LAHORE UNIVERSITY OF MANAGEMENT SCIENCES
Department of Electrical Engineering

EE240 Circuits I
Quiz 08

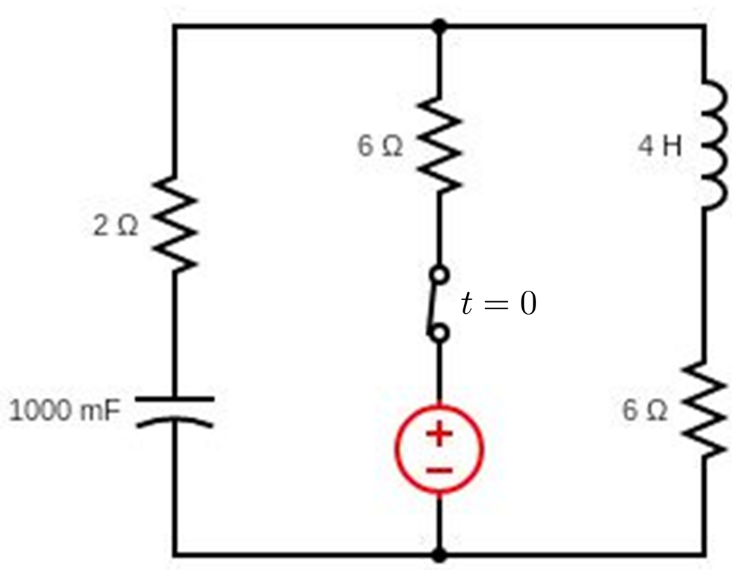
Name: ______________________

Campus ID: ______________________

Total Marks: 10
Time Duration: 20 minutes

Question 1 (10 marks)

Consider the following circuit. The voltage source is 24 V DC. The switch is opened at $t = 0$.



(a) [2 marks] Draw a snapshot of the circuit at $t = 0^-$ and determine the voltage across capacitor and current through inductor at $t = 0^-$.

Solution: At $t = 0^-$, the capacitor acts as an open circuit since it will be fully charged and the inductor acts as a short circuit. The circuit at $t = 0^-$ comprised of 24 V voltage source in series with two 6 Ω resistors.
$i_L(0^-) = 24/12 = 2\,A$ and $v_c(0^-) = 12\,V$.

(b) [3 marks] Formulate a differential equation in terms of $i_L(t)$ for $t \geq 0$. (i_L is the current through inductor from top to bottom).

Solution: Once the switch is operated, we have a series RLC circuit with equation in terms of $i_L(t)$ given by

$$L\frac{di_L}{dt} + Ri_L + \frac{1}{C}\int i_L dt = 0$$

$$\frac{di_L}{dt} + \frac{8}{4}i_L + \frac{1}{4 \times 1}\int i_L dt = 0$$

$$\frac{di_L}{dt} + 2i_L + \frac{1}{4}\int i_L dt = 0 \qquad (1)$$

(c) [5 marks] Determine $i_L(t)$ for all times.

Solution: Initial Conditions First:

$i_L(0^+) = i_L(0^-) = 2\,A$. To find, $\frac{di_L}{dt}$ at $t = 0^+$, we use

$$L\frac{di_L}{dt} + Ri_L + \frac{1}{C}\int i_L dt = 0$$

Note here $\frac{1}{C}\int i_L dt = -v_C(t)$ and therefore $\frac{1}{C}\int i_L dt = -12\,V$ at $t = 0^+$. Consequently, we have

$$4\frac{di_L}{dt}(0^+) + 8i_L(0^+) - 12 = 0,$$

which yields

$$\frac{di_L}{dt}(0^+) = -1\,A/s.$$

Solution of Differential Equation:

Characteristic equation for (1) is given by

$$s^2 + 2s + \frac{1}{4} = 0$$

The roots are given by $s_1 = -1 + \frac{\sqrt{3}}{2}$ and $s_2 = -1 - \frac{\sqrt{3}}{2}$. Consequently, we have

$$i(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t}, \quad t \geq 0$$

Using initial conditions, we obtain $K_1 = 1 + \frac{1}{\sqrt{3}}$ and $K_2 = 1 - \frac{1}{\sqrt{3}}$.