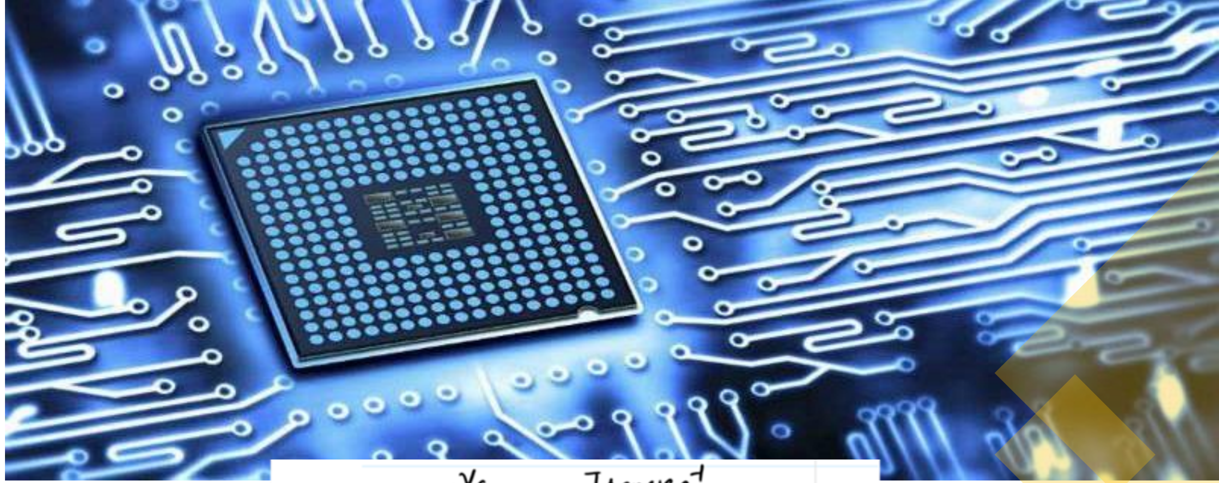
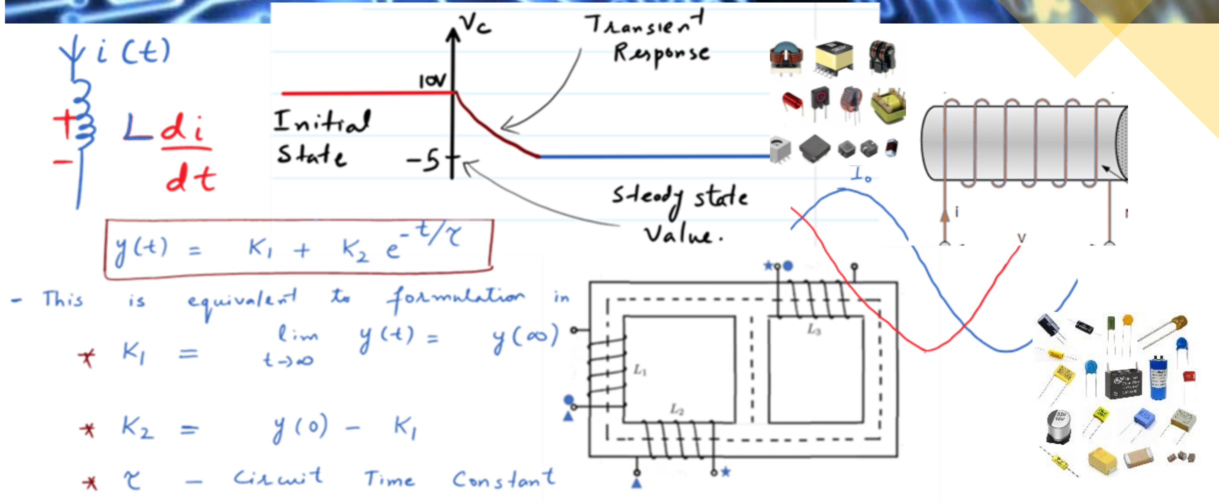
EE 240 Circuits I

Dr. Zubair Khalid

Department of Electrical Engineering
School of Science and Engineering
Lahore University of Management Sciences

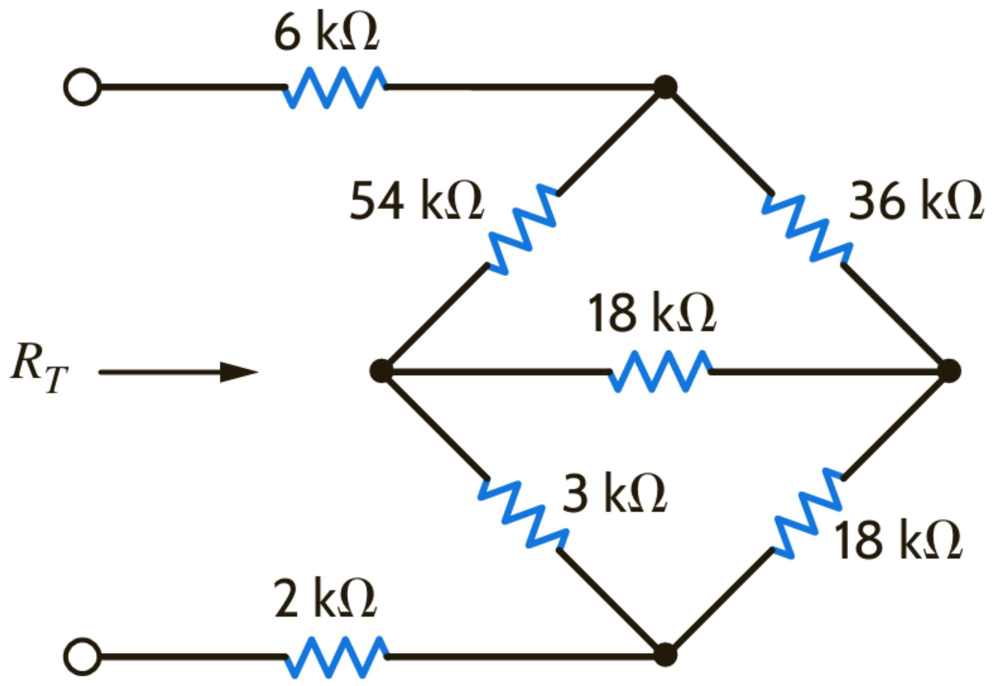
- Delta-Wye Transformation
- Nodal Analysis





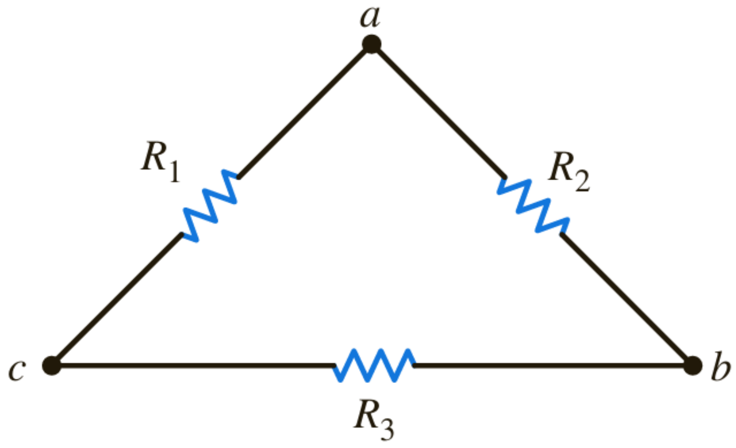
Delta-Wye Transformation

Motivation – Example: Find Equivalent Resistance)

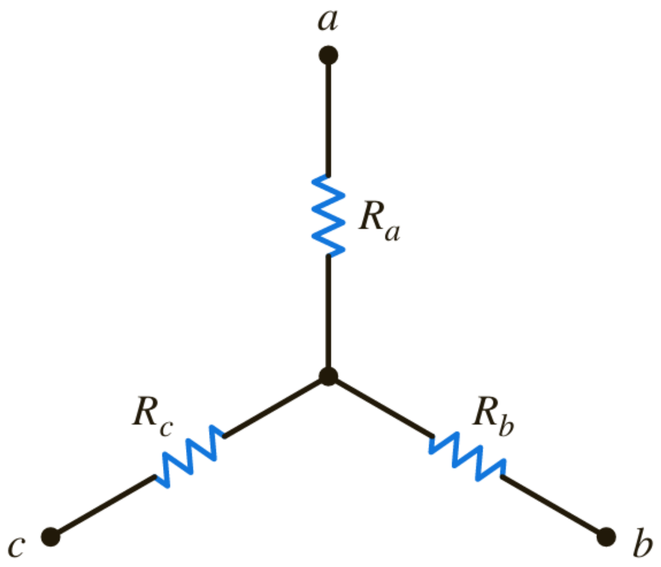


Delta-Wye Transformation

Transformation:



Delta



Wye (Y)

Delta-Wye Transformation

Transformation:

Equivalent Resistance Between Points A and B

The equivalent resistance between points A and B in the Delta configuration is given by:

$$R_{AB}^{\Delta} = \frac{R_1 \cdot (R_2 + R_3)}{R_1 + R_2 + R_3}$$

The equivalent resistance between points A and B in the Wye configuration is:

$$R_{AB}^{Y} = R_a + R_b$$

Equating yields:

$$R_a + R_b = \frac{R_1 \cdot (R_2 + R_3)}{R_1 + R_2 + R_3}$$

Delta-Wye Transformation

Transformation:

Equivalent Resistance Between Points B and C

$$R_{BC}^{\Delta} = \frac{R_2 \cdot (R_1 + R_3)}{R_1 + R_2 + R_3}$$

$$R_b + R_c = \frac{R_2 \cdot (R_1 + R_3)}{R_1 + R_2 + R_3}$$

Equivalent Resistance Between Points C and A

$$R_{CA}^{\Delta} = \frac{R_3 \cdot (R_1 + R_2)}{R_1 + R_2 + R_3}$$

$$R_c + R_a = \frac{R_3 \cdot (R_1 + R_2)}{R_1 + R_2 + R_3}$$

Delta-Wye Transformation

Transformation:

The Wye resistances R_a, R_b, R_c in terms of the Delta resistances R_1, R_2, R_3 are:

$$R_a = \frac{R_1 \cdot R_3}{R_1 + R_2 + R_3}$$

$$R_b = \frac{R_1 \cdot R_2}{R_1 + R_2 + R_3}$$

$$R_c = \frac{R_2 \cdot R_3}{R_1 + R_2 + R_3}$$

Nodal Analysis using Kirchhoff's Current Law

Circuits with Independent Current Sources:

For Node V_1:

$$\frac{V_1 - V_2}{2\,\mathrm{k}\Omega} + \frac{V_1 - V_3}{1\,\mathrm{k}\Omega} = 4\,\mathrm{mA}$$

For Node V_2:

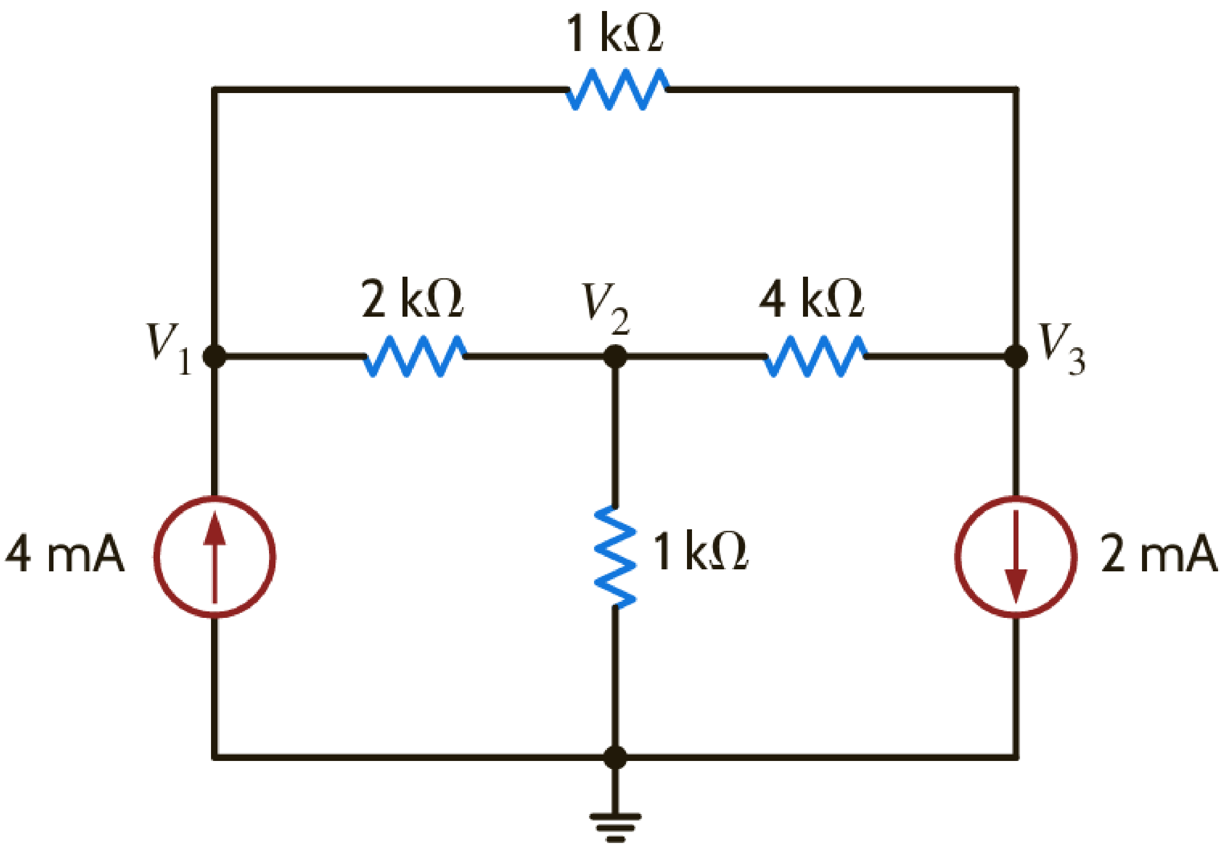
$$\frac{V_2 - V_1}{2\,\mathrm{k}\Omega} + \frac{V_2 - V_3}{4\,\mathrm{k}\Omega} + \frac{V_2}{1\,\mathrm{k}\Omega} = 0$$

For Node V_3:

$$\frac{V_3 - V_2}{4\,\mathrm{k}\Omega} + \frac{V_3 - V_1}{1\,\mathrm{k}\Omega} = -2\,\mathrm{mA}$$

The solution for the node voltages is:

$$V_1 = \frac{38}{7}\,\mathrm{V}, \quad V_2 = 2\,\mathrm{V}, \quad V_3 = \frac{22}{7}\,\mathrm{V}$$



The system of equations can be written in matrix form as:

$$\begin{pmatrix} \frac{1}{2}+1 & -\frac{1}{2} & -1 \\ -\frac{1}{2} & \frac{1}{2}+\frac{1}{4}+1 & -\frac{1}{4} \\ -1 & -\frac{1}{4} & \frac{1}{4}+1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix}$$

(Inverse of Resistances) × (Voltges) = Currents (Ohm's Law)

Nodal Analysis using Kirchhoff's Current Law

Circuits with Independent Current Sources:

The equations for the circuit using nodal analysis are:

For node V_1:

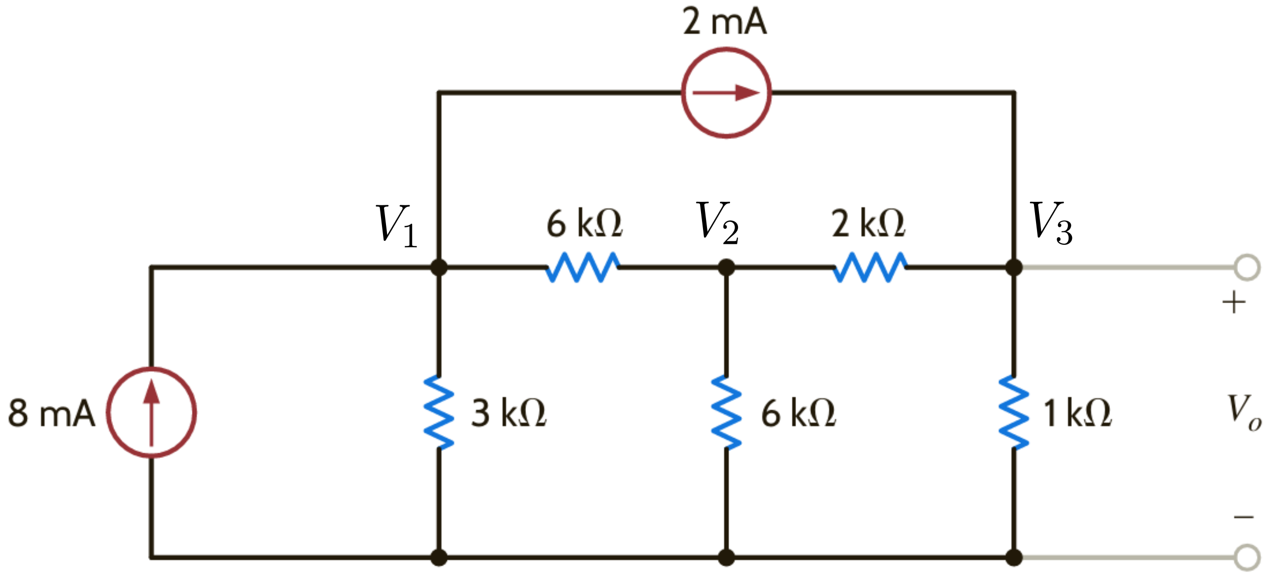
$$\frac{V_1}{3\,\mathrm{k}\Omega} + \frac{V_1 - V_2}{6\,\mathrm{k}\Omega} = 6\,\mathrm{mA}$$

For node V_2:

$$\frac{V_2 - V_1}{6\,\mathrm{k}\Omega} + \frac{V_2}{6\,\mathrm{k}\Omega} + \frac{V_2 - V_3}{2\,\mathrm{k}\Omega} = 0$$

For node V_3:

$$\frac{V_3 - V_2}{2\,\mathrm{k}\Omega} + \frac{V_3}{1\,\mathrm{k}\Omega} = 2\,\mathrm{mA}$$



The nodal equations can be written in matrix form as:

$$\begin{pmatrix} \frac{1}{3} + \frac{1}{6} & -\frac{1}{6} & 0 \\ -\frac{1}{6} & \frac{1}{6} + \frac{1}{6} + \frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} + 1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \\ 2 \end{pmatrix}$$

Solving either matrix form or equations yields:

$$V_1 = 18\,\mathrm{V}, \quad V_2 = 12\,\mathrm{V}, \quad V_3 = 4\,\mathrm{V}$$

The output voltage is $V_o = V_3$