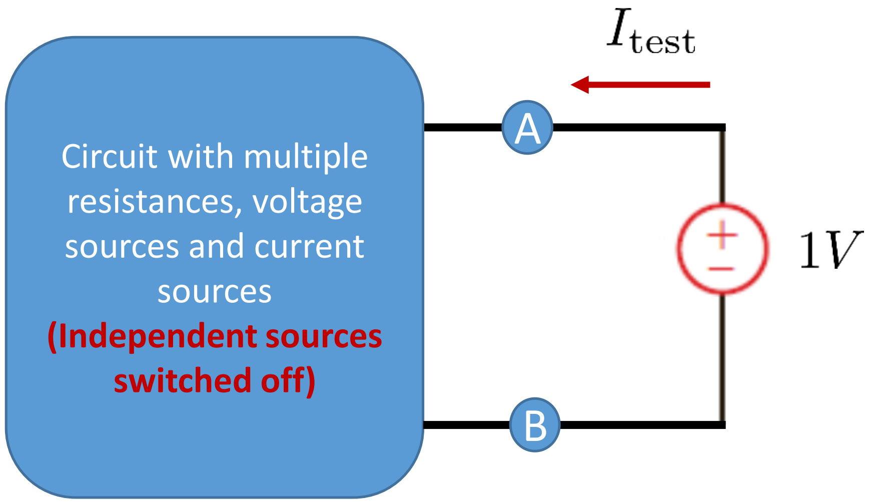
- Switch off the independent sources
- Connect a test source across terminals A-B

- **If 1V (known) voltage test-source is connected:**

Determine the current I_{test} supplied by voltage source.

We can determine R_{th} as follows:

$$R_{\text{th}} = \frac{1}{I_{\text{test}}}.$$

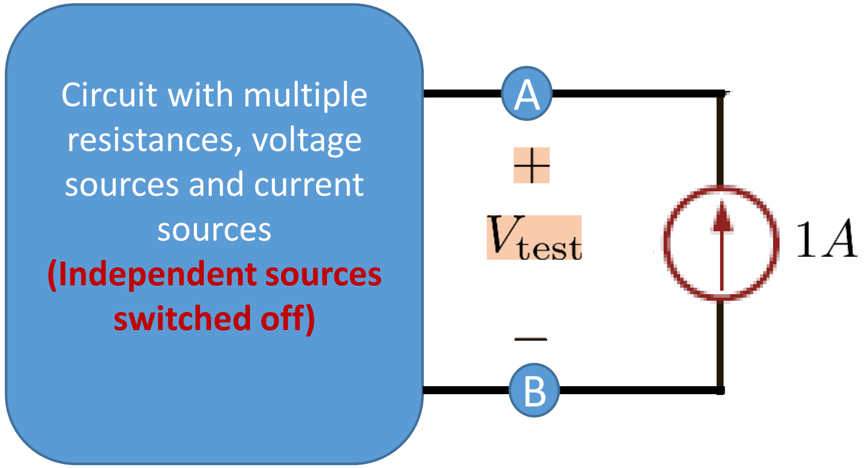


- **If 1A (known) current test-source is connected:**

Determine the voltage V_{test} developed across the current source.

We can determine R_{th} as follows:

$$R_{\text{th}} = \frac{V_{\text{test}}}{1}.$$



Thevenin’s Theorem

How to Obtain Thevenin Equivalent

Determine R_{th}:

Which method to use?

Independent sources	Dependent sources	Method – can be used	Justification
✓	✗	Methods 1, 2 and 3	
✓	✓	Methods 2 and 3	Method 1 cannot be used due to the presence of dependent sources
✗	✓	Method 3	No independent source driving V_{th} or I_{SC}
✗	✗	Methods 1 and 3	No independent source driving V_{th} or I_{SC}

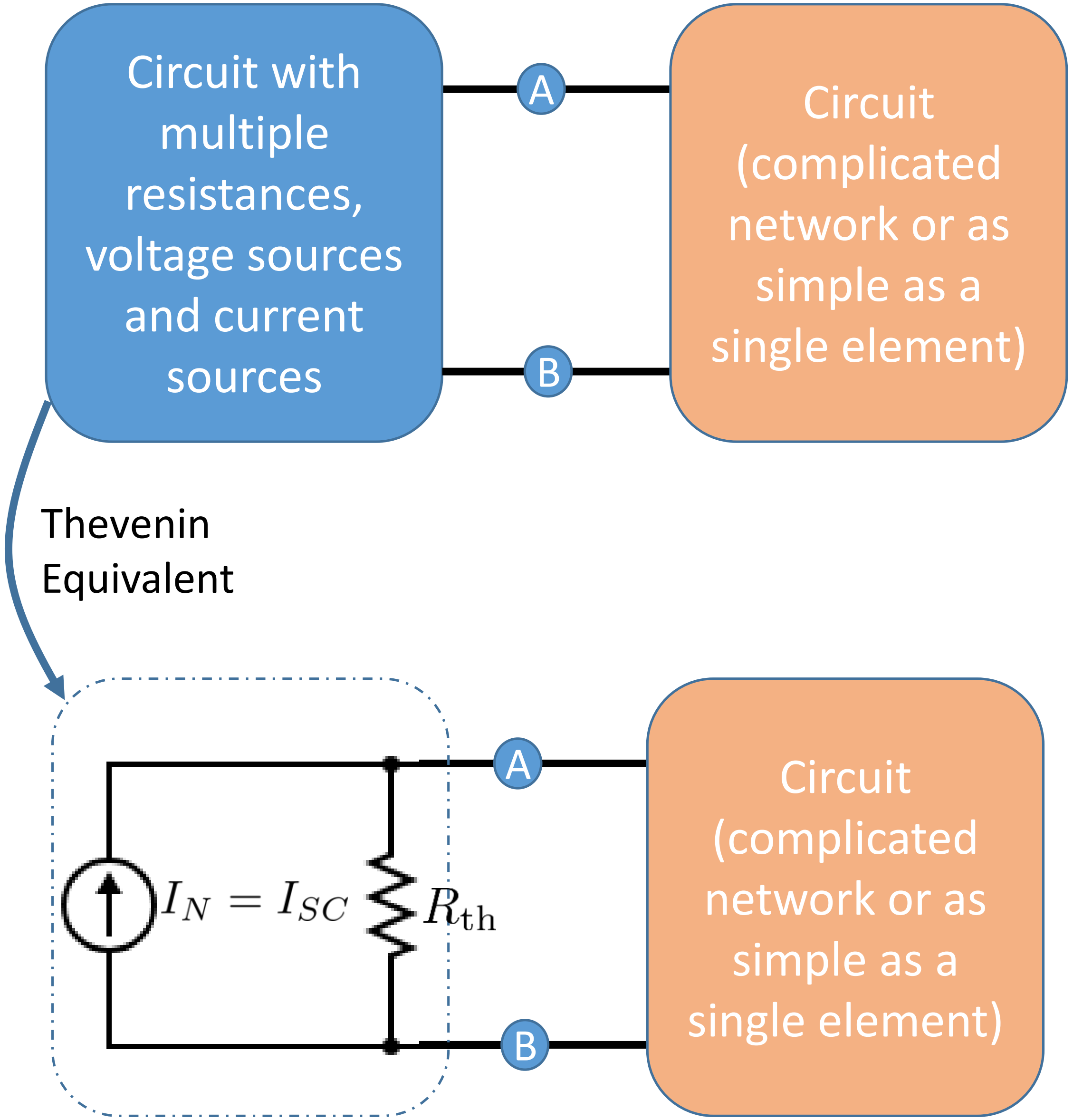
Note: Equivalent circuit does not have the voltage source if there is no independent source in the circuit.

Norton's Theorem

Norton theorem is used to change a complicated circuit into a simple equivalent circuit consisting of a single current source, referred to as Norton current voltage I_N in parallel with a single resistance (the same as Thevenin Resistance R_{th} , explained below).

- *The value of the Norton current is one that flows from terminal A to B when the two terminals are shorted together. This is in fact I_{sc}, that is short-circuit current.*

- *The resistance represents the resistance looking back into the terminals when source is switched off. This is in fact Thevenin Resistance.*

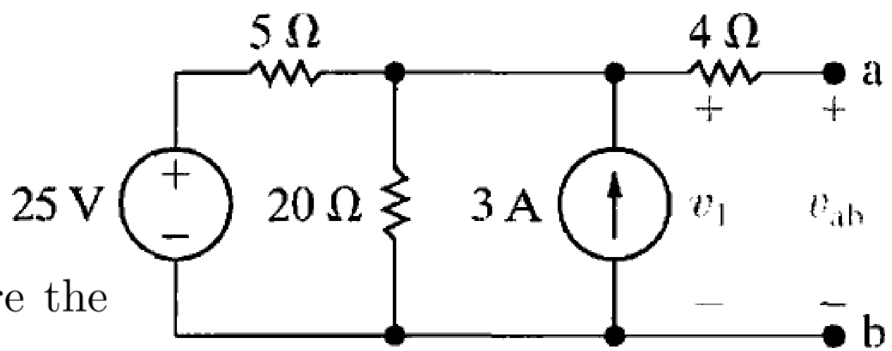


Connection with the Thevenin's Theorem

The source transformation of Thevenin's equivalent yields Norton's equivalent and vice versa.

Thevenin's Theorem

Example 1: Find the Thevenin's equivalent circuit for the following circuit across terminals a-b.

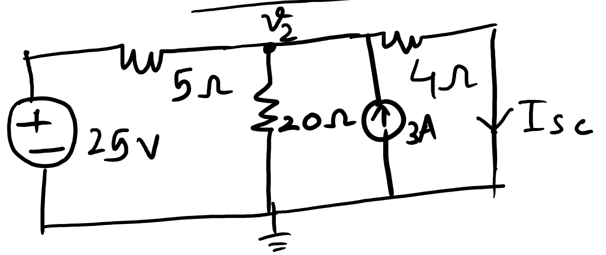


Taking bottom node as ground node and applying KCL at a node where the voltage is v_1

$$* \quad \frac{v_1 - 25}{5} + \frac{v_1}{20} = 3$$

$$\frac{v_1}{5} + \frac{v_1}{20} = 8 \Rightarrow \boxed{v_1 = 32V}$$

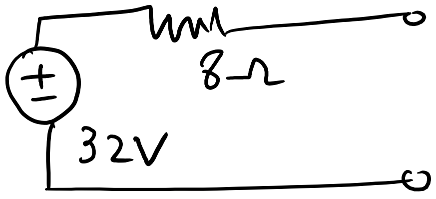
R_{th}: Find I_{sc}:



$$-3 + \frac{v_2 - 25}{5} + \frac{v_2}{20} + \frac{v_2}{4} = 0$$

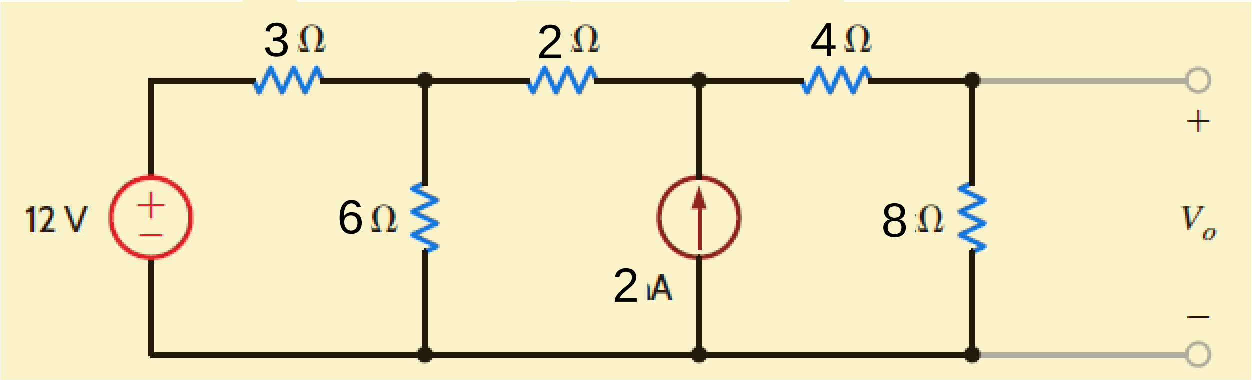
$$v_2 = 16V \Rightarrow \boxed{I_{sc} = 4A}$$

$$R_{th} = \frac{v_{th}}{I_{sc}} = 8\Omega$$



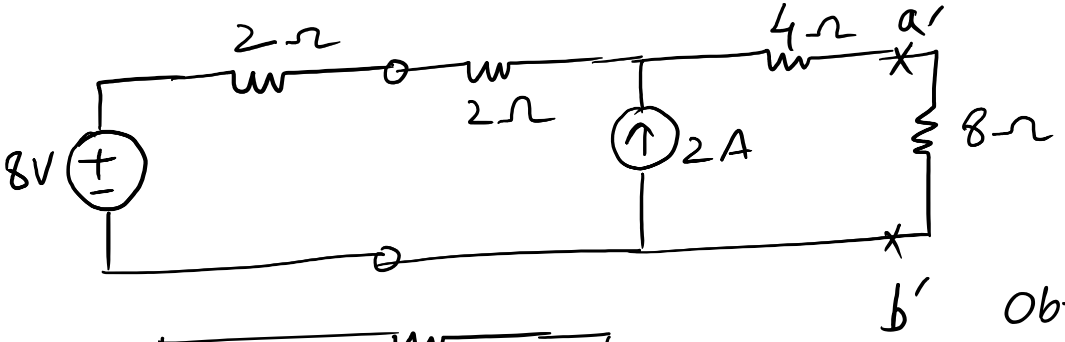
Thevenin's Theorem

Example 2: Find V_o using Thevenin's theorem.

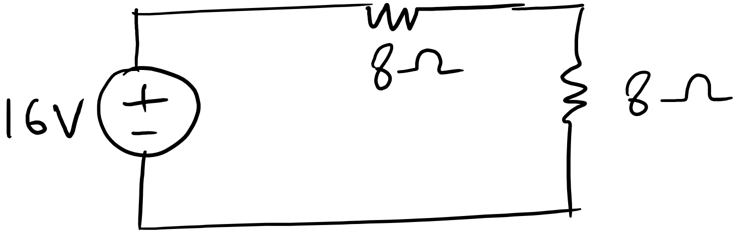


We will use Thevenin's theorem twice here.

Obtaining Thevenin equivalent across terminals a-b:



Obtaining Thevenin equivalent across terminals a'-b':

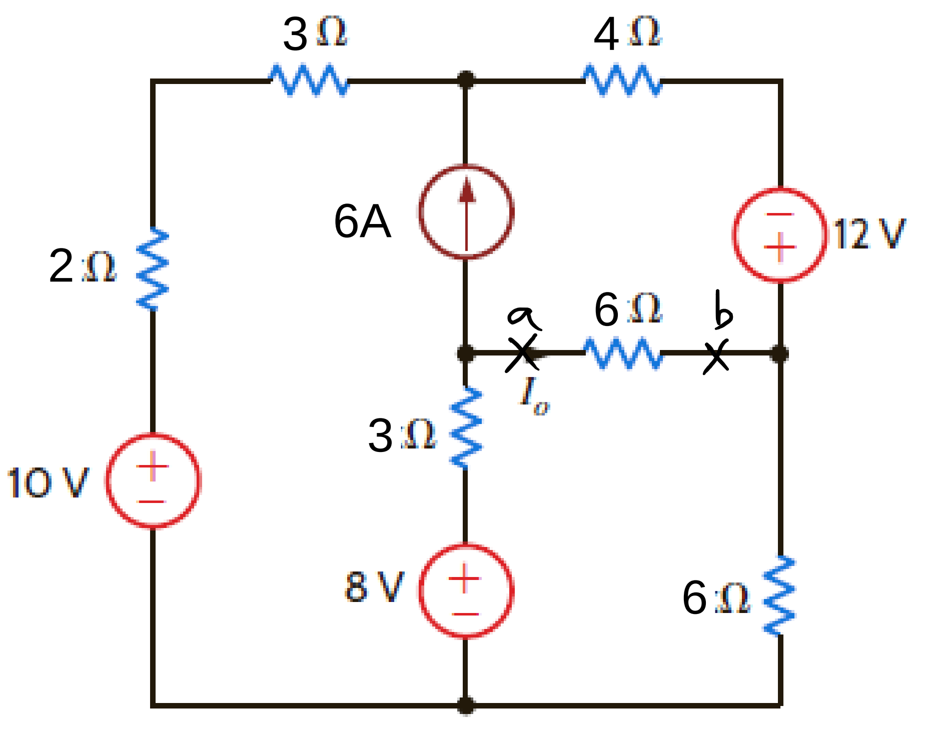


$$V_o = 16 \times \frac{8}{16} = \boxed{8V}$$

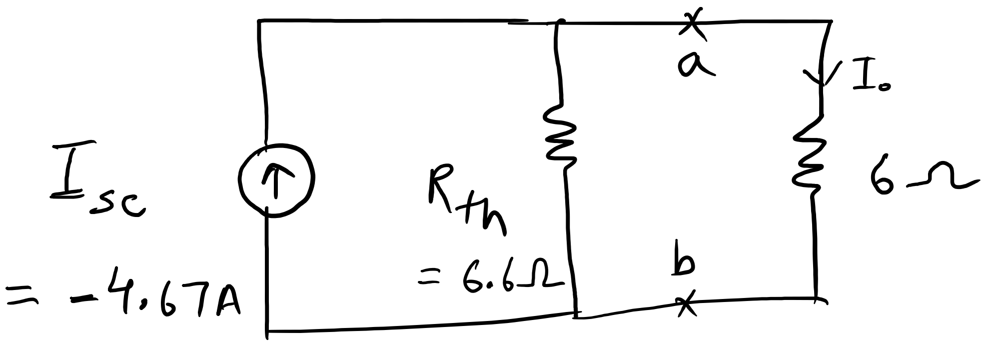
Thevenin's and Norton's Theorems

Problems – In class

Example 3: Find I_o using Thevenin's or Norton's theorem



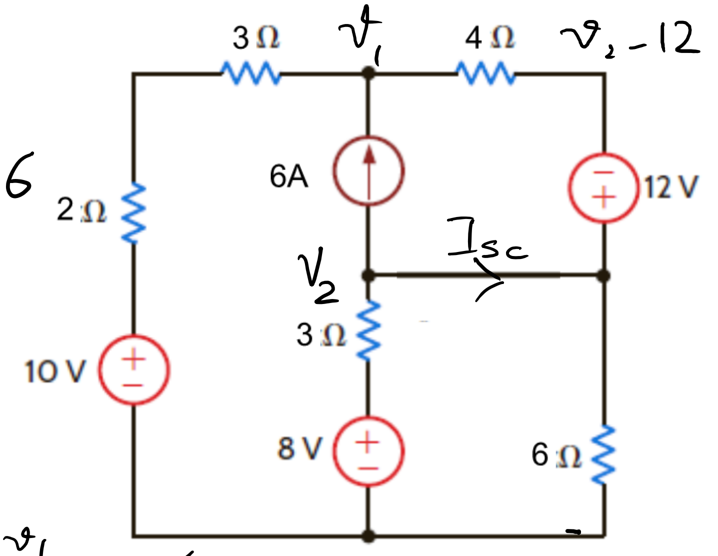
Equivalent: (computed on next page)



$$I_o = \frac{6.6}{12.6} \times (-4.67) = \boxed{-2.44A}$$

Thevenin’s and Norton's Theorems

Problems – In class



$$\frac{v_1 - 10}{5} + \frac{v_1 - (v_2 - 12)}{4} = 6$$

$$\boxed{9v_1 - 5v_2 = 100}$$

$$\frac{v_2 - 8}{3} + \frac{v_2}{6} + \frac{v_2 - 12 - v_1}{4} = -6$$

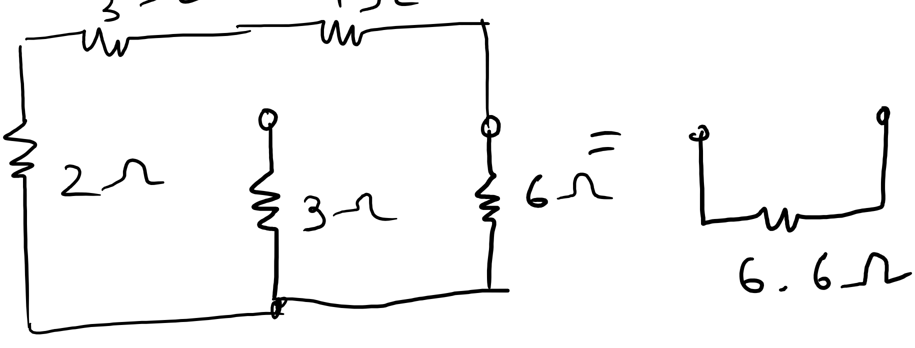
$$\Rightarrow \frac{4v_2 - 24 + 2v_2 + 3v_2 - 36 - 3v_1}{12} = -6$$

$$\boxed{-3v_1 + 9v_2 = -4} \Rightarrow -9v_1 + 27v_2 = -12$$

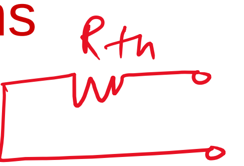
$$\Rightarrow 22v_2 = 88 \Rightarrow \boxed{v_2 = 4V}$$

$$\frac{v_2 - 8}{3} + 6 + I_{sc} = 0 \Rightarrow \boxed{I_{sc} = -4.667\,A}$$

R_{th} :

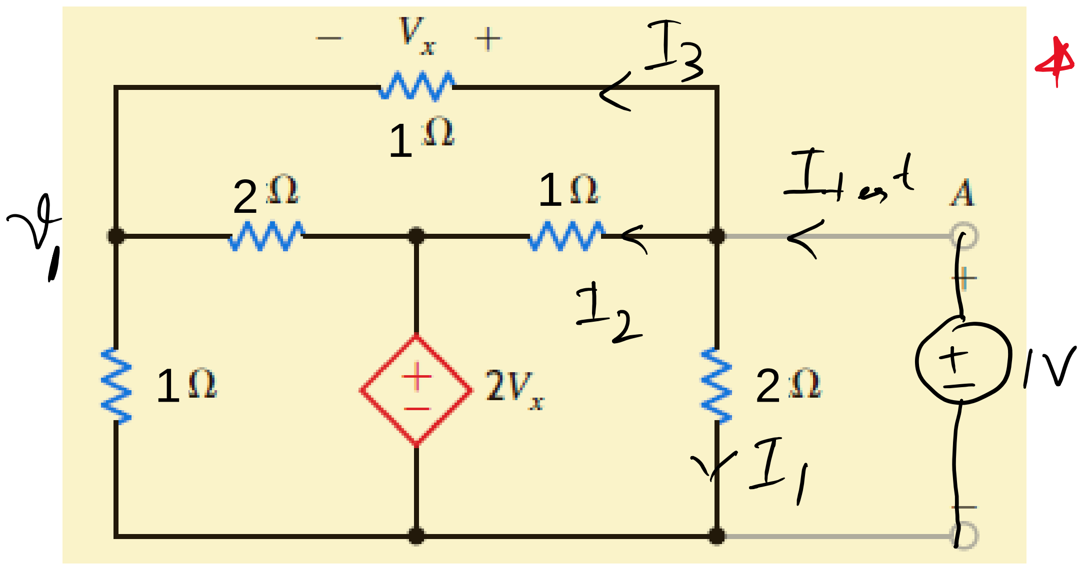


Thevenin’s and Norton's Theorems



Problems – In class

Example 4: Find the Thevenin equivalent circuit for the following circuit with respect to the terminals AB (Irwin – Example 5.8)



$* \; V_{th} = 0V$

We have

$$v_x + v_1 = 1V \quad \Rightarrow \quad v_1 = 1 - v_x$$

KCL ;

$$\frac{v_1}{1} + \frac{v_1 - 2v_x}{2} + \frac{v_1 - 1}{} = 0$$

$$\Rightarrow \quad 1 - v_x + \frac{1 - 3v_x}{2} - v_x = 0$$

$$\Rightarrow \quad 3 = 7v_x \quad \Rightarrow \quad \boxed{v_x = \frac{3}{7} V}$$

$$I_1 = \frac{1}{2} A, \quad I_2 = \frac{1 - 2(3/7)}{1} = 1/7 \, A$$

$$I_3 = 3/7 \, A$$

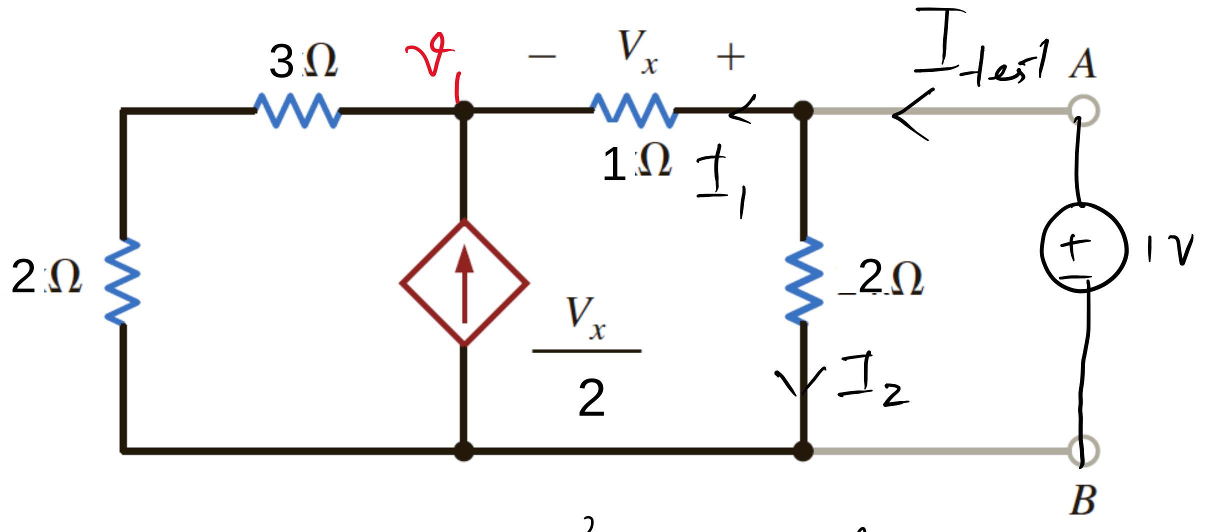
$$I_{test} = I_1 + I_2 + I_3 = \boxed{\frac{15}{14} A}$$

$$\Rightarrow R_{th} = \frac{1}{I_{test}} = \frac{14}{15} \Omega$$

Thevenin's and Norton's Theorems

Problems – In class

Example 5: Find the Thevenin equivalent circuit for the following circuit with respect to the terminals AB (Irwin – E 5.13)



$$\frac{\vartheta_1}{5} + \frac{\vartheta_1 - 1}{1} = \frac{V_x}{2}$$

$$\vartheta_x + \vartheta_1 = 1 \Rightarrow \vartheta_x = 1 - \vartheta_1$$

$$\Rightarrow \boxed{\vartheta_1 = \frac{15}{17}\ V}$$

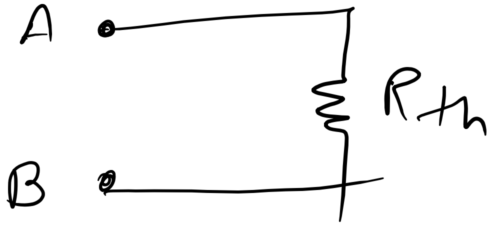
$$I_{test} = I_1 + I_2 = \frac{1}{2} + \frac{2}{17} = \frac{21}{34}\ A$$

$$I_1 = \frac{1 - \vartheta_1}{1} = \frac{2}{17}\ A$$

$$I_2 = \frac{1}{2}$$

$$\Rightarrow R_{th} = \frac{1}{I_{test}} = \frac{34}{21}\ \Omega$$

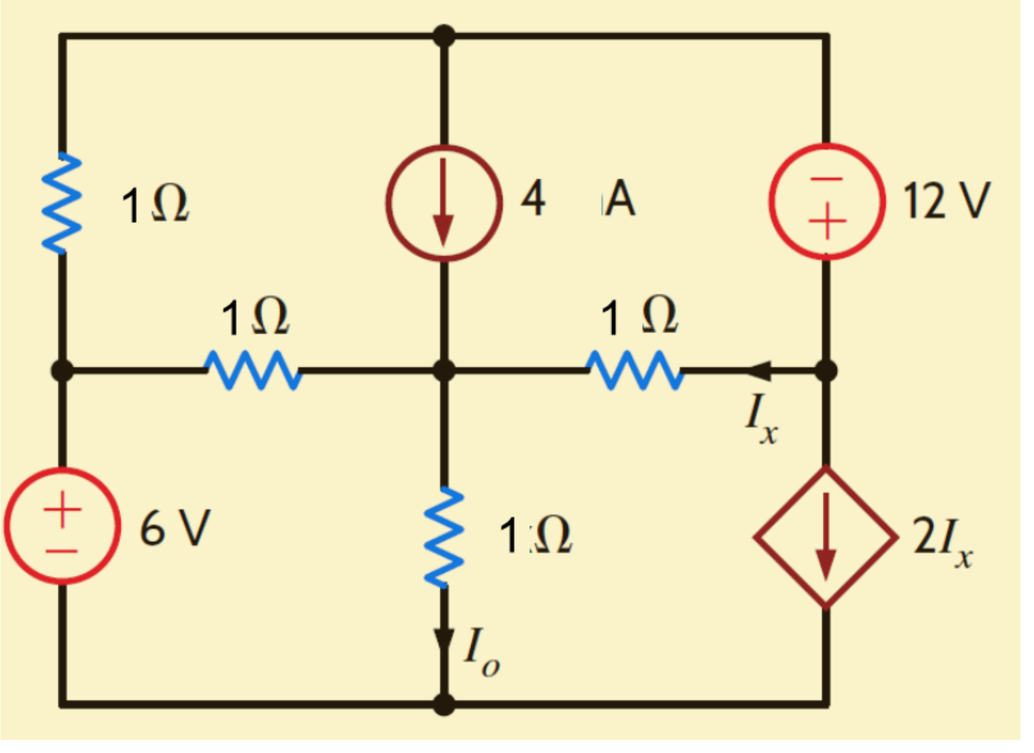
Equivalent :-



Thevenin’s and Norton's Theorems

Problems – In class

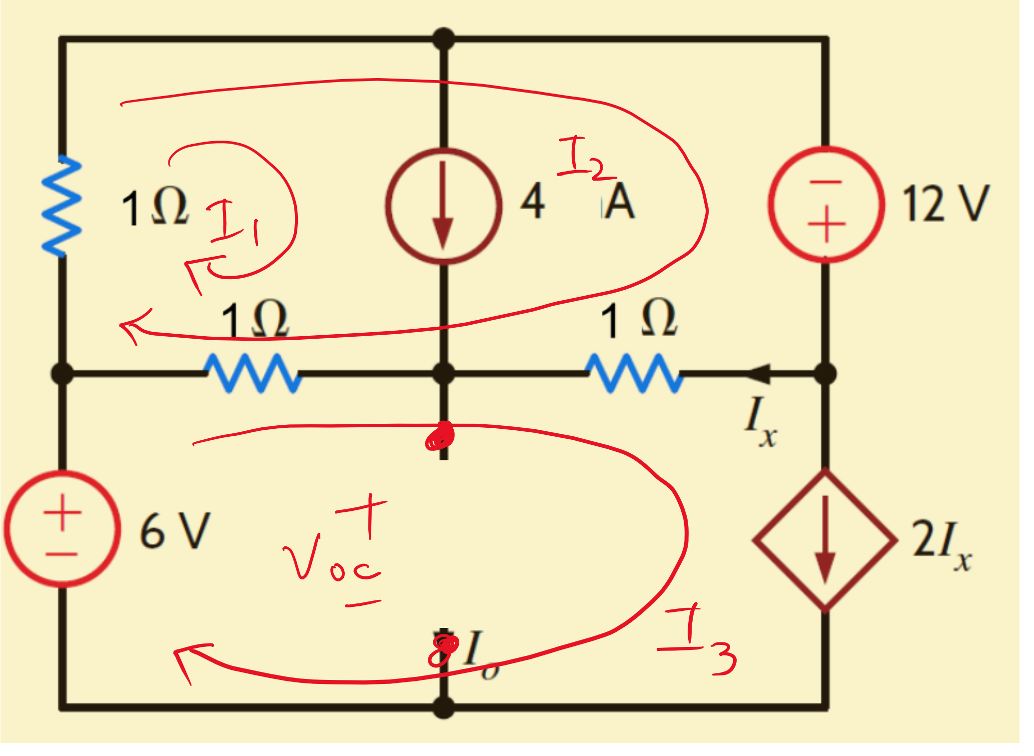
Example 6: Find I_o using Thevenin’s theorem or Norton’s theorem



Thevenin's and Norton's Theorems

Problems – In class

Determine V_{oc} first:



$I_1 = 4A, \quad I_3 = 2I_x$

Loop 2 $\quad -12 + 1(I_2 - I_3) + 1(I_1 + I_2 - I_3) + (I_1 + I_2)1 = 0$

$I_x = I_2 - I_3 \Rightarrow \boxed{3I_x = I_2}$

$\boxed{\text{Solving}}$

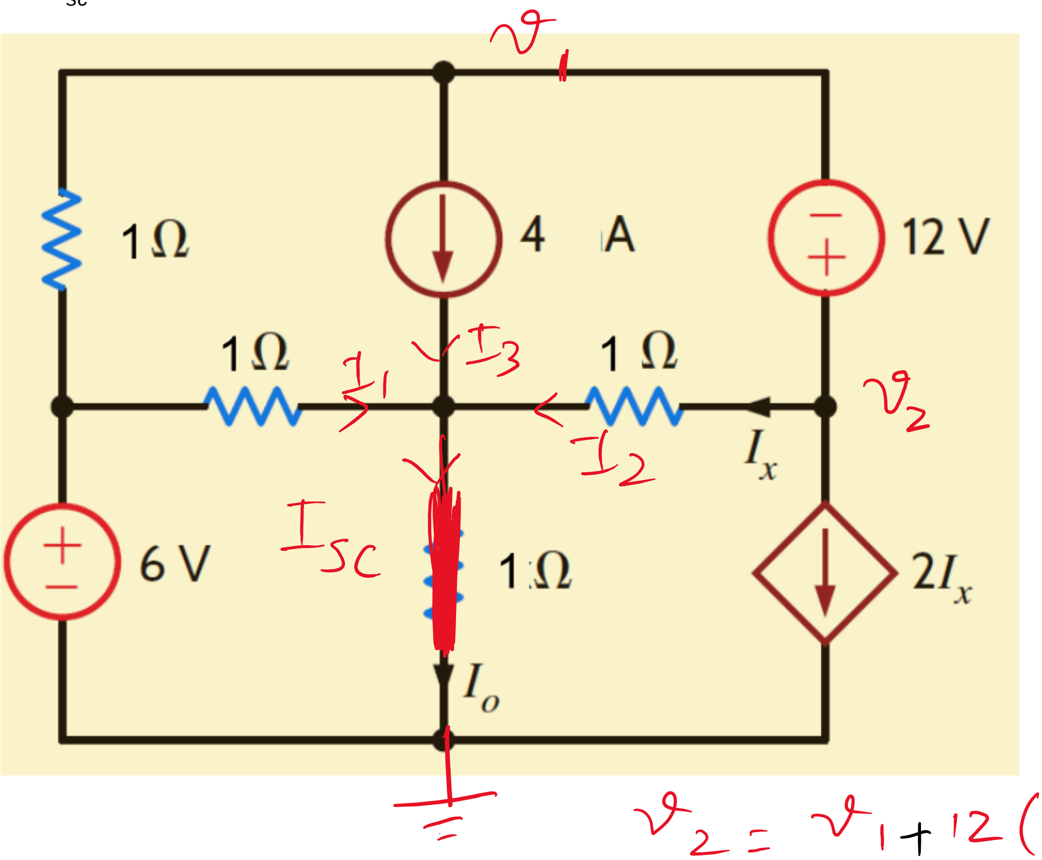
$I_3 = \frac{8}{5}A, \quad I_2 = \frac{12}{5}A, \quad I_x = \frac{4}{5}A$

$V_{oc} = 6 + (I_1 + I_2 - I_3)1 = \frac{54}{5}V$

Thevenin's and Norton's Theorems

Problems – In class

Determine I_{sc}



$v_2 = v_1 + 12$ (Super Node)

$$\frac{v_1 - 6}{1} + 4 + \frac{v_2}{1} + 2I_x = 0 \quad \Bigg\} \quad I_x = \frac{v_2}{1}$$

$\Rightarrow v_1 - 6 + 4 + 3v_1 + 36 = 0$

$\Rightarrow v_1 = -\frac{17}{2}$ V, $v_2 = 7/2$

$$I_{sc} = \frac{6}{1} + \frac{7}{2} + 4 = \frac{27}{2} \text{ A}$$

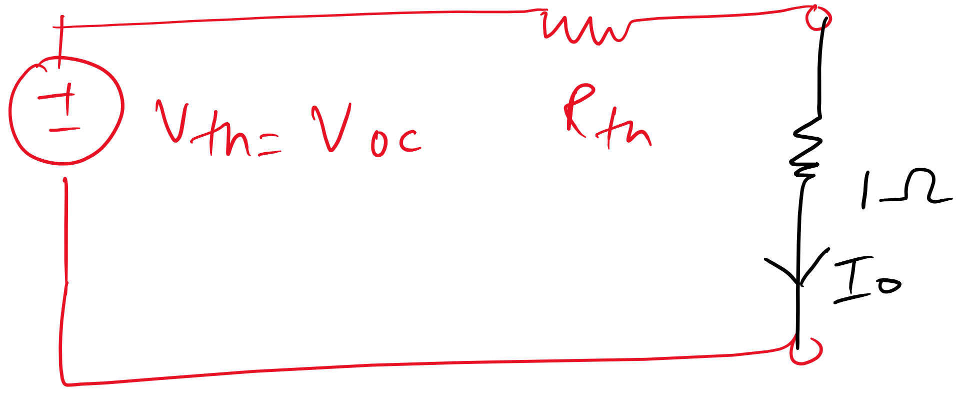
$$\Rightarrow R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{54/5}{27/2} = \frac{4}{5} \, \Omega$$

Thevenin’s and Norton's Theorems

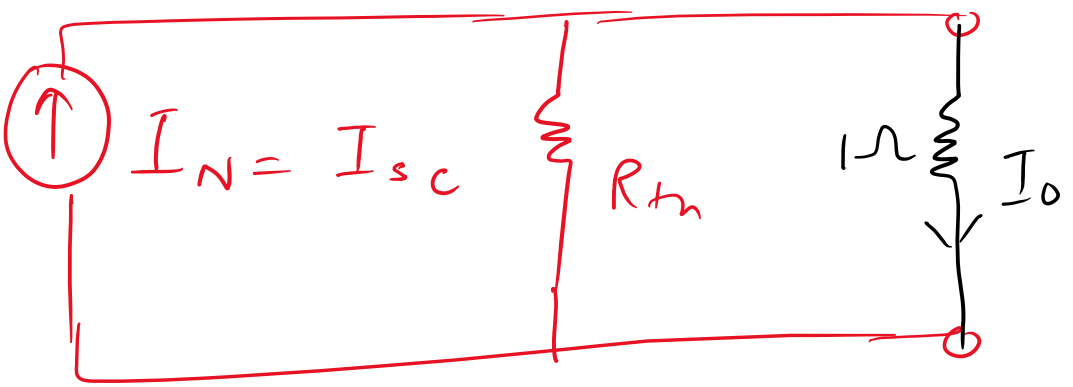
Problems – In class

Using equivalent circuits to determine I_o

Thevenin Equivalent



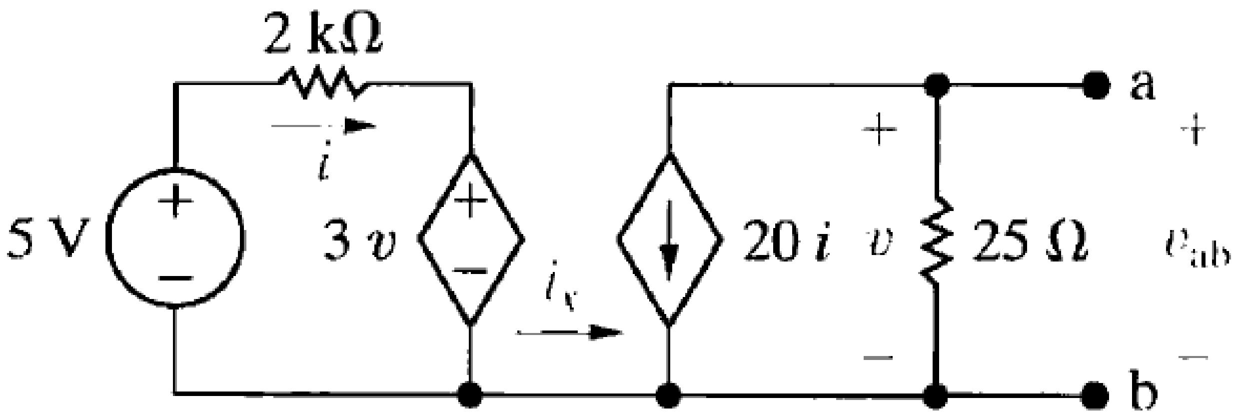
Norton Equivalent



$$I_o = \frac{54/5}{1+\frac{4}{5}} = \boxed{6\ A}$$

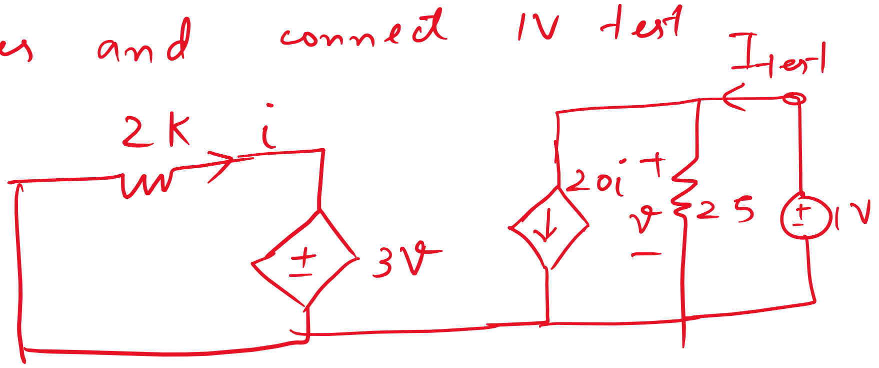
Problems – In class

Example 7: Find the Thevenin equivalent circuit for the following circuit with respect to the terminals a,b



First find R_{th} :-

Switch off sources and connect 1V test source



$v = 1V$

$3v = 3V$

$i = -3/2k$

$\Rightarrow \quad I_{test} = \frac{1}{25} - (20)(i) = 0.04 - 0.03 = 0.01A$

$\Rightarrow \boxed{R_{th} = 100\Omega} \leftarrow \frac{1}{I_{sc}}$

V_{th} :- $V_{ab} = v = v_{th}$

$V_{th} = -20i \times 25$ — ①

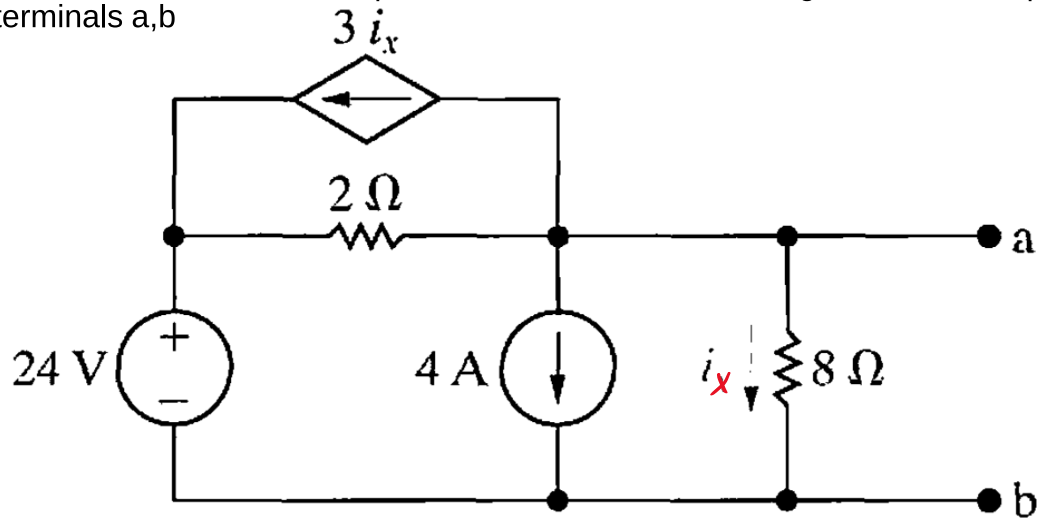
$i = \frac{5 - 3V_{th}}{2k}$ — ②

① + ② $\Rightarrow \boxed{V_{th} = -5V}$

Thevenin's and Norton's Theorems

Problems – In class

Example 8: Find the Thevenin equivalent circuit for the following circuit with respect to the terminals a,b



* Circuit contains both dependent and Independent Sources; we can use either of the following techniques
1) Determine V_{ab} and I_{sc}
2) Determine V_{ab} ; Determine R_{th} by switching off independent sources and applying test current (or voltage) source at a–b.

Let's apply 2)

V_{ab} : Using KCL;

$$\frac{V_{ab}}{8} + \frac{V_{ab}-24}{2} + 3\,i_x + 4 = 0$$

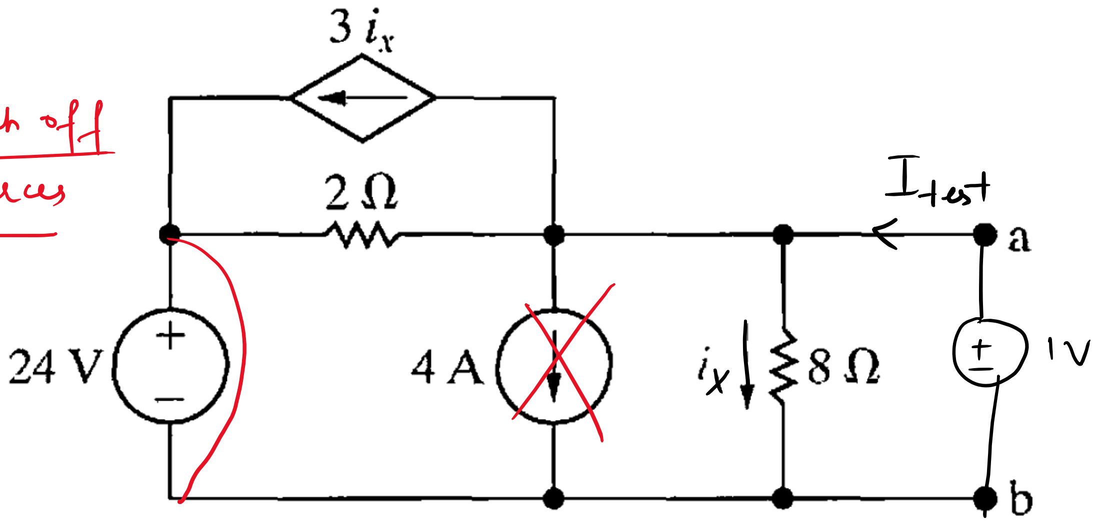
where $i_x = \dfrac{v_{ab}}{8} \Rightarrow \boxed{v_{ab} = 8\,i_x}$

Solving $\boxed{v_{ab} = 8V}$

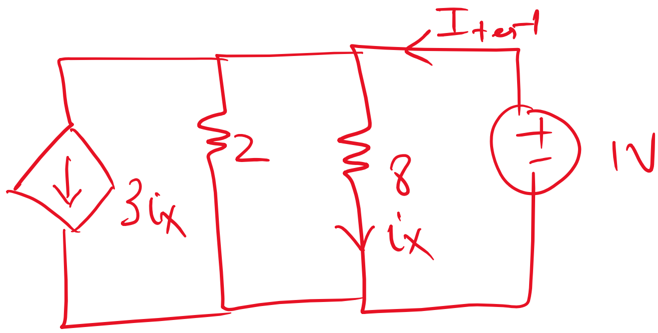
10

R_{th} : Apply 1V voltage source :

Switch off Sources



$i_x = \frac{1}{8}$ A



$$I_{test} = \frac{1}{8} + \frac{1}{2} + \frac{3}{8}$$

$$= 8/8 = 1A$$

$$\Rightarrow \quad R_{th} = \frac{1V}{I_{test}} = \boxed{1\,\Omega}$$