EE 240 Circuits I

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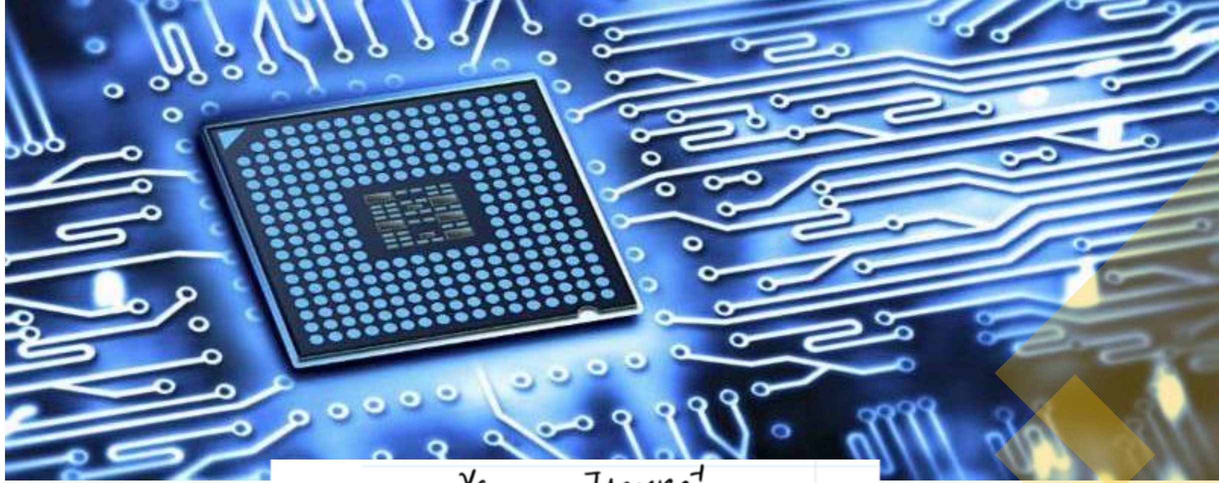
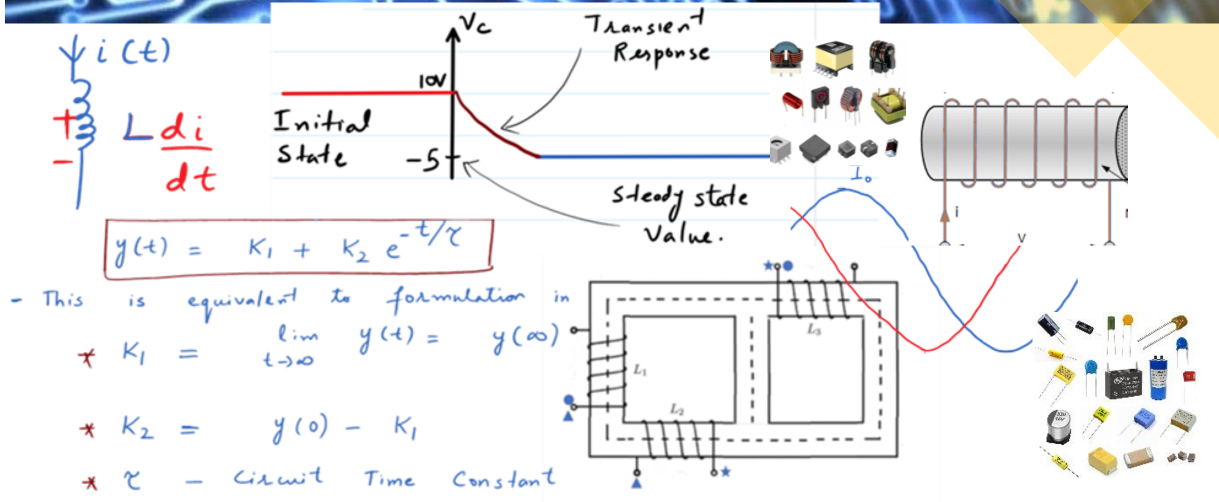
- Circuit Analysis
- Kirchhoff’s Current Law (Nodal Analysis)
- Kirchhoff’s Voltage Law (Loop Analysis)
- Examples





Circuit Analysis

Determine the current through or the voltage across every element in the circuit.

Big Picture: We will learn techniques to carry out circuit analysis in the following order.

Kirchhoff's Laws

- Formulation of equations for
  - Single node, single loop
  - Multiple nodes or loops
- Nodal Analysis
  - Super Node
  - Resistive networks
- Loop Analysis
  - Super Loop
  - Resistive networks
- Matrix formulation of equations
- Equivalent Networks
  - Series parallel equivalent (already covered)
  - Source transformation
  - Thevenin Equivalent
  - Norton Equivalent
- Superposition Principle
- Concept of Duality

Circuit Analysis

Circuit:

Interconnection of circuit elements; passive elements and energy sources such that the energy flows between elements.

We analyse circuit by formulating network equations that is governed by Kirchhoff's Laws. These laws can be intelligently employed to formulate minimum number of independent equations. Before we formally define these laws, we quickly review the concepts node, loop, branch and ground node.

Node (Junction):

A point of connection of two or more circuit elements.

Loop:

Any closed path through the circuit in which no node is encountered more than once.

Branch:

Part of circuit containing one element and nodes on both sides.

Ground, Zero potential node, Reference or Datum Node:

Since voltage in a circuit is a relative quantity, it is useful to have one reference node such that all the voltages are expressed with respect to the reference node.

Circuit Analysis

Independent Loop:

A loop is said to be independent if it contains at least one branch which is not a part of any other independent loop.

Network Topology (Graphical Representation of a Circuit):

Topology is the branch of Geometry (helps us in analyzing the circuit).

Graph is a collection of nodes and lines connecting the nodes.

- Nodes are also called as vertices.
- Lines are also called edges or branches.
- If the path from one node to the other node is uni-directional, such path is represented by an arrow and the graph is referred to as directed graph or oriented graph.

For a given electrical circuit, we construct graph by replacing network elements with branches.

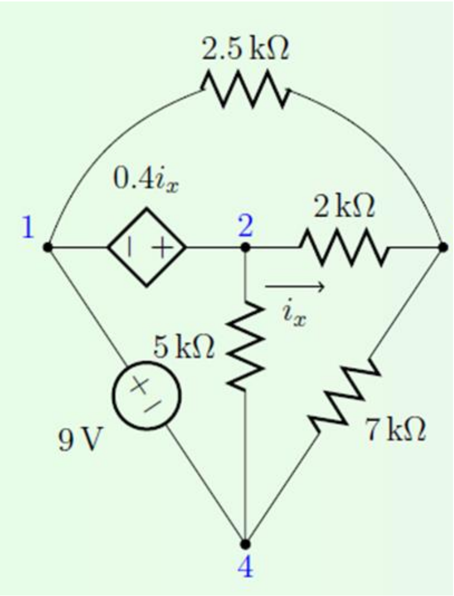
A network with **b** branches, **n** nodes, and **l** independent loops will satisfy the fundamental theorem of network topology:

number of nodes + number of independent loops = number of branches + 1

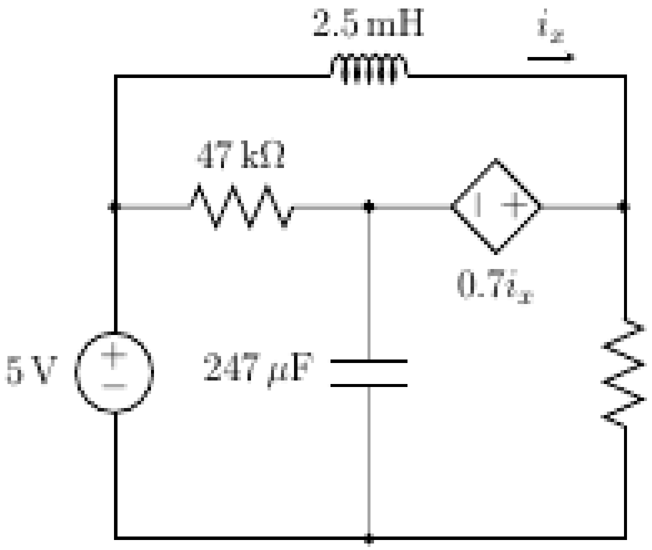
$$n + l = b + 1$$

Graphical Representation of Electric Circuits (Networks)

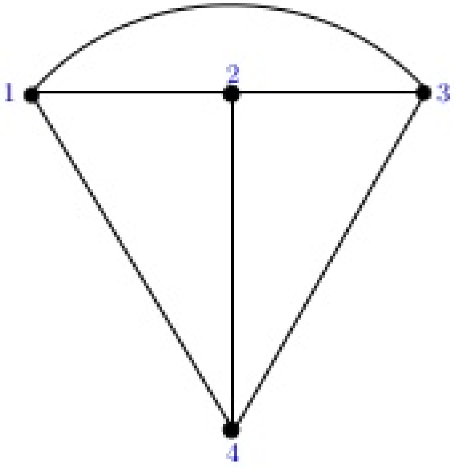
Example:



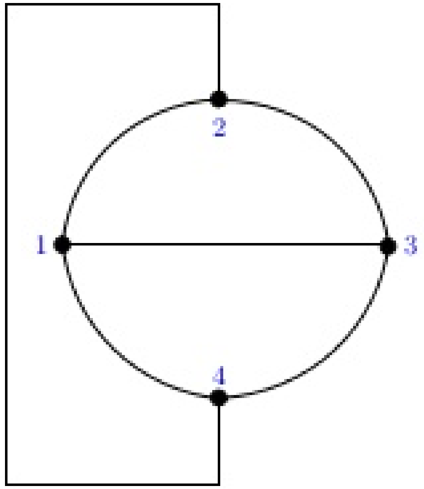
Circuit 1



Circuit 2



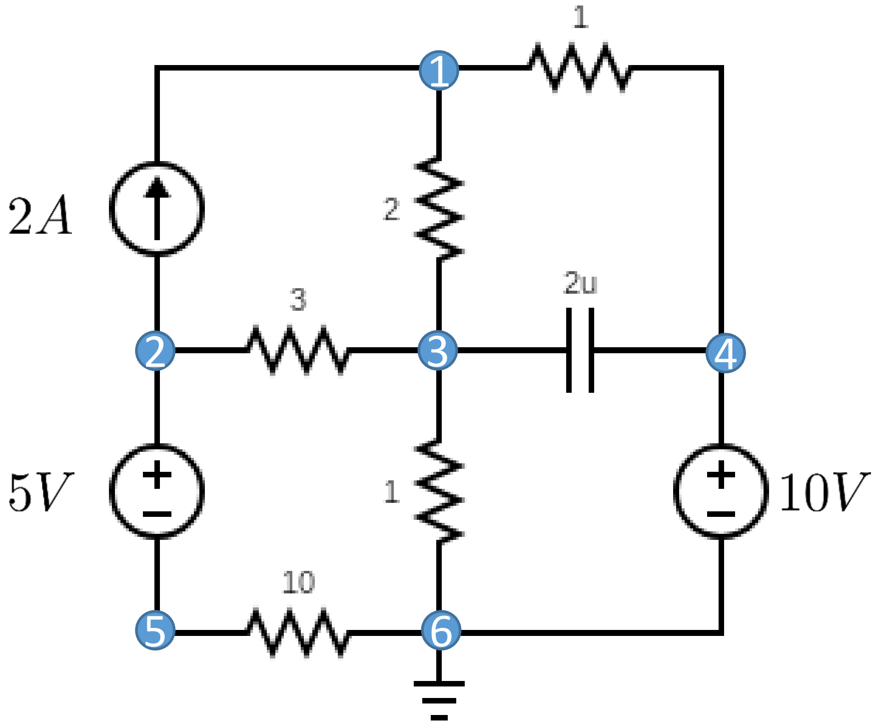
Graph



Graph Equivalent

Circuit Analysis

Example:



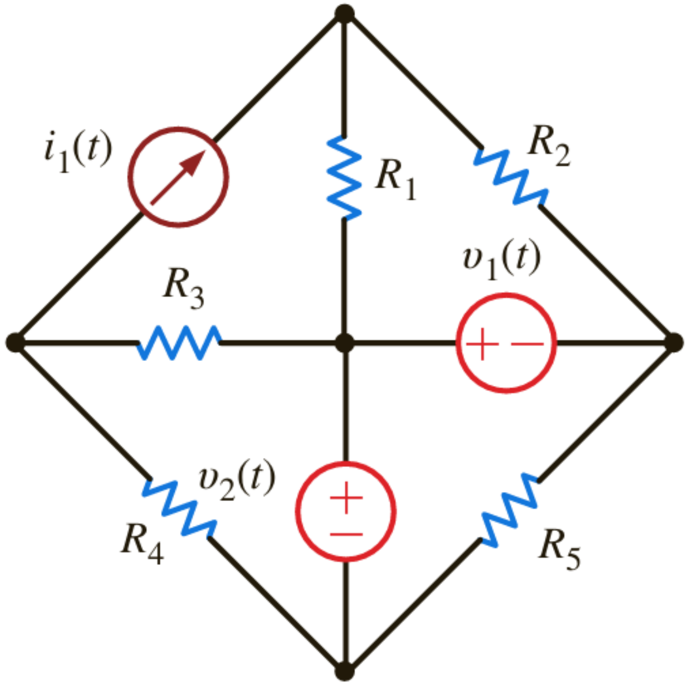
Interpretations:

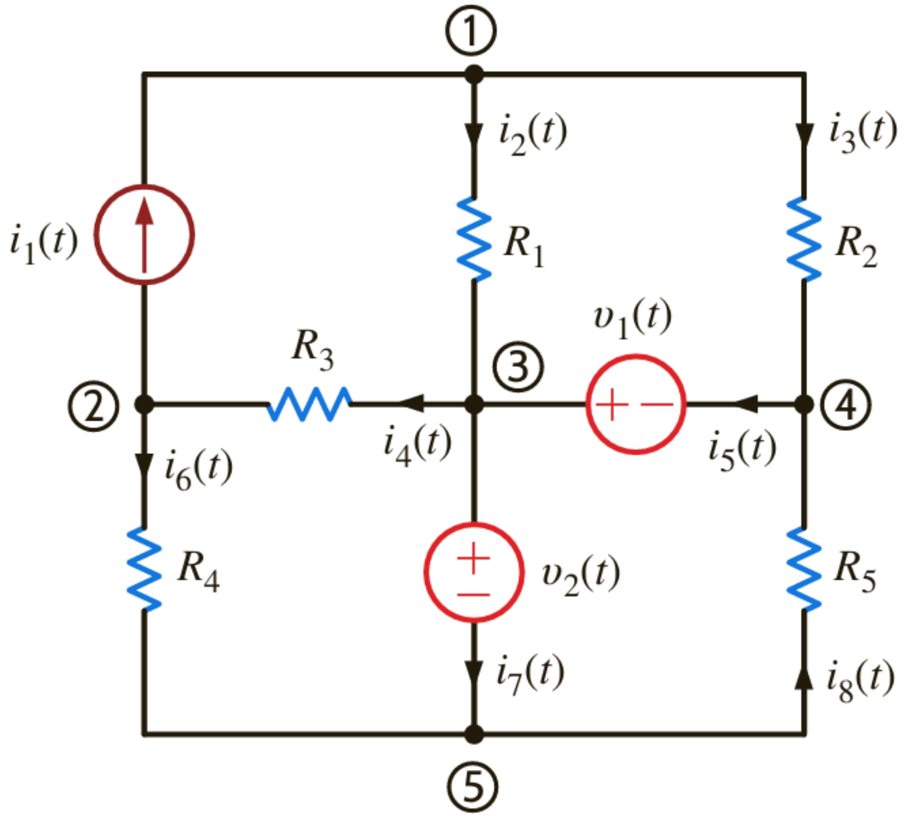
- There are 6 nodes
- There are 9 branches
- We have considered node 6 as ground node
- Node 4 is at 10 V (with respect to ground node)
- 2A current is being drawn from node 2 and is being fed to node 1 through the branch connecting nodes 1 and 2.
- Node 2 is at 5V with respect to node 5

Circuit Analysis

Example:

Figure 2.5 (Irwin and Nelms):





- *Both are same circuits*

Kirchhoff’s Current Law (KCL)

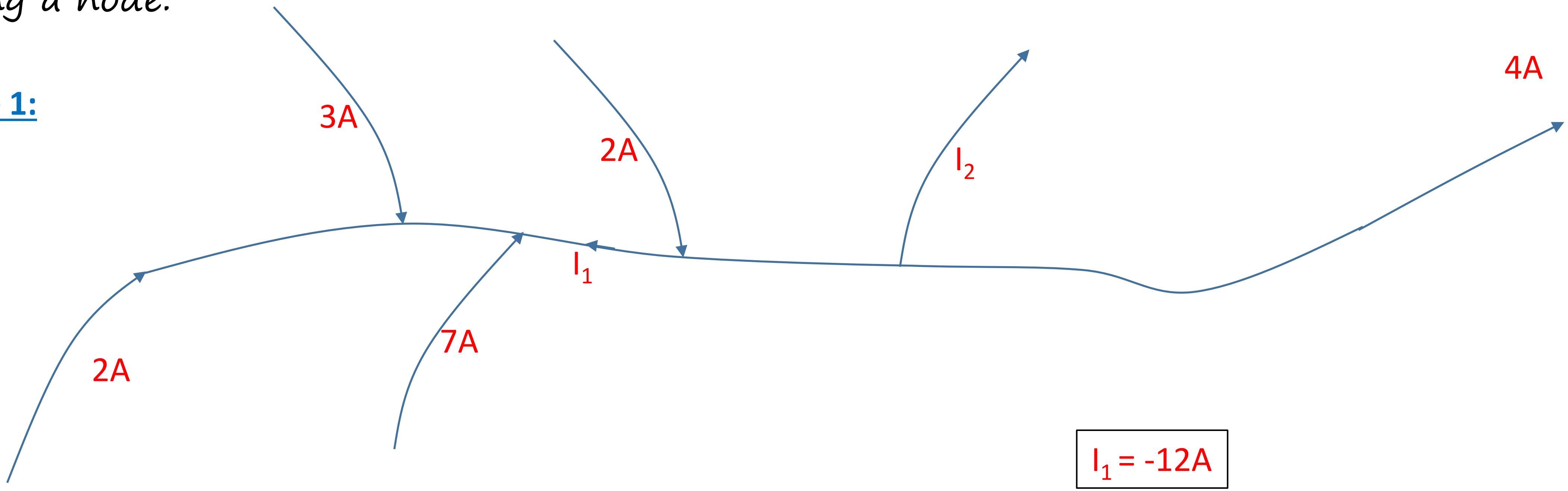
Statement:

Algebraic sum of all the currents entering (or leaving) a node is equal to zero.

OR

Algebraic sum of the currents leaving a node is equal the algebraic sum of the currents entering a node.

Example 1:



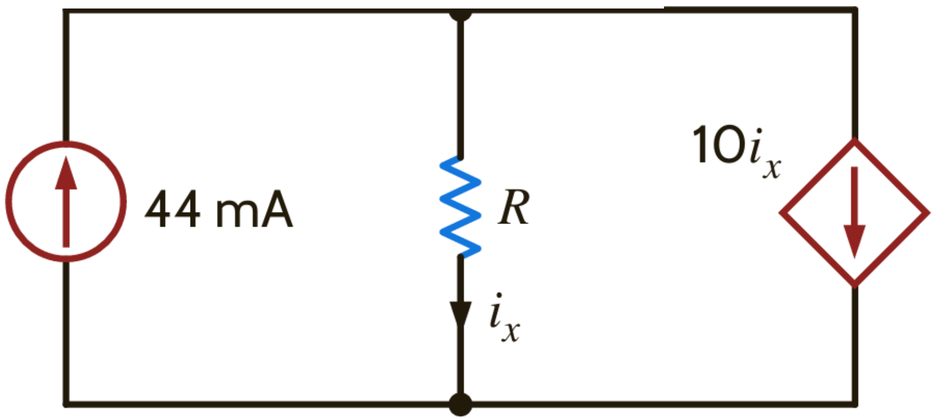
$I_1 = -12A$

$I_2 = 10A$

Kirchhoff's Current Law (KCL)

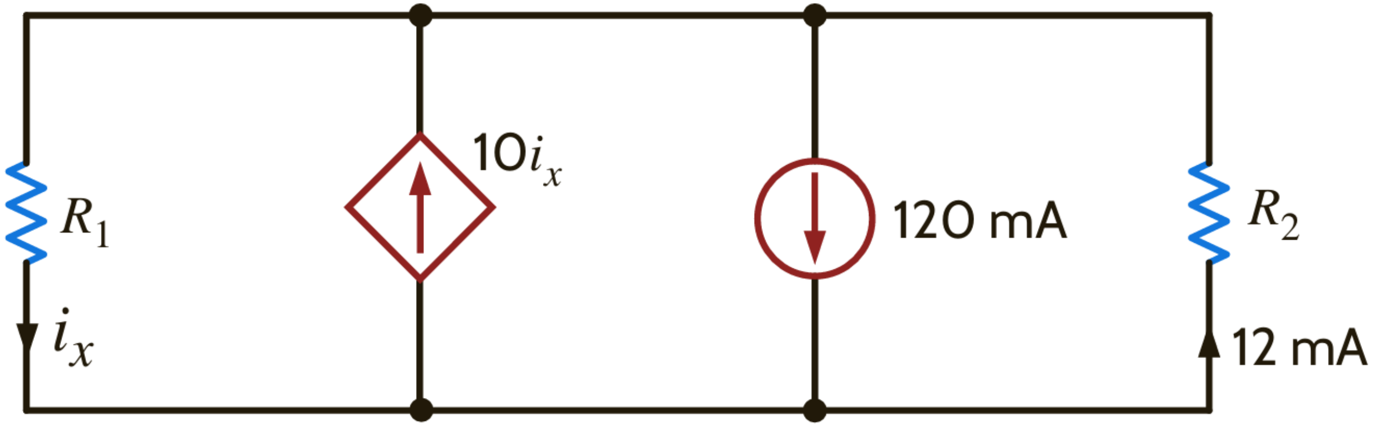
Example 2 (E2.6 Irwin and Nelms):

Find the current i_x in the circuits



$$11i_x = 44 \text{ mA}$$

$$i_x = 4 \text{ mA}$$



$$10i_x + 12 \text{ mA} = i_x + 120 \text{ mA}$$

$$i_x = 12 \text{ mA}$$

Nodal Analysis using Kirchhoff's Current Law

Steps:

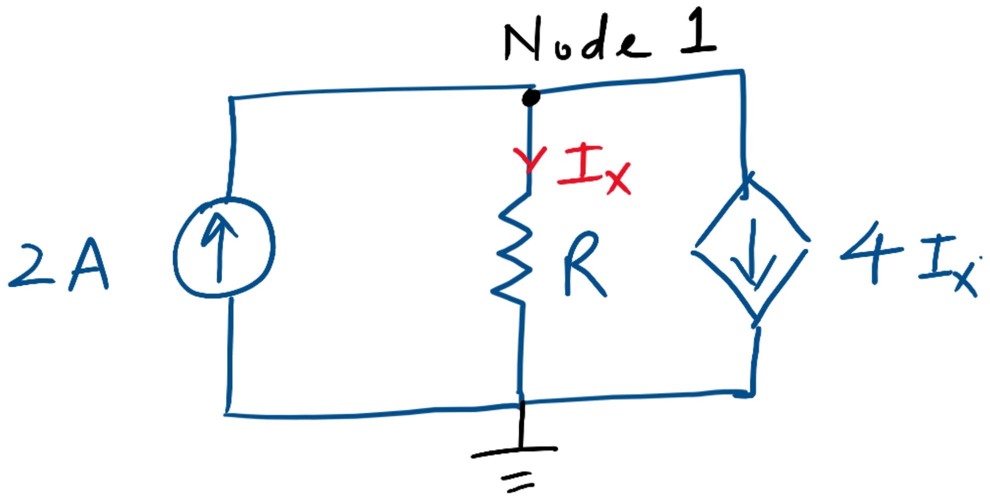
- *Define ground (reference) node*
- *Define the variables to indicate voltages at other nodes*
- *Apply KCL at each node to obtain equation associated with the node.*
- *Number of equations = number of nodes -1*

Example:

Single Node-Pair Circuit:

Determine I_x.

$$I_x = \frac{2}{5}\ \text{A}$$

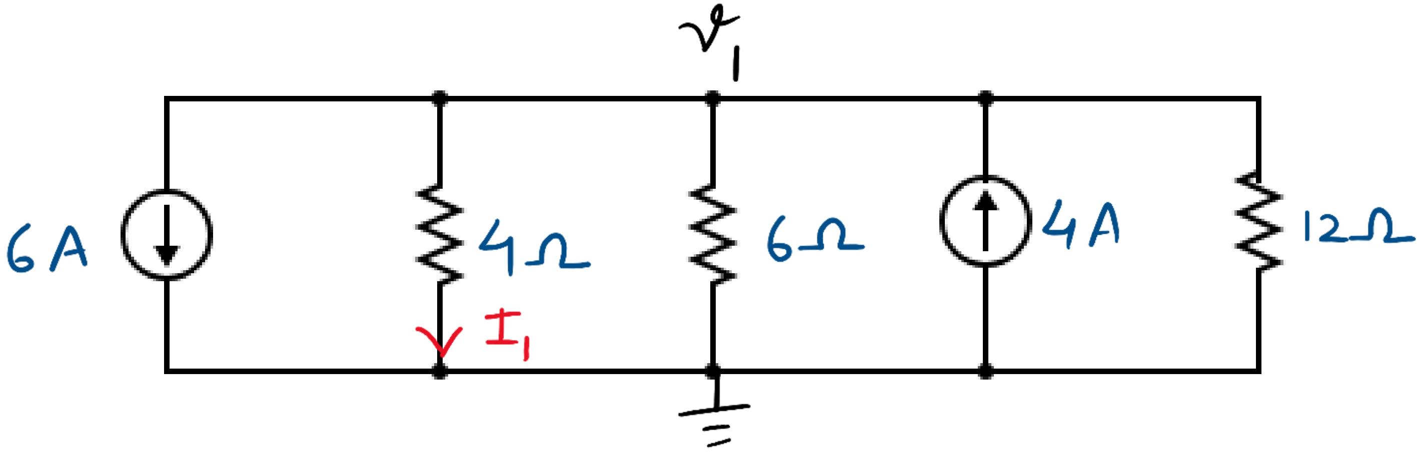


Nodal Analysis using Kirchhoff's Current Law

Example:
Single Node-Pair Circuit:

Determine I_1 and power absorbed by $6\,\Omega$ resistor.

Applying KCL at node 1 yields



$$\frac{v_1}{4} + \frac{v_1}{6} + \frac{v_1}{12} + 6 - 4 = 0$$

$$v_1 = -4\,V$$

$$I_1 = \frac{v_1}{4} = -1\,A$$

Power absorbed by $6\,\Omega$ resistor

$$P_6 = \frac{v_1^2}{R} = \frac{16}{6}\,W$$

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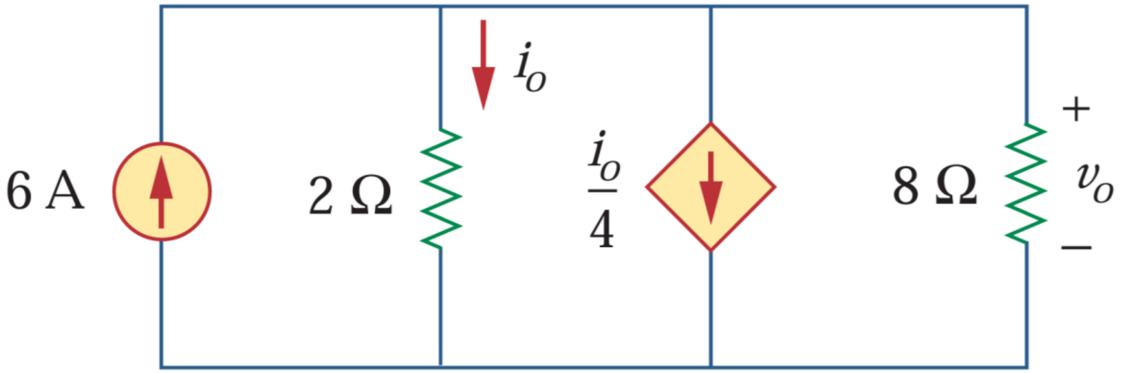
Nodal Analysis using Kirchhoff's Current Law

Example:

Single Node-Pair Circuit:

Determine v_o and i in the circuit



Step 1: Apply KCL at the top node

Using Kirchhoff's Current Law (KCL) at the top node, the total current entering and leaving the node is:

$$6 = i_o + \frac{i_o}{4} + \frac{v_o}{8}$$

Step 2: Express v_o in terms of i_o using Ohm's Law

$$v_o = 8 \cdot \left(i_o + \frac{i_o}{4} \right) = 10 i_o$$

Step 3: Substitute v_o into the KCL equation

Substitute $v_o = 10 i_o$ into the KCL equation:

$$6 = i_o + \frac{i_o}{4} + \frac{10 i_o}{8} \quad \Rightarrow i_o = \frac{6}{2.5} = 2.4\,A$$

Now, calculate v_o

$$v_o = 10 i_o = 10 \times 2.4 = 24\,V$$

Kirchhoff's Voltage Law (KVL)

Statement:

Algebraic sum of all the drop (or gain) of voltages around any closed-path is equal to zero.

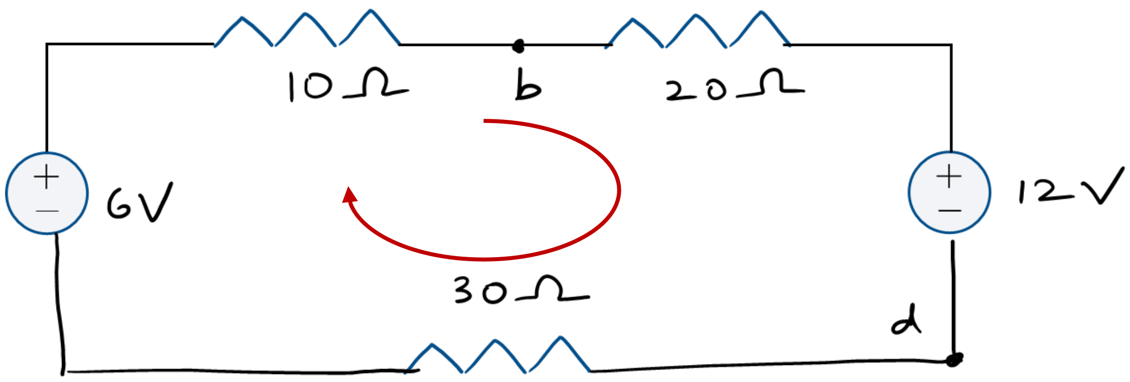
OR

Algebraic sum of the drop of voltages is equal the algebraic sum of the gain of voltages around a closed path.

Example:

Single Loop Circuit:

Determine V_{bd}.



$$10i + 20i + 12 + 30i - 6 = 0$$

$$\Rightarrow \quad i = \boxed{-0.1\,A}$$

$$v_{bd} = v_{bc} + v_{cd} = (20)(i) + 12 = \boxed{10V}$$

Loop Analysis using Kirchhoff's Voltage Law

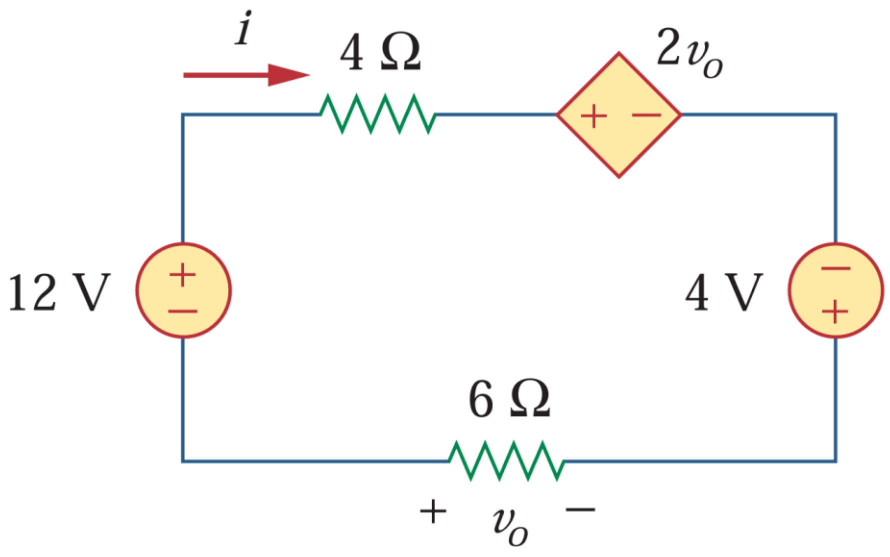
Steps:

- *Define Loop (or mesh) currents (variable + direction)*
- *Apply KVL around each loop to obtain equation associated with the loop.*

Example:

Single Loop Circuit:

Determine v_o and i in the circuit



Apply Kirchhoff's Voltage Law (KVL) around the loop in direction of i

$$-12\,V + (4\,\Omega)\cdot i + 2v_o - 4\,V + (6\,\Omega)\cdot i = 0$$

Using Ohm's law $v_o = -6i$, substitute into the KVL equation yields:

$$i = -8\,A$$

Calculate v_o

$$v_o = 48\,V$$