Inductor

A passive, two terminal, electrical component which stores energy in the form of magnetic flux.

Solenoid - Air core

It is also referred to as coil or choke.

Inductors appear in different shapes and sizes but the basic configuration is ***'A simple insulated wire that is coiled up - coil of insulated wire* (around the core)'.**

Solenoid - Iron core

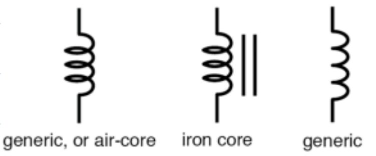
When the current is passed through the inductor, magnetic flux is produced proportional to the current around the inductor. This gives the wire interesting property which we refer to as 'inductance'.

Toroid

Recall; Capacitor resists change in the voltage.
Inductor opposes the change of current flowing through it due to the energy self-induced in its magnetic field

Inductors do not carry any magnetic field or flux if no current is flowing through the inductor.

Symbols:



Inductance:

We characterize an inductor by its inductance which quantifies the tendency of an electrical conductor to oppose a change in the electric current flowing through it.

Inductance is the property that relates the magnetic flux with the current

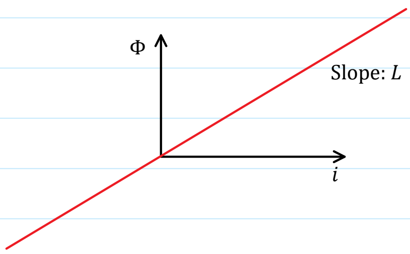
We denote the inductance by L and is defined as $L = \frac{\Phi}{i}$, that is, the magnatic flux Φ per unit ampere of the current

Unit of inductance is Henry (Webers per Ampere), abbreviated as H. Typical inductors have the inductance mH.

Weber is a unit of magnetic flux.

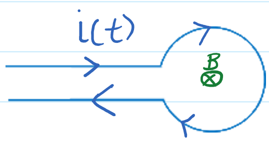
Φ -i Characteristics:

Linear characteristics (ideal inductor)



How to Compute Inductance?

Inductance of a Single Loop



Consider a loop of wire of length ℓ and area A.

Current i(t) is flowing in the direction indicated.

This current will produce magnetic field density that is given by Ampere's Law.

Ampere's Law

$$\oint_C B \cdot \mathrm{d}l = \mu_o I_{enc}$$

B Magnetic field density (Teslas)

$\oint_C$ ⬚ Closed integral around closed curve

$\mu_o = 4\pi \text{ x } 10^{-7} \mathrm{H/m}$ Permeability of free space, vaccum

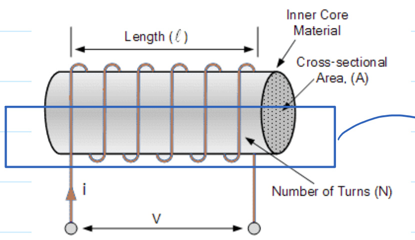
$B\ell = \mu_o\, i$

Magnetic flux is given by surface integral of magnetic field density

$$\Phi = \iint_S B.ds = BA = \mu_o \frac{A}{\ell}\, i = L\, i$$

$$\text{Inductance}, L = \mu_o \frac{A}{\ell} \quad \text{(Henry)}$$

Inductance of Solenoid:



Ampere's law integral path

Consider a coil forming a loop of area A, length ℓ, and number of turns N .

Assuming air core (no core).

Ampere's Law

$$\oint_C B \cdot dl = \mu_o I_{enc}$$

Choosing ampere's law integral path

$B\ell = \mu_o\, N\, i$ (Current is multiplied by N as we have N number of turns over the length l)

$B = \mu_o \frac{N}{\ell}\, i = \mu_o\, n\, i$ (n is the turn density, that is, the number of turns per unit length)

Magnetic field density depends on the turn density.

Magnetic flux is given by surface integral of magnetic field density

$$\Phi = \iint_S B.ds = BA = \mu_o \frac{N}{\ell}\, i\, A$$

Total flux linkage is given by

$$\Psi = N\Phi = \mu_o \frac{N^2}{\ell} A\, i$$

$$\text{Inductance}, L = \mu_o \frac{N^2}{\ell} A \quad \text{(Henry)}$$

Role of Iron (Ferro-magnetic) Core in Practical Inductor:

The use of core provides the following benefits
- provides mechanical support for the windings
- Keeps magnetic flux concentrated around the core
- Improves permeability (inversely related of reluctance that is the resistance of the material to magnetic flux). μ_o is replaced with $\mu_r \mu_o$, where μ_r is the relative permeability of the magenetic material used for the core. Practical values of relative permeability can be in the range of 100 to 1000.

Mid-term summary: **Inductance quantifies the energy stored in the form of magnetic flux due to current i.**

Here, inductance *L* quantifies flux linkage in the coil due to the current in the same coil, that is, $\Psi = L\, i$.
We refer to *L* as self inductance.

If we place another loop of wire (coil 2) in the vicinity of the coil 1 carrying current i_1 , the flux Ψ_2 in coil 2 will be linked to i_1 as $\Psi_2 = M_{21}\, i_1$.

Here M_{21} is referred to as Mutual Inductance and the two inductors are said to be mutually coupled. M_{21} quantifies the flux in coil 2 due to the current in coil 1. The value of mutual inductance depends on the geometry and construction of the inductors. We will revisit this concept in detail.

Relationship Between Voltage and Current:

We have

$$\Phi = L\,i$$

We use Faraday's law to establish the relationship between current and voltage in an inductor

$$v = \frac{d\Phi}{dt} = L\frac{di}{dt}$$

$$v(t) = L\frac{di(t)}{dt}$$

This is the voltage developed in the inductor due to the change in the magnetic flux or current. Polarity of the voltage is given by *Lenz's law* such that the current produced due to the voltage across conductor opposes the current or magnetic flux producing it.

This relationship governs the behaviour of the inductor in electric circuits.

We can have different interpretation. But the most important is that

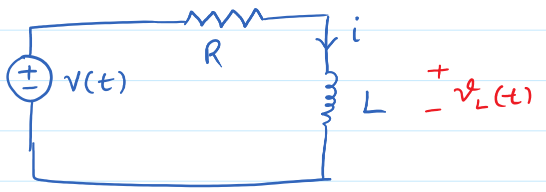
- *Inductor does not allow instantaneous change in the current*
- *Flux cannot change instantaneously*
- *To change the current instantaneously, we require infinite amount of voltage across the inductor'*

We can also express current in terms of voltage as

$$i(t) = \frac{1}{L}\int_{\tau=-\infty}^{t} v(\tau)d\tau$$

Let's illustrate with the help of couple of examples.

Example 01



* Establish polarity of voltage across inductor

* We assume $v_L(t)$ as indicated and show that this is the correct polarity. Note that this polarity is consistent with the 'passive sign convention'.

Using Ohm's Law

$$i(t) = \frac{v(t) - v_L(t)}{R} \quad \text{— (01)}$$

* When $v(t)$ is increasing

$\Rightarrow$ $i(t)$ is increasing

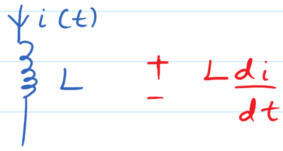
$\Rightarrow$ $\frac{di}{dt}$ is positive $\Rightarrow$ $\underbrace{L\frac{di}{dt}}$ is positive

* This is infact voltage across inductor

* Polarity of $L\frac{di}{dt}$ should be such that it reduces the current $i(t)$.

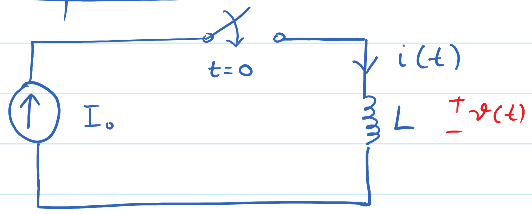
$\therefore$ $v_L(t) = L\frac{di}{dt}$ (positive) will decrease the current (From (01))

Hence;



$\longleftrightarrow$ Passive sign Convention

Example 02 :



$*$ $i(t) = I_0\, u(t)$

$*$ $v(t) = L\dfrac{di}{dt}$

$= L\, I_0\, \delta(t)$

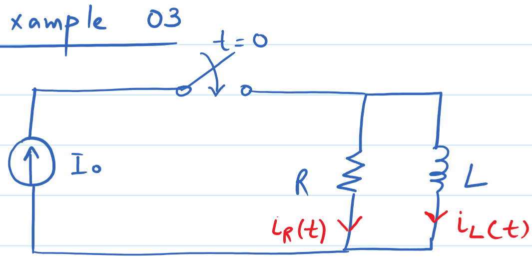
(Impulse)

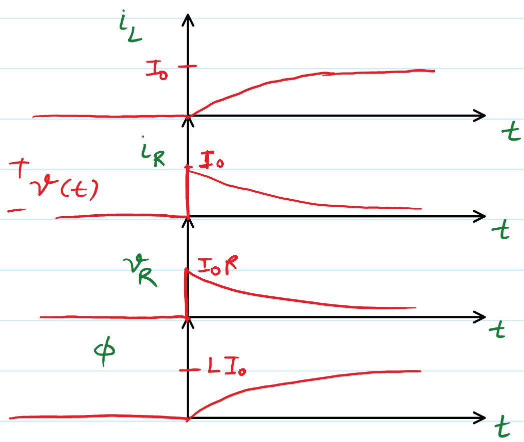
$*$ Due to instantaneous change in current, infinite voltage (impulse) develops across conductor.

$*$ Due to this, inductors are not connected directly (in series) with the DC current source.

$*$ Analog of this for the case of C; C is not connected in parallel to DC voltage source (Recall spark due to impulse of current).

Example 03

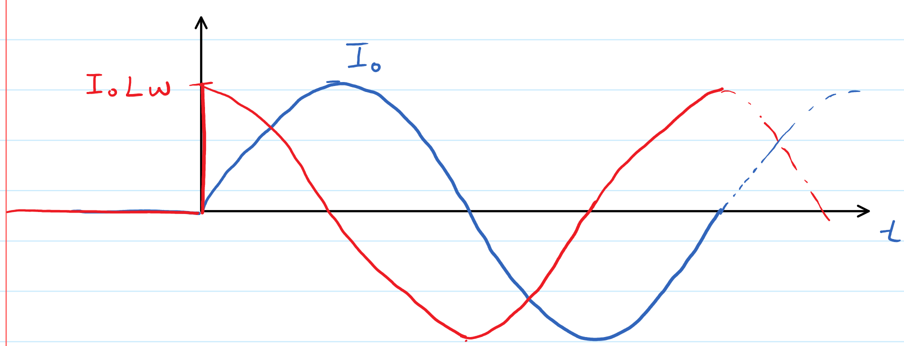




Example 04

Current through inductor ; $i_L = I_0 \sin\omega t\, u(t)$

Voltage across inductor ; $v_L = L\dfrac{di_L}{dt} = I_0 L\omega \cos\omega t\, u(t)$



$*$ Voltage leads current

$*$ current lags voltage

In capacitor, current leads voltage
voltage lags current

Energy in Inductors:

To find the energy or work in establishing the current I over time 't' in the inductor, we can integrate the power expression and use the relationship between current and the voltage in the inductor as follows

$$w = \int_o^t p(\tau)d\tau = \int_0^t v(\tau)\, i(\tau)d\tau = L\int_0^t \frac{di}{d\tau}\, i(\tau)\, d\tau = L\int_0^I i\, di\ = \frac{1}{2}\, LI^2$$

We note again that the energy is stored in the magnetic field (flux) produced by the current flowing through the inductor.