EE 240 Circuits I

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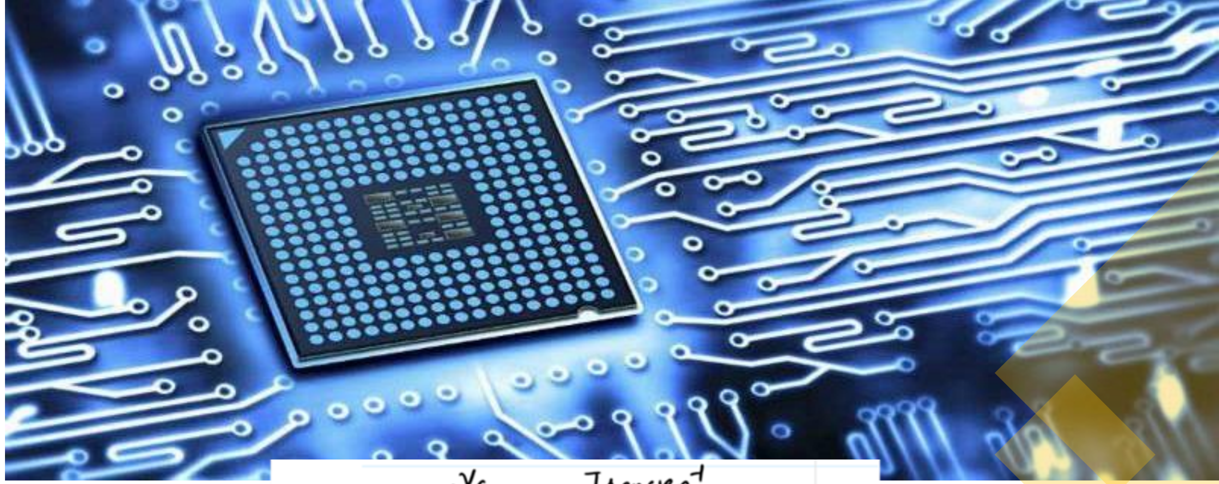
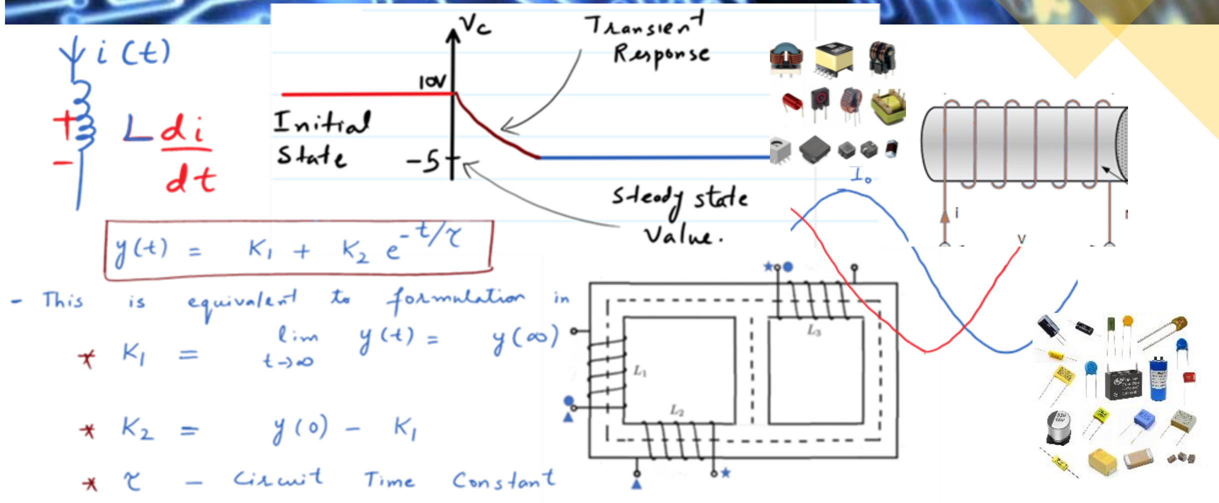
- Super Node
- Super Loop

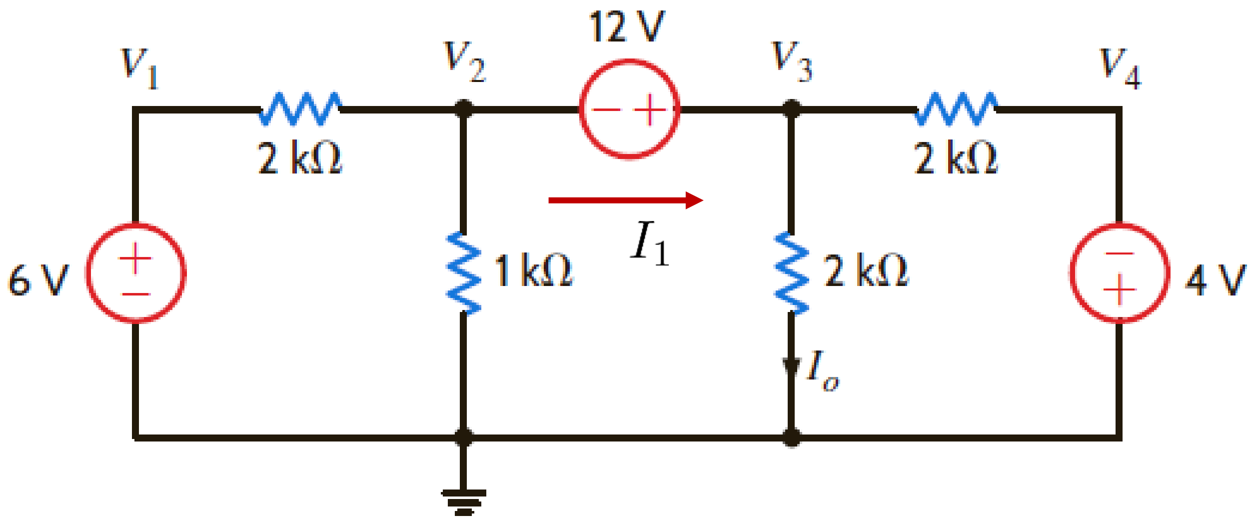




Nodal Analysis – Super-node Concept

Example: Carry out nodal analysis

$V_1 = 6V$ $\quad\quad$ $V_4 = -4V$



When we attempt to write nodal equation for either node 2 or node 3, we face a problem.

As we have a 12 voltage source in the branch connecting node 2 and node 3, the current in this branch is certainly not known and it cannot be directly expressed.

To tackle this problem, we assume that this current is denoted by I_1 as indicated in the figure.

Now we can write equations node 2 and node 3 as follows:

Node 2 Equation:

$$\frac{V_2 - V_1}{2k} + \frac{V_2}{1k} + I_1 = 0$$

Node 3 Equation:

$$\frac{V_3 - V_4}{2k} + \frac{V_3}{2k} - I_1 = 0$$

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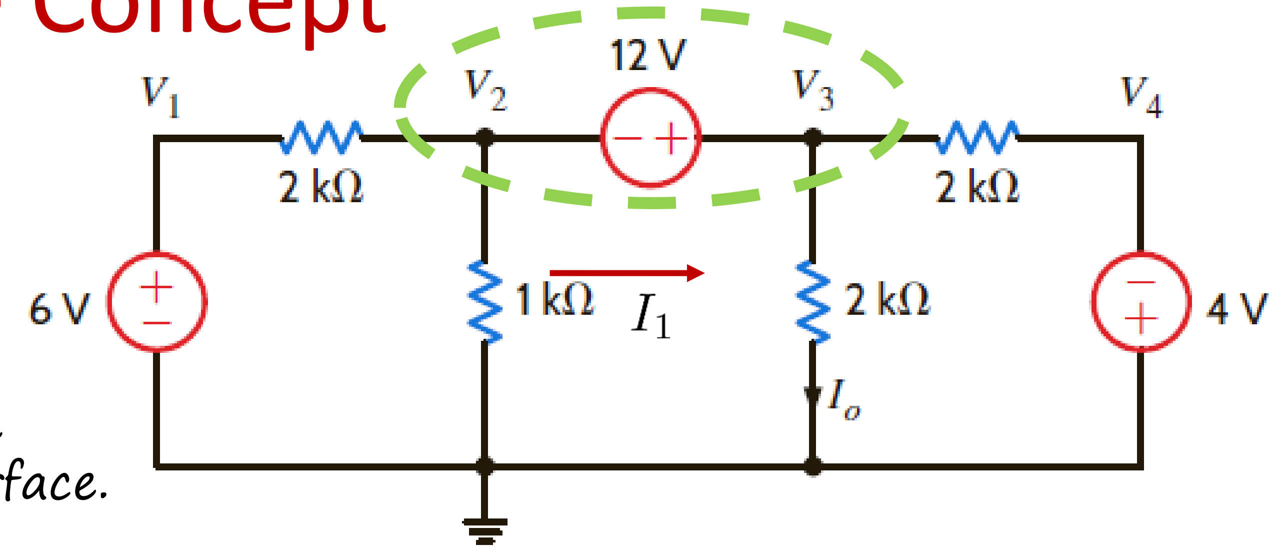
Nodal Analysis – Super-node Concept

Example: Carry out nodal analysis



Adding the equations for Nodes 2 and 3 yields:

$$\frac{V_2 - V_1}{2k} + \frac{V_2}{1k} + \frac{V_3 - V_4}{2k} + \frac{V_3}{2k} = 0$$

This equation is indeed an equation of the surface indicated in green when we apply KCL to the surface.

Since this surface is a collection of two nodes with **different voltages** at both the ends, we refer to this as **Super-node.**

If we encounter the voltage source between two nodes, we can form a Super-node and apply KCL to the super-node to obtain the network equation.

Q: We have one equation of the super-node that is in fact a sum of two equations. How do we get one more equation?

A: We obtain one more equation by relating potentials at these nodes using the voltage source between the nodes. 12V voltage source between node 2 and node 3 ensures the potential difference of 12V between node 3 and node 2

$$V_3 - V_2 = 12$$

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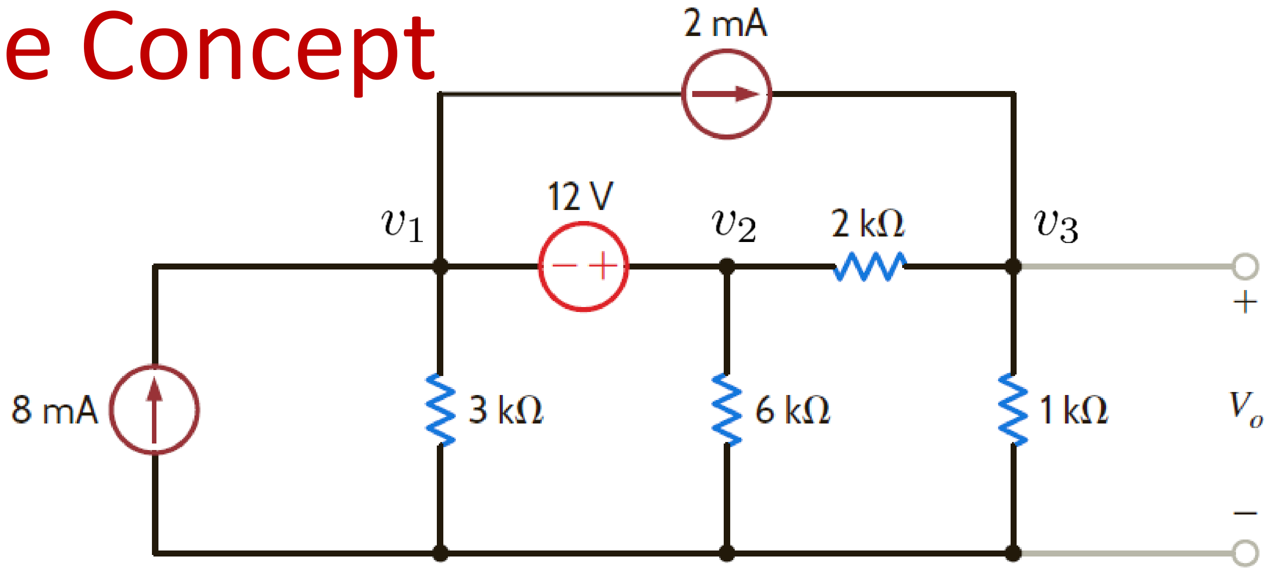
Nodal Analysis – Super-node Concept

Reinforcement Example: Determine V_o.



Super-node Equation:

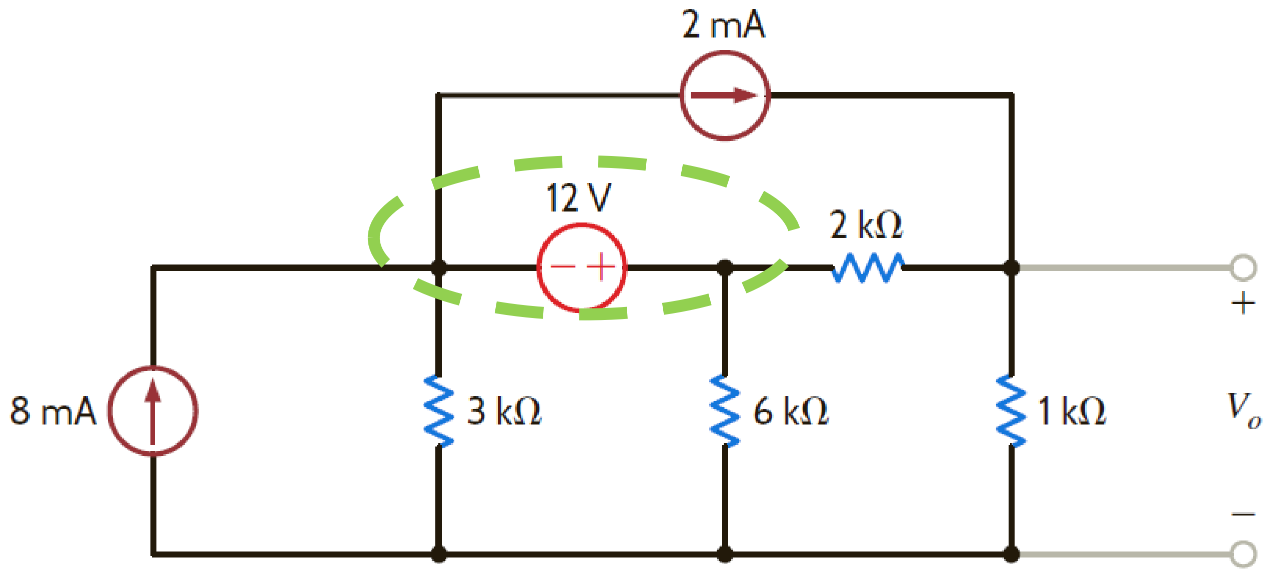
$$\frac{v_1}{3k} + \frac{v_2 - v_3}{2k} + \frac{v_2}{6k} - 8 + 2 = 0$$

Node 3 Equation:

$$\frac{v_3 - v_2}{2k} + \frac{v_3}{1k} - 2 = 0$$

Equation relating voltages at super-node:

$$v_2 - v_1 = 12$$

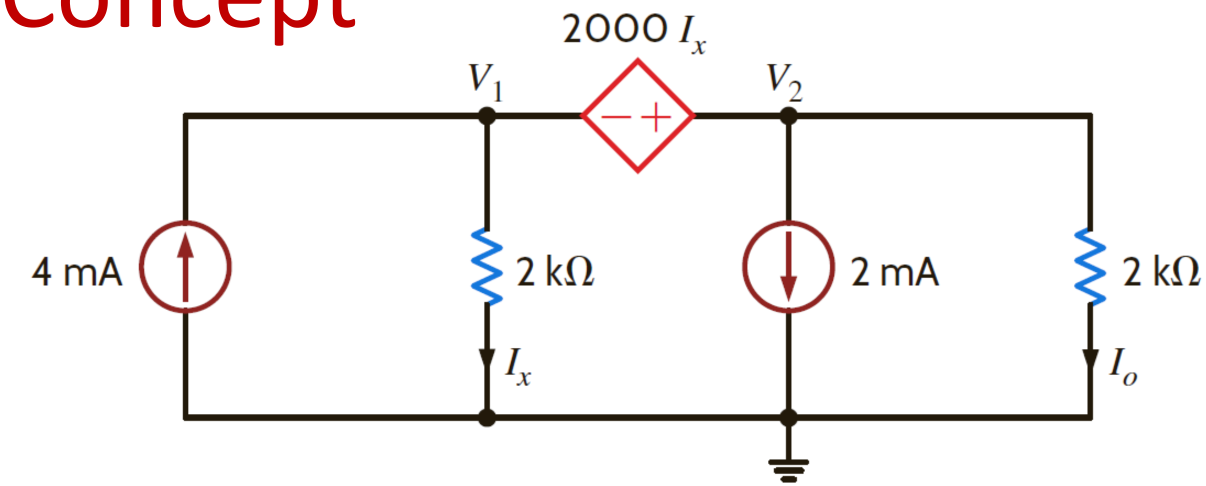


By solving these equations, we obtain

$$v_3 = V_o = 5.6V$$

Nodal Analysis – Super-node Concept

Reinforcement Example: Determine the current I_o.



Super-node Equation:

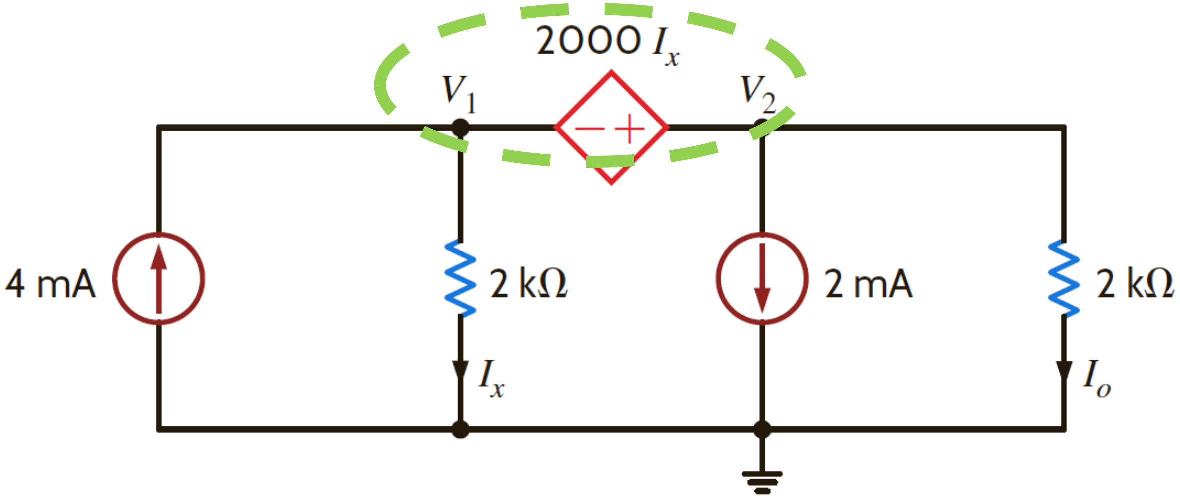
$$\frac{V_1}{2k} + \frac{V_2}{2k} - 4 + 2 = 0$$

Controlled Source Equation:

$$I_x = \frac{V_1}{2k}$$

Equation relating voltages at super-node:

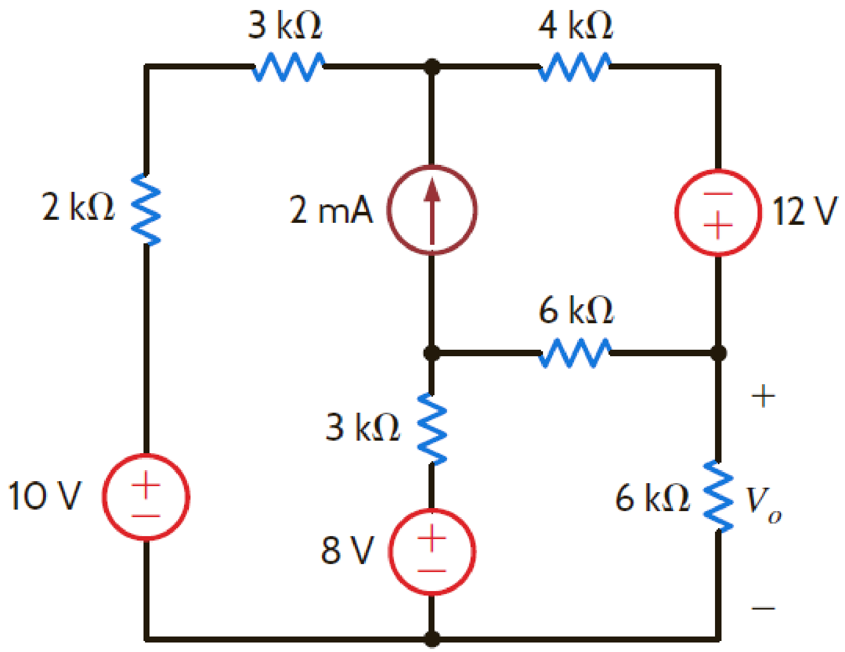
$$V_2 - V_1 = 2000 I_x$$



By solving these equations, we obtain

$$I_o = \frac{V_2}{2k} = \frac{4}{3}\text{mA}$$

Loop Analysis – Super-loop Concept



Example: Carry out loop analysis Determine V_o.

We first mark the loop currents.

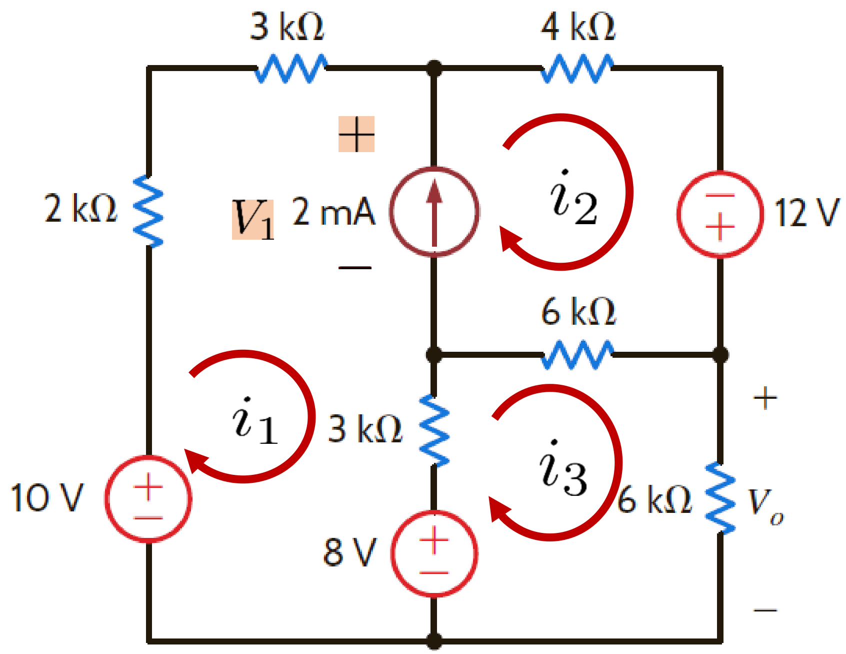
We see that we have a current source shared between loop 1 and loop 2.

When we attempt to write loop equation for either loop 1 or loop 2, we face a problem.

As we have a 2mA current source in the branch shared between loop 1 and loop 2, the voltage across current source is certainly not known and it cannot be directly expressed.

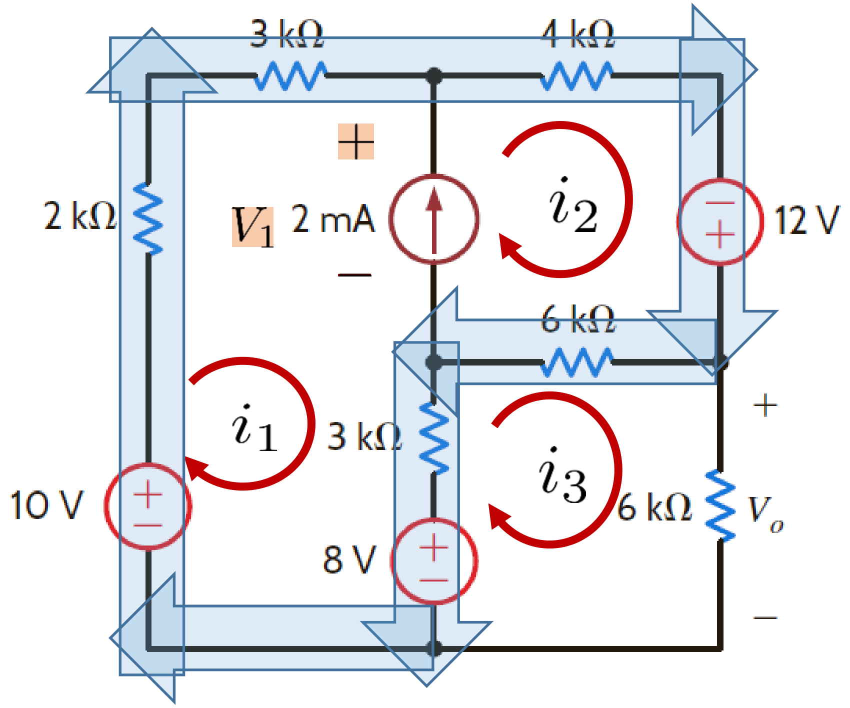
To tackle this problem, we assume that this voltage is denoted by V_1 as indicated in the figure.

Now we can write equations for loop 1 and loop 2 as follows:



Loop Analysis – Super-loop Concept

Example: Carry out loop analysis



Loop 1 Equation: $(8k)i_1 - (3k)i_3 + 8 - 10 + V_1 = 0$

Loop 2 Equation: $(10k)i_2 - (6k)i_3 - 12 - V_1 = 0$

Adding the equations for Loops 1 and 2 yields:

$$(8k)i_1 + (10k)i_2 - (9k)i_3 - 14 = 0$$ **(Super-loop Equation)**

This equation is indeed an equation of the loop indicated in the blue shade.

Since this loop is a collection of two loops with **different loop currents in these loops**, we refer to this as **Super-loop.** The use of super loop is useful when the current source is shared between two loops.

Q: We have one equation of the super-loop that is in fact a sum of two equations. How do we get one more equation?

A: We obtain one more equation by relating the two loop currents that form the super-loop using the current source between the loops. 2mA current source in the branch shared by loop 1 and loop 2 currents 2mA current in the branch, that is,

$$i_2 - i_1 = 2\,\mathrm{mA}$$ **(Equation relating current in the super-loop)**

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Loop Analysis – Super-loop Concept

Example: Carry out loop analysis

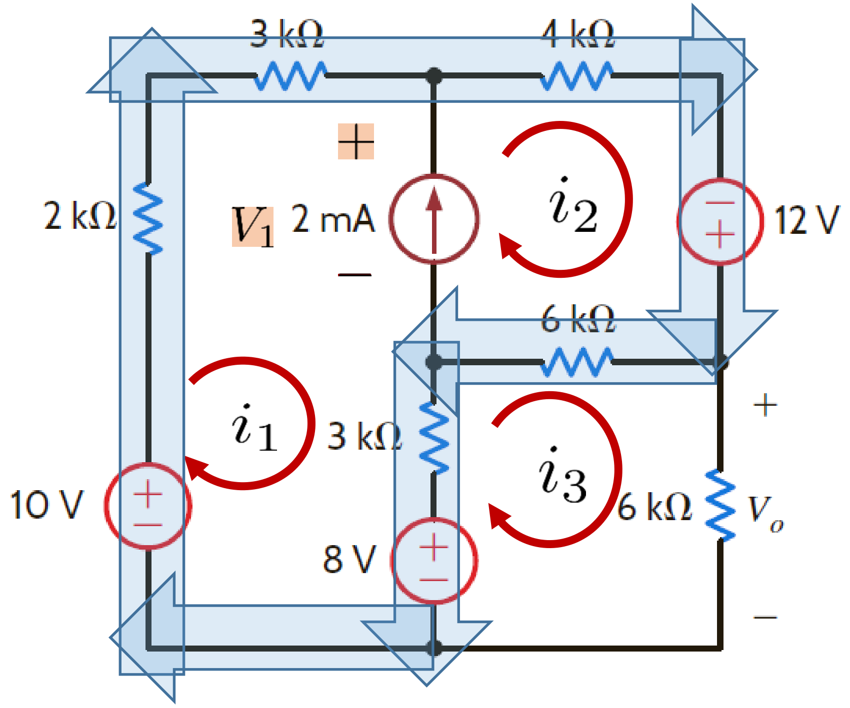
Loop 3 Equation: $-(3k)i_1 - (6k)i_2 + 15i_3 = 8$

Solving (Super-loop Equation), (Equation relating current in the super-loop) and Loop 3 equation, we obtain

$$i_3 = \frac{34}{21}\,\mathrm{mA}$$

Using which we can determine

$$V_o = (6k)(\tfrac{34}{21}m) = \frac{68}{7}\,\mathrm{V}$$



Node Analysis – Summary

Steps to Carry out Circuit Analysis:

- Determine the number nodes; mark the ground node and assign node voltages to other nodes.
- For an N-node circuit, there are N-1 node voltages, and we need (N-1) linearly independent equations to determine these node voltages.
- If voltage source is connected between a node and the ground, the voltage at the node is equal to the voltage of the source.
- If voltage source (dependent or independent) is connected between two nodes, we employ the super-node concept to obtain two equations: 1) equation of the super-node using KCL and 2) equation relating the node voltages and the voltage source.
- Formulate the remaining node equations by employing KCL.
- For each dependent source, write a constraint equation, which is one of the necessary linearly independent equations. While writing the constraint equation, express the controlling quantity in terms of nodal voltages.

Loop Analysis – Summary

Steps to Carry out Circuit Analysis:

- Assign a loop current to analyze each independent loop.
- For an N-loop circuit, there are N loop currents, and we need N linearly independent equations to determine the loop currents.
- If current source is connected in a branch carrying only one loop current, the current is equal to the current of the source.
- If current source (dependent or independent) is shared between two loops, we employ the super-loop concept to obtain two equations: 1) equation of the super-loop using KVL and 2) Equation relating the currents in the two loops and the current source.
- Formulate the remaining loop equations by employing KVL.
- For each dependent source, write a constraint equation, which is one of the necessary linearly independent equations. While writing the constraint equation, express the controlling quantity in terms of loop currents.