LUMS
A Not-for-Profit University

School of Science and Engineering

EE 240 Circuits-I

ASSIGNMENT 1 – SOLUTIONS

Due Date: 1 pm, Tuesday, October 1, 2024.
Format: 9 problems, for a total of 100 marks
Instructions:

- You are allowed to collaborate with your peers but copying your colleague's solution is strictly prohibited. This is not a group assignment. Each student must submit his/her own assignment.
- Solve the assignment on blank A4 sheets and staple them before submitting.
- Submit in-class or in the dropbox labeled EE-240 outside the instructor's office.
- **Write your name and roll no. on the first page.**
- Feel free to contact the instructor or the teaching assistants if you have any concerns.

- You represent the most competent individuals in the country, do not let plagiarism come in between your learning. In case any instance of plagiarism is detected, the disciplinary case will be dealt with according to the university's rules and regulations.
- We require you to acknowledge any use or contributions from generative AI tools. Include the following statement to acknowledge the use of AI where applicable.

  I have used [insert Tool Name] to [write, generate, plot or compute; explain specific use of generative AI] [number of times].

(a) $i = \frac{dq}{dt} = 40\pi \cos 4\pi t \ \text{mA}$

$p = vi = 80\pi \cos^2 4\pi t \ \text{mW}$

At t=0.3s,

$$p = 80\pi \cos^2(4\pi x 0.3) = \underline{164.5 \ \text{mW}}$$

(b) $W = \int p dt = 80\pi \int_0^{0.6} \cos^2 4\pi t dt = 40\pi \int_0^{0.6} [1 + \cos 8\pi t] dt \ \text{mJ}$

$$W = 40\pi \left[0.6 + \frac{1}{8\pi} \sin 8\pi t \Big|_0^{0.6} \right] = \underline{78.34 \ \text{mJ}}$$

Problem 1 (8 marks)

Problem 2 (15 marks)

$$q = it = 8x10^{3}x15x10^{-6} = \underline{\mathbf{120\ mC}}$$

(a) [**2 marks**]

$$q = it = 85\ x10^{-3}\ x\ 12\ x\ 60\ x\ 60 = 3{,}672\ C$$

$$E = pt = ivt = qv = 3672\ x1.2 = 4406.4\ J$$

(b) [**2 marks**]

$$\text{(a) } q = \int idt = \int_0^1 10\,dt = \underline{10\ C}$$

$$\text{(b) } q = \int_0^3 idt = 10\times 1 + \left(10 - \frac{5\times 1}{2}\right) + 5\times 1$$
$$= 15 + 7.5 + 5 = \underline{22.5C}$$

$$\text{(c) } q = \int_0^5 idt = 10 + 10 + 10 = \underline{30\ C}$$

(c) [**5 marks**]

(d) [**6 marks**]

For 0 < t < 6s, assuming q(0) = 0,

$$q(t) = \int_0^t idt + q(0) = \int_0^t 3tdt + 0 = 1.5t^2$$

At t=6, $q(6) = 1.5(6)^2 = 54$

For 6 < t < 10s,

$$q(t) = \int_6^t idt + q(6) = \int_6^t 18dt + 54 = 18t - 54$$

At t=10, q(10) = 180 – 54 = 126

For 10<t<15s,

$$q(t) = \int_{10}^t idt + q(10) = \int_{10}^t (-12)dt + 126 = -12t + 246$$

At t=15, q(15) = -12x15 + 246 = 66

For 15<t<20s,

$$q(t) = \int_{15}^t 0dt + q(15) = 66$$

Thus,

$$q(t) = \begin{cases} 1.5t^2 \text{ C}, & 0 < t < 6\text{s} \\ 18t - 54 \text{ C}, & 6 < t < 10\text{s} \\ -12t + 246 \text{ C}, & 10 < t < 15\text{s} \\ 66 \text{ C}, & 15 < t < 20\text{s} \end{cases}$$

Problem 3 (15 marks)

(a) [**5 marks**]

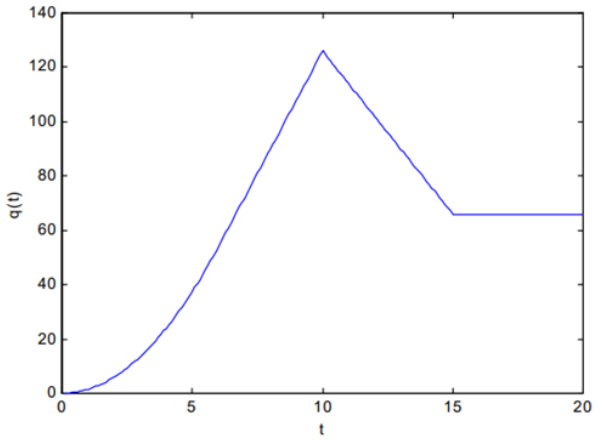
$$w(t) = \int_{t_1}^{t_2} p(t)\,dt = \int_0^{0.25} 2.5e^{-4t}\,dt = 0.395\,\text{J} = 395\,\text{mJ}$$

$$p(t) = vi$$

$$i(t) = \frac{2.5e^{-4t}}{50e^{t}} = 0.05e^{-3t}$$

$$q(t) = \int_{t_1}^{t_2} i(t)\,dt = \int_0^{0.25} 0.05e^{-3t}\,dt = 8.79\,\text{mC}$$

The plot of the charge is shown below.



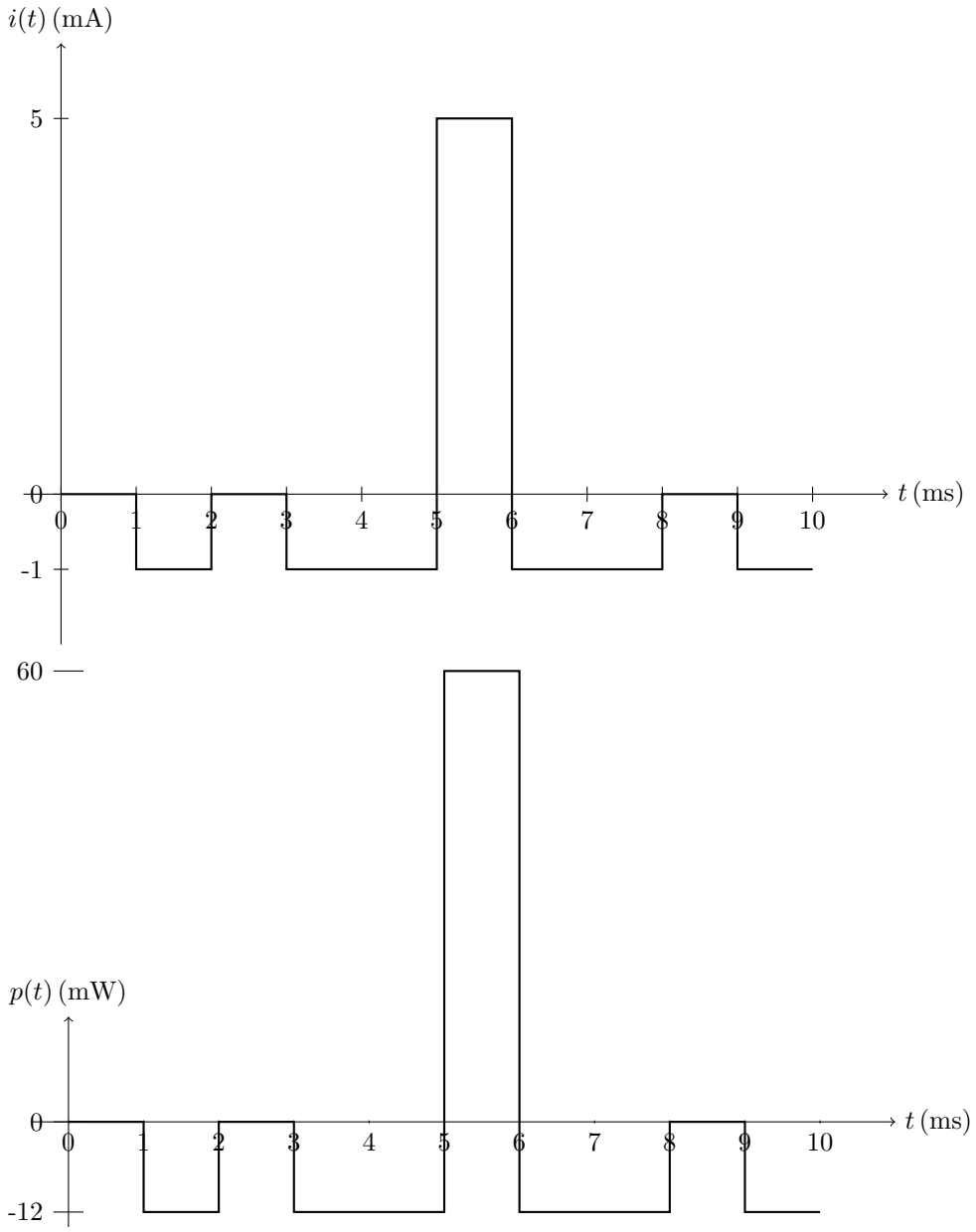
(b) **[10 marks]**

$$
\begin{aligned}
0 \le t \le 1\,\text{ms}: &\quad q = 1\,\text{mC}, \quad i = 0\,\text{mA} \\
1 \le t \le 2\,\text{ms}: &\quad q = -t + 2\,\text{mC}, \quad i = -1\,\text{mA} \\
2 \le t \le 3\,\text{ms}: &\quad q = 0\,\text{mC}, \quad i = 0\,\text{mA} \\
3 \le t \le 5\,\text{ms}: &\quad q = -t + 3\,\text{mC}, \quad i = -1\,\text{mA} \\
5 \le t \le 6\,\text{ms}: &\quad q = 5t - 27\,\text{mC}, \quad i = 5\,\text{mA} \\
6 \le t \le 8\,\text{ms}: &\quad q = -t + 4\,\text{mC}, \quad i = -1\,\text{mA} \\
8 \le t \le 9\,\text{ms}: &\quad q = 1\,\text{mC}, \quad i = 0\,\text{mA} \\
9 \le t \le 10\,\text{ms}: &\quad q = -t + 10\,\text{mC}, \quad i = -1\,\text{mA}
\end{aligned}
$$

$$i = \frac{dq}{dt}$$

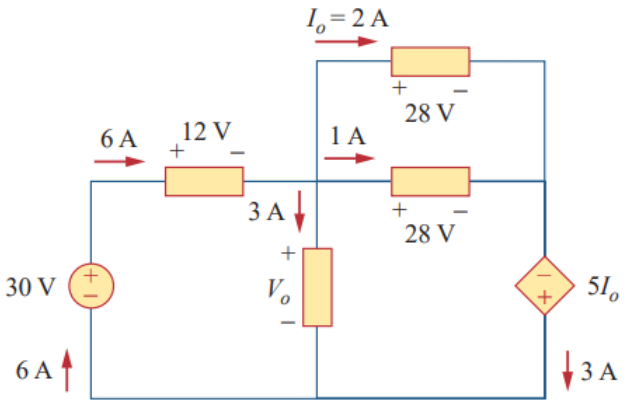
$$v = 12\,\text{V} \quad p = vi$$

$$
p(t) = \begin{cases}
0 & 0 \le t \le 1 \\
-12 & 1 \le t \le 2 \\
0 & 2 \le t \le 3 \\
-12 & 3 \le t \le 5 \\
60 & 5 \le t \le 6 \\
-12 & 6 \le t \le 8 \\
0 & 8 \le t \le 9 \\
-12 & 9 \le t \le 10
\end{cases}
$$

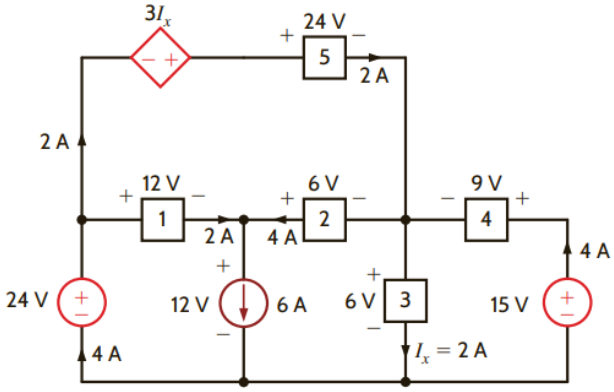

$i(t)$ (mA)
5
0
-1
0 1 2 3 4 5 6 7 8 9 10
t (ms)
60
$p(t)$ (mW)
0
-12
0 1 2 3 4 5 6 7 8 9 10
t (ms)

Problem 4 (15 marks)

(a) **[7 marks]** Find V_o in the circuit below using Tellegen's theorem.



(b) **[8 marks]** Calculate the power absorbed by each element in the circuit below. Also, verify that Tellegen's theorem is satisfied by this circuit.



$$P_1 = 12 * 2 = 24 \text{ W absorbed}$$
$$P_2 = -6 * 4 = -24 \text{ W supplied}$$
$$P_3 = 6 * 2 = 12 \text{ W absorbed}$$
$$P_4 = 9 * 4 = 36 \text{ W absorbed}$$
$$P_5 = 24 * 2 = 48 \text{ W absorbed}$$
$$P_{3I_x} = -6 * 2 = -12 \text{ W supplied}$$
$$P_{24V} = -24 * 4 = -96 \text{ W supplied}$$
$$P_{6A} = 12 * 6 = 72 \text{ W absorbed}$$
$$P_{15V} = -15 * 4 = -60 \text{ W supplied}$$

According to Tellegen's Theorem:

$$\text{Power supplied} = \text{Power absorbed}$$
$$24 + 12 + 96 + 60 = 24 + 12 + 36 + 48 + 72$$
$$192 = 192$$

Verified

$V_1 = 30V$

At node 1:

$$i = \frac{V_1 - V_4}{4} + \frac{V_1 - V_2}{2}$$

At node 2:

$$\frac{V_1 - V_2}{2} = 4 + \frac{V_2}{3} + \frac{V_2 - V_3}{6}$$

At node 3:

$$\frac{V_2 - V_3}{6} = \frac{V_3}{1} + \frac{V_3 - V_4}{8}$$

At node 4:

$$4 = \frac{V_4 - V_1}{4} + \frac{V_4 - V_3}{8}$$

After simplifying:

1. $4i = 90 - 2V_2 - V_4$
2. $-6V_2 + V_3 = -66$
3. $4V_2 - 3V_3 + 3V_4 = 0$
4. $3V_4 - V_3 = 92$

Solving these equations gives:

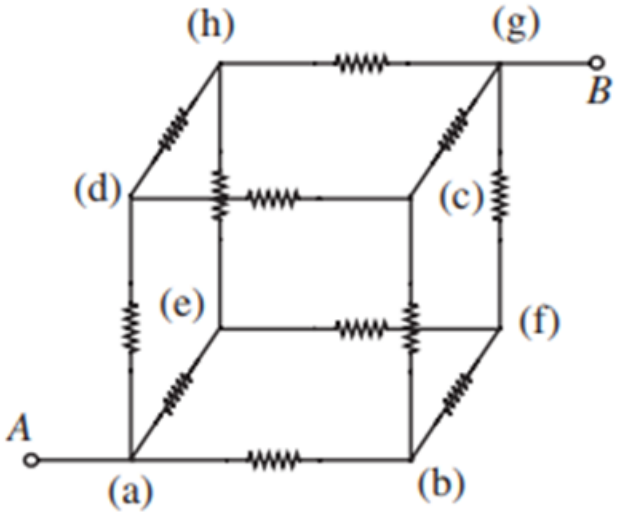
$$V_2 = 11.77\,\text{V}, \quad V_3 = 4.64\,\text{V}, \quad V_4 = 32.2\,\text{V}$$

And the current i is:

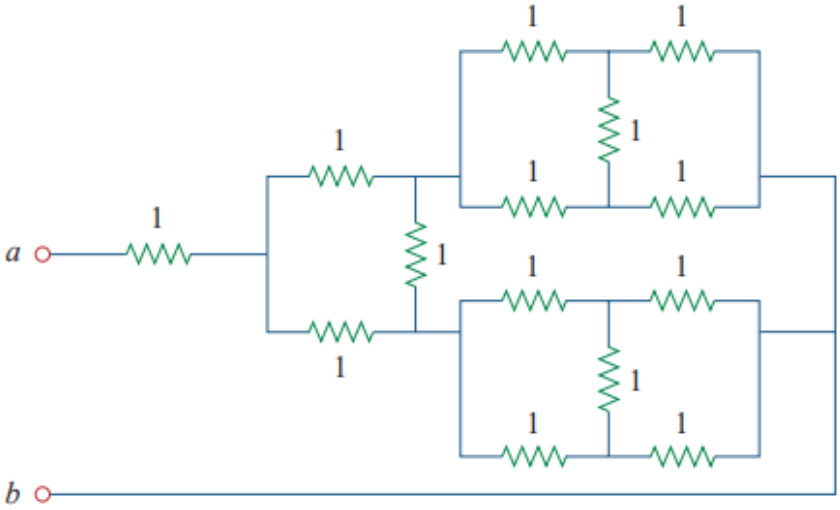
$$i = 8.56\,\text{A}$$

Problem 5 (12 marks)

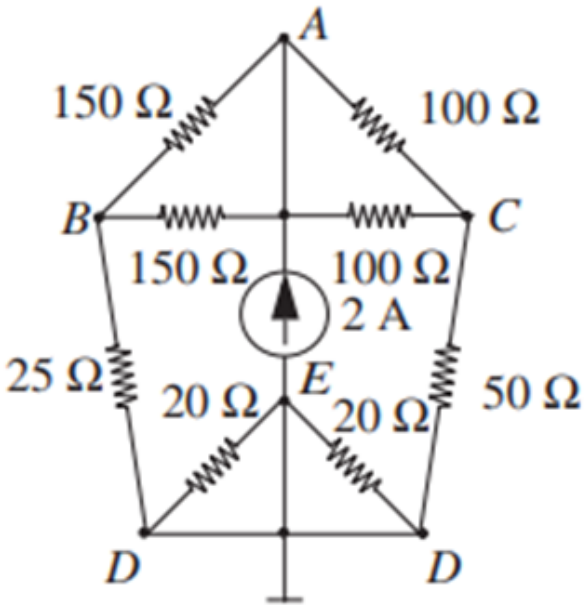
(a) [**7 marks**] For the figure below, find the equivalent resistance of the network when looking into the A-B terminal. All resistors have the R = 2 Ω.



(b) [**5 marks**] Find R_{ab}. Assume all resistances to be 1 Ω.

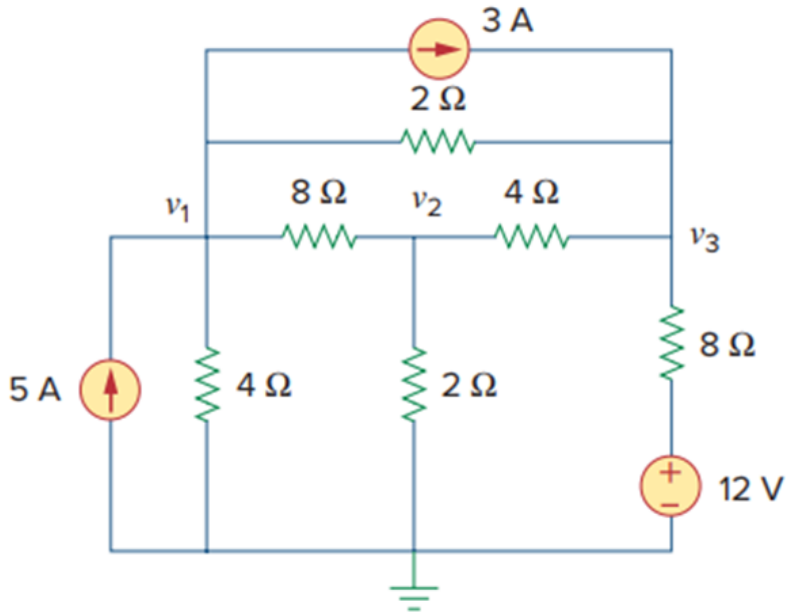


Problem 6 (10 marks) For the figure shown below, find the voltages between each of the mentioned nodes and ground e.g. V_{ad}

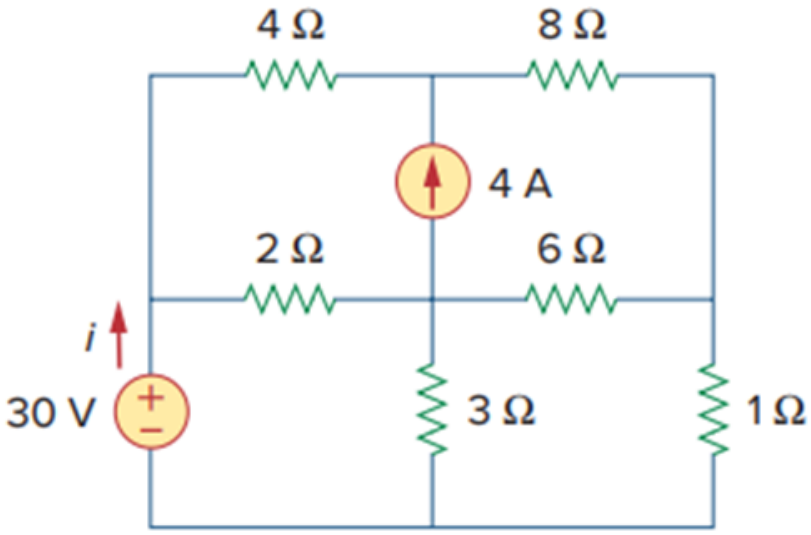


Problem 7 (10 marks)

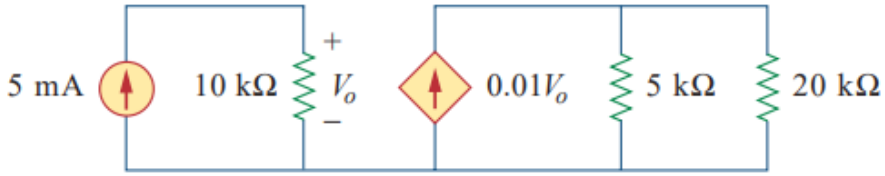
(a) [**5 marks**] For the circuit shown, find the voltages v_1, v_2, and v_3.



(b) [**5 marks**] For the circuit shown, find the current i.



Problem 8 (5 marks) For the figure below, find the current, voltage, and power associated with the 20k resistor.



Problem 9 (10 marks) Find R_{eq} and I in the circuit below.