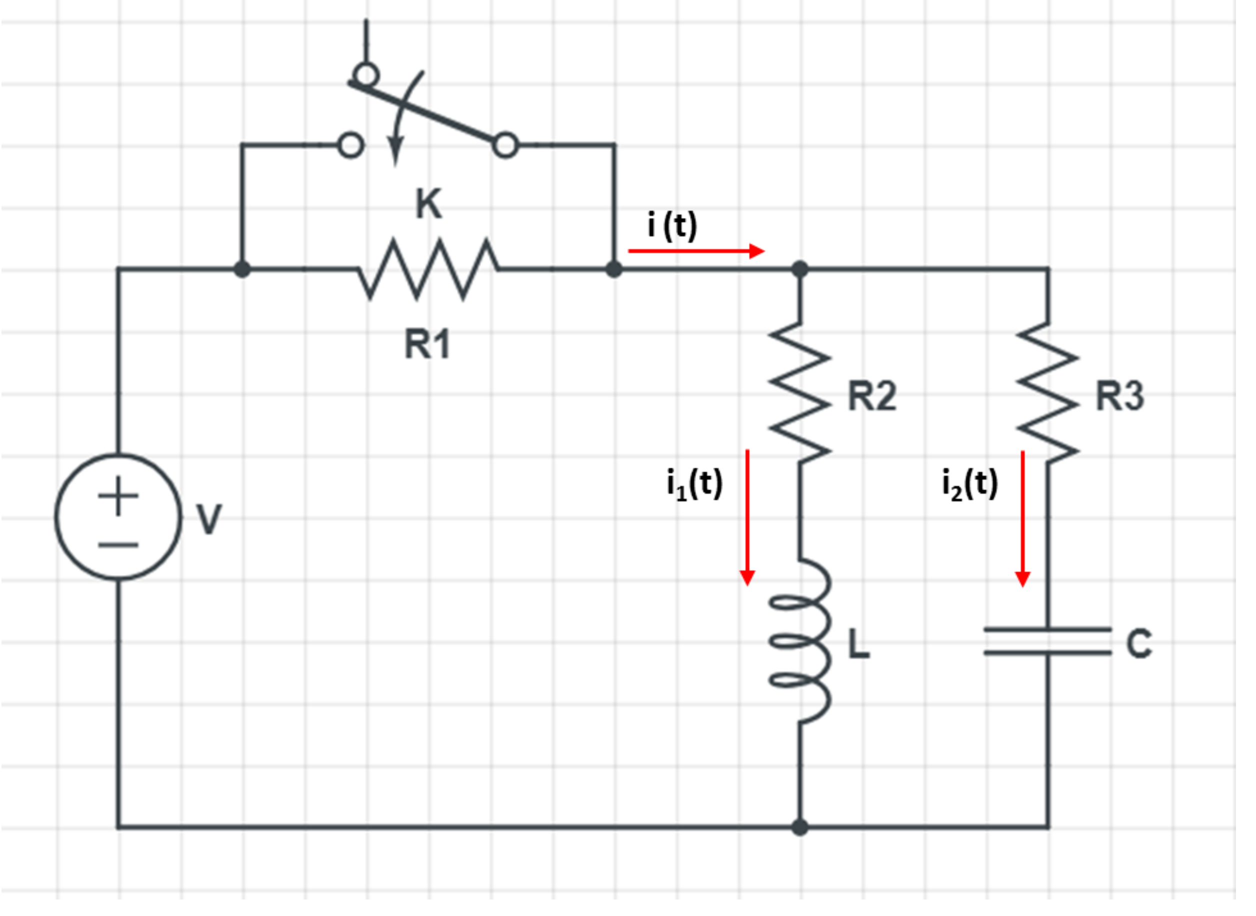
Evaluation of Initial Conditions

Problems – In class

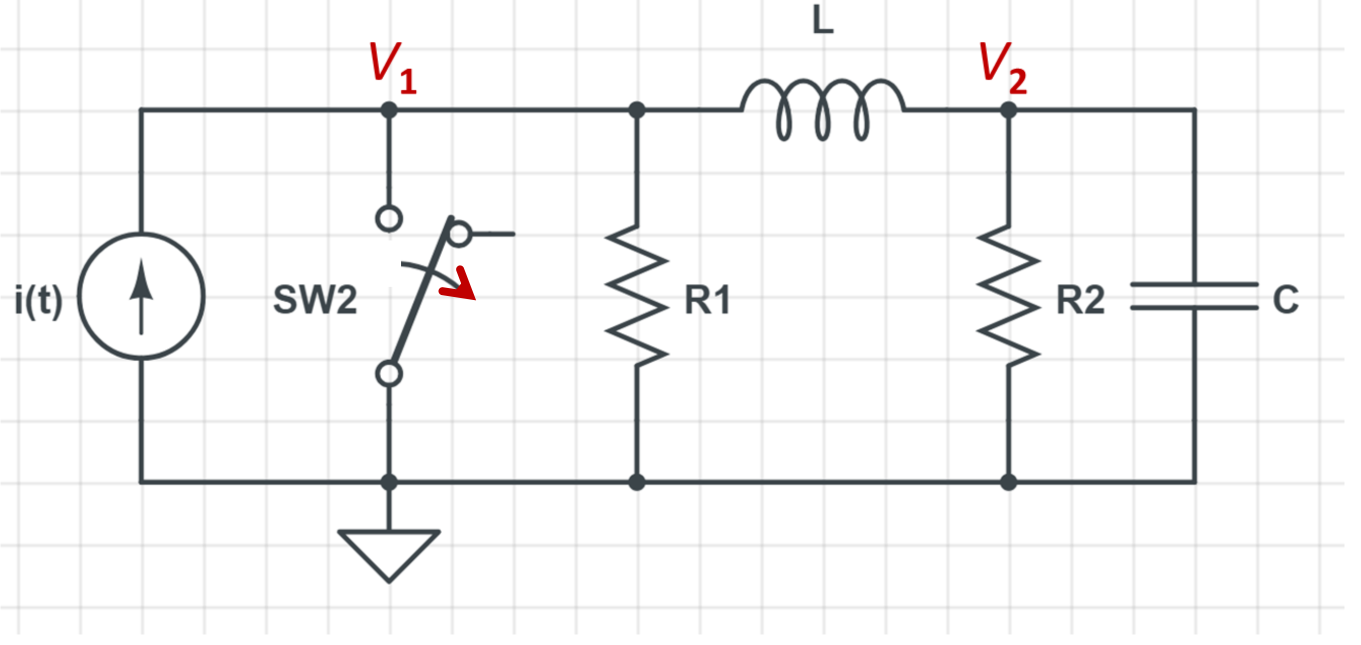
Problem 1 (5-20): In the circuit below, we have $R_1 = 10\Omega$, $R_2 = R_3 = 20\Omega$, $L = 1H$ and $C = 1\mu F$. Assume that the steady state is reached with switch K open. At time $t = 0$, the switch is closed. Determine $i_1(0^+)$, $i_2(0^+)$, $di_1/dt(0^+)$ and $di_2/dt(0^+)$.



Evaluation of Initial Conditions

Problems – In class

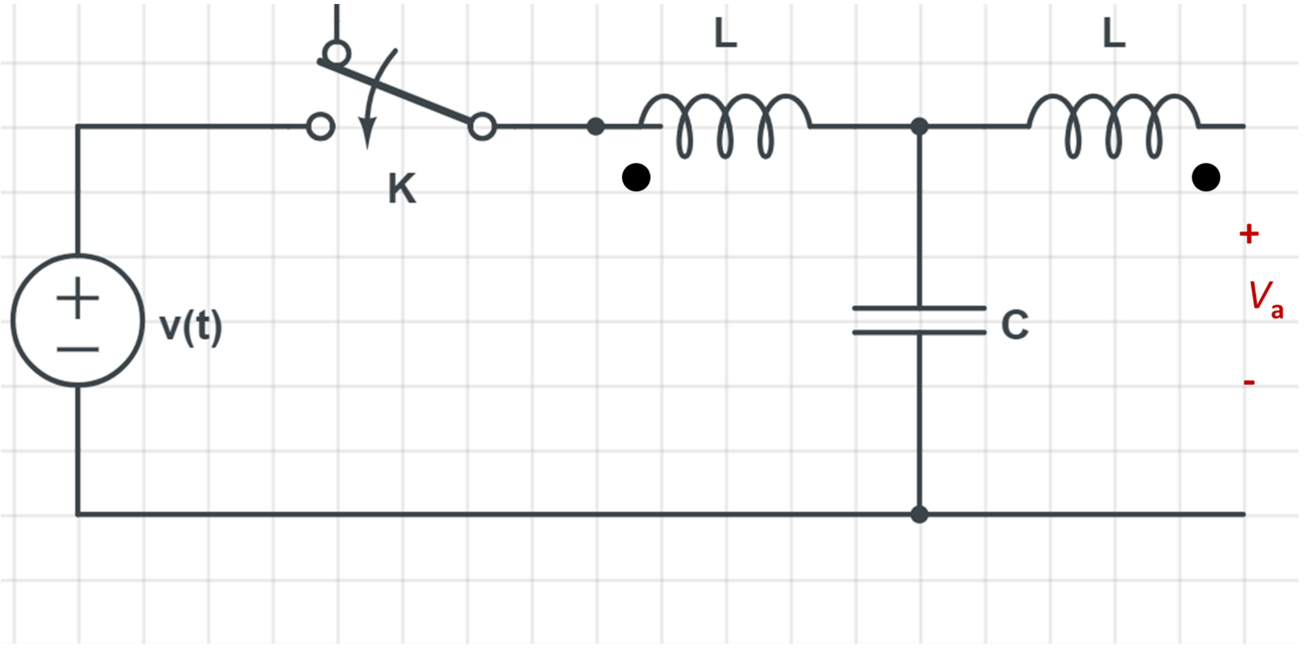
Problem 2 (5-21): In the circuit below, the steady state is reached with switch SW2 in closed state. At time $t = 0$, the switch is closed. Determine $V_1(0^+)$, $V_2(0^+)$, $dV_1/dt(0^+)$ and $dV_2/dt(0^+)$.



Evaluation of Initial Conditions

Problems – In class

Problem 3 (5-24): In the circuit below, the steady state is reached with switch K in opened state. At time $t = 0$, the switch is closed connected a voltage source $v(t) = V\sin(t/\sqrt{MC})$, where M denotes the mutual inductance between the coupled inductors. Determine $V_a(0^+)$, $dV_a/dt(0^+)$ and $d^2V_a/dt^2(0^+)$.

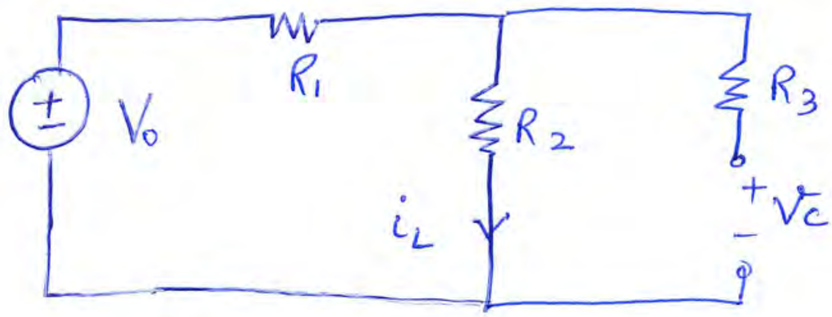


PROBLEM 01

5-20

EVALUATION OF INITIAL CONDITIONS

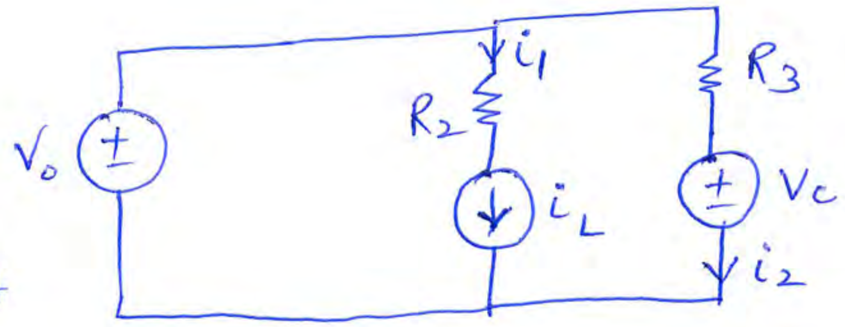
At $t = 0^-$



$$i_L = \frac{V_0}{R_1+R_2}$$

$$v_c = \frac{R_2 V_0}{R_1+R_2}$$

At $t = 0^+$



$$\Rightarrow \quad i_1(0^+) = i_L \quad A$$

$$i_2(0^+) = \frac{V_0 - V_c}{R_3} = \frac{V_0(R_1)}{R_3(R_1+R_2)} \quad A.$$

Loop 1: $(V_0,\ R_2$ and $L)$

Equation: $$V_0 = i_1 R_2 + L\frac{di_1}{dt}$$

$$\Rightarrow \quad L\frac{di_1}{dt} = V_0 - i_1 R_2$$

$$\Rightarrow \quad \frac{di_1}{dt}(0^+) = \frac{V_0}{L} - \frac{V_0 R_2}{L(R_1+R_2)} = \frac{V_0}{L}\frac{R_1}{(R_1+R_2)} \quad A/sec$$

Loop 2: V_0, R_3 and C

Equation: $$V_0 = i_2 R_3 + \frac{1}{C}\int i_2\,dt \Rightarrow 0 = R_3\frac{di_2}{dt} + \frac{i_2}{C}$$

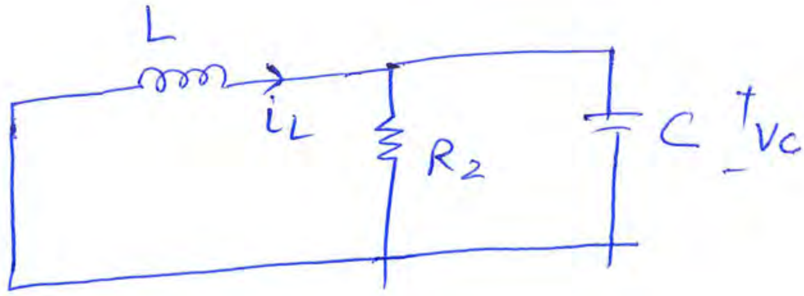
$$\Rightarrow \quad \frac{di_2}{dt} = -\frac{i_2}{R_3 C}$$

$$\Rightarrow \quad \frac{di_2}{dt}(0^+) = \frac{-V_0 R_1}{R_3^2 (R_1+R_2)} \quad A/sec.$$

Problem 02

5-21

At $t = 0^-$



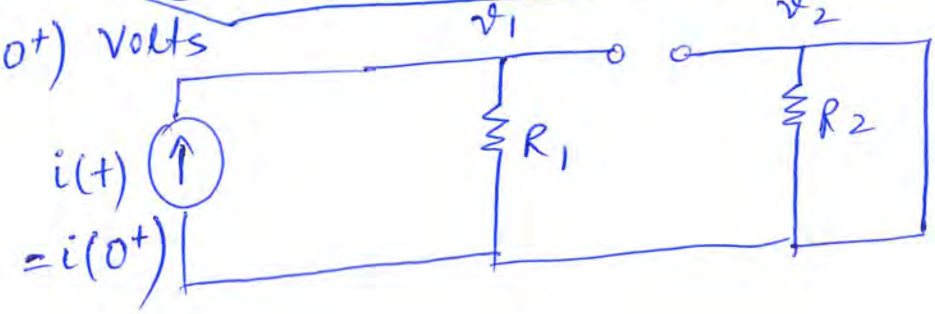
$i_L = 0$

$v_c = 0$

At $t = 0^+$; Replace L with open circuit
C with short circuit

$v_1 = R_1\, i(t) = R_1 i(0^+)$ Volts

$v_2 = 0$ Volts



Node ① Equation

$$\frac{v_1}{R_1} + \frac{1}{L}\int (v_1 - v_2)\,dt = i(t)$$

$$\frac{1}{R_1}\frac{dv_1}{dt} + \frac{v_1 - v_2}{L} = \frac{di}{dt}$$

$$\Rightarrow \frac{dv_1}{dt} = R_1\frac{di}{dt} + R_1\frac{v_2 - v_1}{L}$$

$t = 0^+$ $$\frac{dv_1}{dt} = R_1\frac{di(0^+)}{dt} - \frac{R_1^2\, i(0^+)}{L} \quad \text{V/sec}$$

Node ② Equation

$$\frac{v_2}{R_2} + \frac{C\,dv_2}{dt} + \underbrace{\frac{1}{L}\int (v_2 - v_1)\,dt}_{i_L = 0 \text{ at } t = 0^+} = 0$$

At $t = 0^+$

$$\Rightarrow \boxed{\frac{dv_2}{dt} = 0}$$
at $t = 0^+$

Problem 03

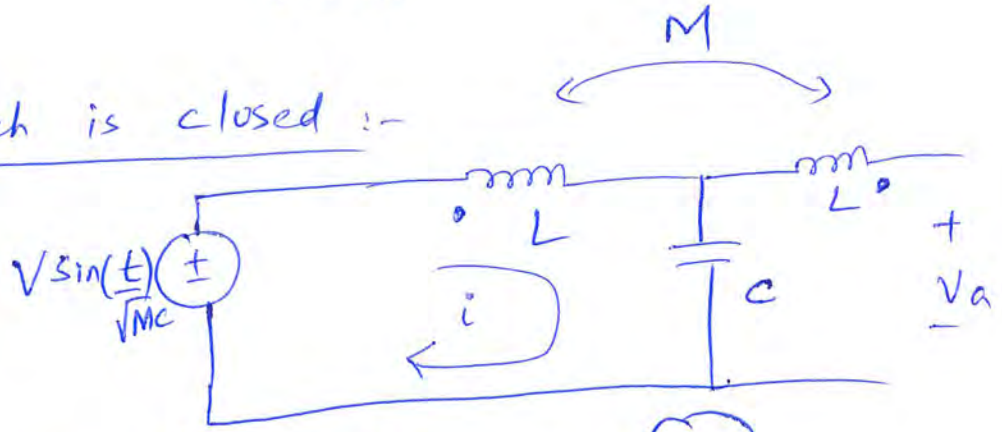
5_24

~~Att~~ At $t = 0^-$; * current through inductors is zero
* voltage across capacitor is zero

After the switch is closed :-

$i(0^+) = 0$

Loop Equation



$$V\sin\frac{t}{\sqrt{MC}} + L\frac{di}{dt} + \underbrace{\frac{1}{C}\int i\,dt}_{0 \text{ at } t=0^+} = 0 \quad - (01)$$

$\Rightarrow$

$$\Rightarrow \frac{di}{dt}(0^+) = -\frac{V}{L}\sin\left(\frac{t}{\sqrt{MC}}\right)\Bigg|_{t=0^+} = 0$$

V_a :- $$V_a = \frac{1}{C}\int i\,dt + M\frac{di}{dt} \quad - (02)$$

$t=0^+$ $\boxed{V_a = 0}$

Taking derivative of (01) $\Rightarrow \frac{d^2 i}{dt^2}(0^+) = \frac{V}{\sqrt{MC}}$

Taking ——— (02) $\Rightarrow \frac{dV_a}{dt}(0^+) = \frac{V}{L}\sqrt{\frac{M}{C}}$

Taking ——— twice

$$\frac{d^2V_a}{dt^2} = \frac{1}{C}\frac{di}{dt} + \underbrace{M\frac{d^3 i}{dt^3}}_{0 \text{ at } t=0^+} \Rightarrow \boxed{\frac{d^2V_a}{dt^2}(0^+) = 0}$$