EE 240 Circuits I

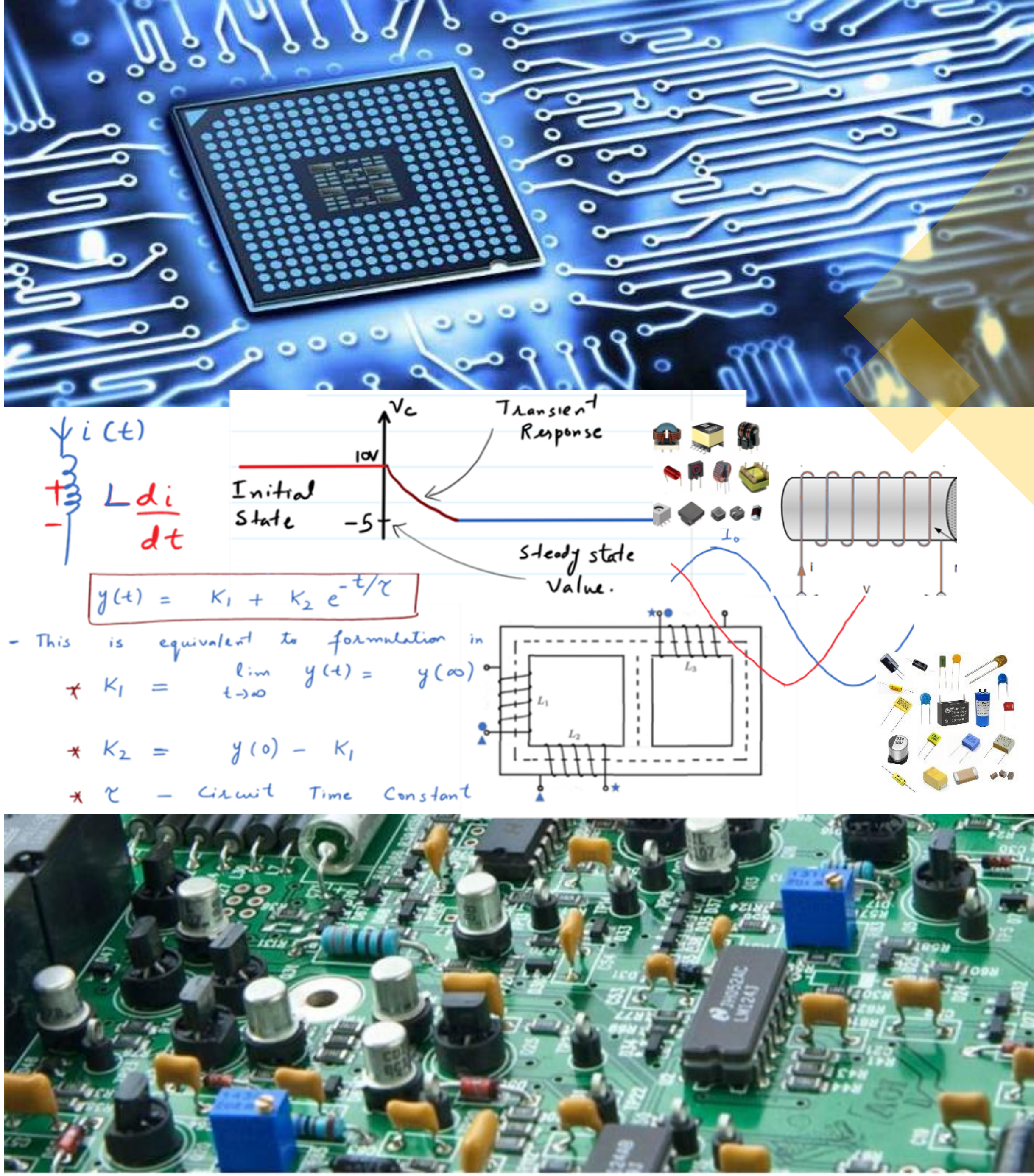
Dr. Zubair Khalid

Department of Electrical Engineering

School of Science and Engineering

Lahore University of Management Sciences

- Nodal Analysis (Super Node)
- Loop Analysis



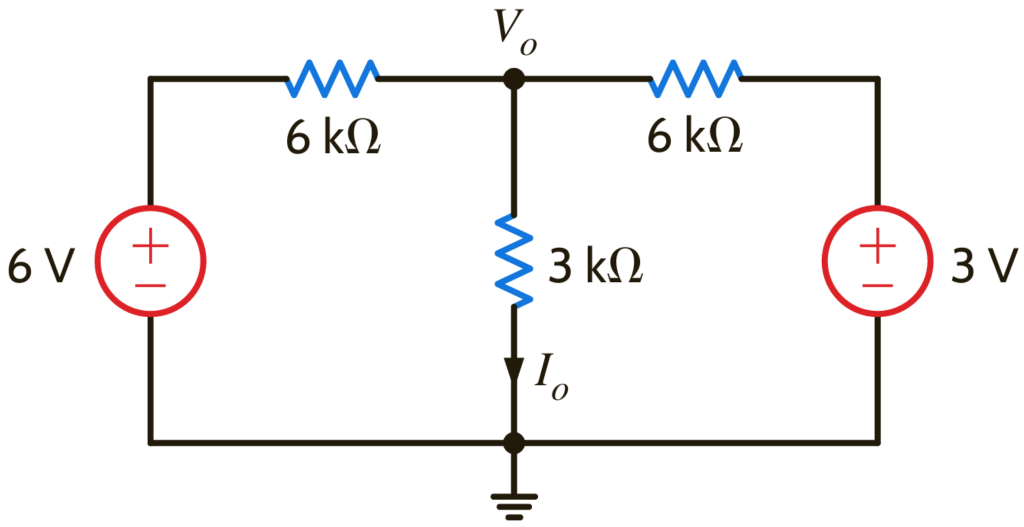
Nodal Analysis using Kirchhoff's Current Law

Circuits with Voltage Source Between the Nodes (Super Node) – Motivation:

Step 1: Apply KCL at Node V_o

The sum of currents leaving the node is zero:

$$\frac{V_o - 6}{6\,\mathrm{k\Omega}} + \frac{V_o}{3\,\mathrm{k\Omega}} + \frac{V_o - 3}{6\,\mathrm{k\Omega}} = 0$$



Step 2: Solve the Equation

$$4V_o - 9 = 0 \Rightarrow V_o = 2.25\,\mathrm{V}$$

Step 3: Find I_o

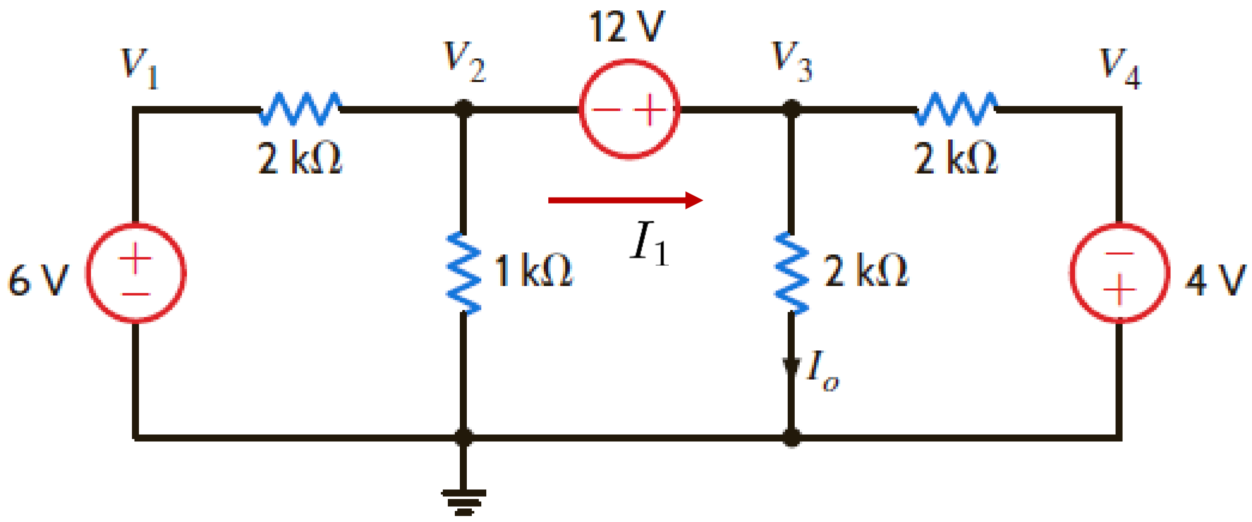
I_o is the current flowing through the $3\,\mathrm{k\Omega}$ resistor to the ground:

$$I_o = \frac{V_o}{3\,\mathrm{k\Omega}} == 0.75\,\mathrm{mA}$$

Nodal Analysis – Super-node Concept

Example: Carry out nodal analysis

$V_1 = 6V$ $\qquad$ $V_4 = -4V$



When we attempt to write nodal equation for either node 2 or node 3, we face a problem.

As we have a 12 voltage source in the branch connecting node 2 and node 3, the current in this branch is certainly not known and it cannot be directly expressed.

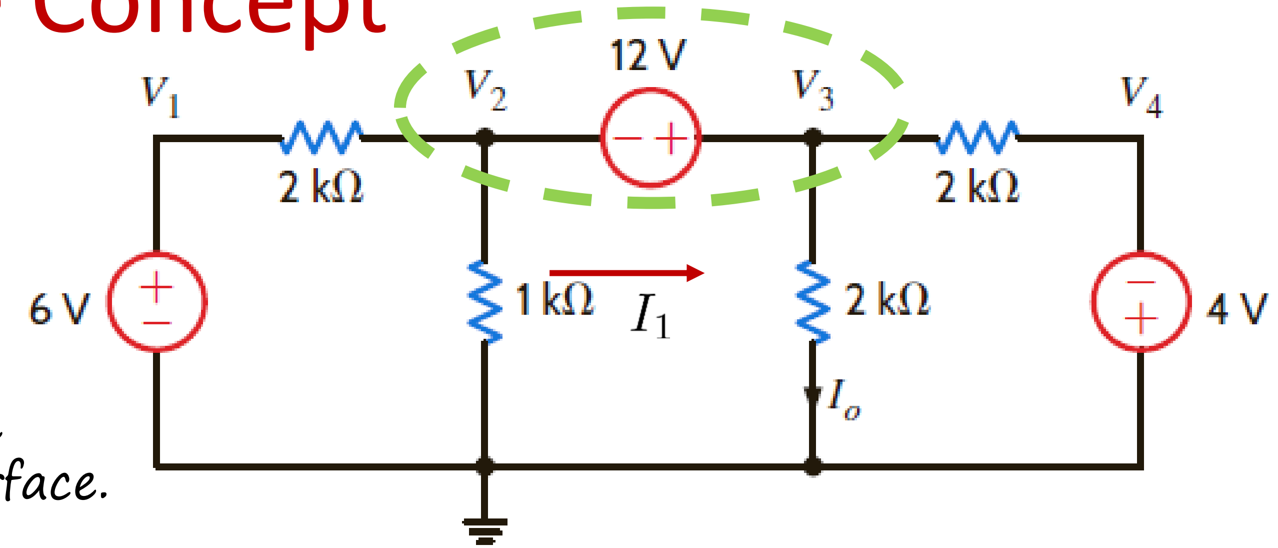
To tackle this problem, we assume that this current is denoted by I_1 as indicated in the figure.

Now we can write equations node 2 and node 3 as follows:

Node 2 Equation: $\frac{V_2 - V_1}{2k} + \frac{V_2}{1k} + I_1 = 0$

Node 3 Equation: $\frac{V_3 - V_4}{2k} + \frac{V_3}{2k} - I_1 = 0$

Nodal Analysis – Super-node Concept



Example: Carry out nodal analysis

Adding the equations for Nodes 2 and 3 yields:

$$\frac{V_2 - V_1}{2k} + \frac{V_2}{1k} + \frac{V_3 - V_4}{2k} + \frac{V_3}{2k} = 0$$

This equation is indeed an equation of the surface indicated in green when we apply KCL to the surface.

Since this surface is a collection of two nodes with **different voltages** at both the ends, we refer to this as **Super-node.**

If we encounter the voltage source between two nodes, we can form a Super-node and apply KCL to the super-node to obtain the network equation.

Q: We have one equation of the super-node that is in fact a sum of two equations. How do we get one more equation?

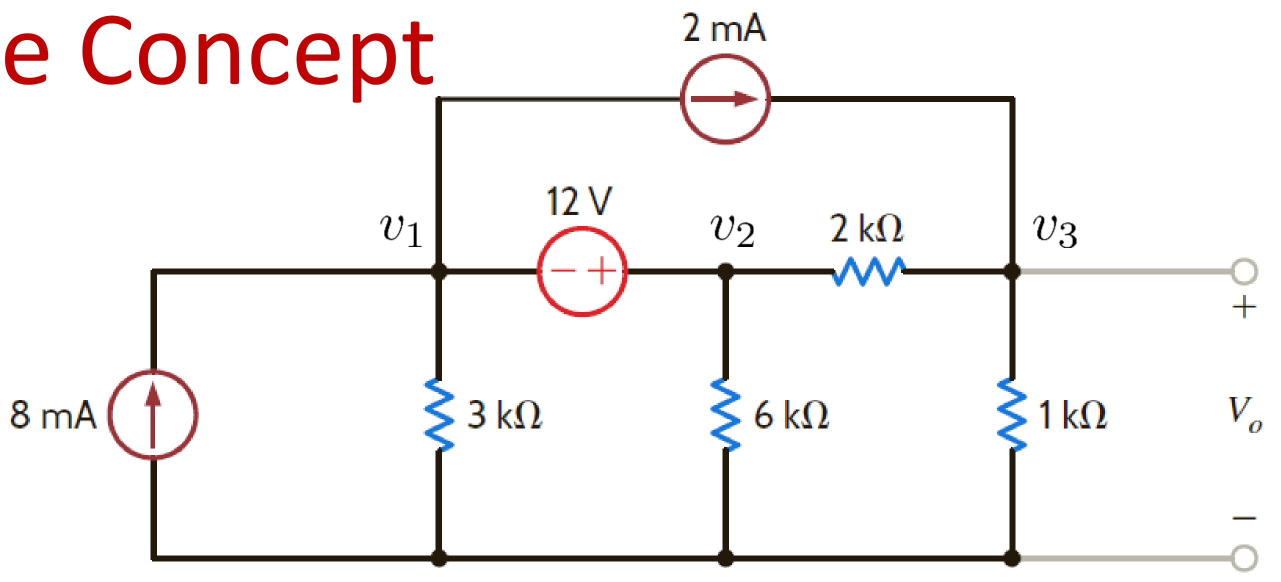
A: We obtain one more equation by relating potentials at these nodes using the voltage source between the nodes. 12V voltage source between node 2 and node 3 ensures the potential difference of 12V between node 3 and node 2

$$V_3 - V_2 = 12$$

LUMS
A Not-for-Profit University

Nodal Analysis – Super-node Concept

Reinforcement Example: Determine V_o.



Super-node Equation:

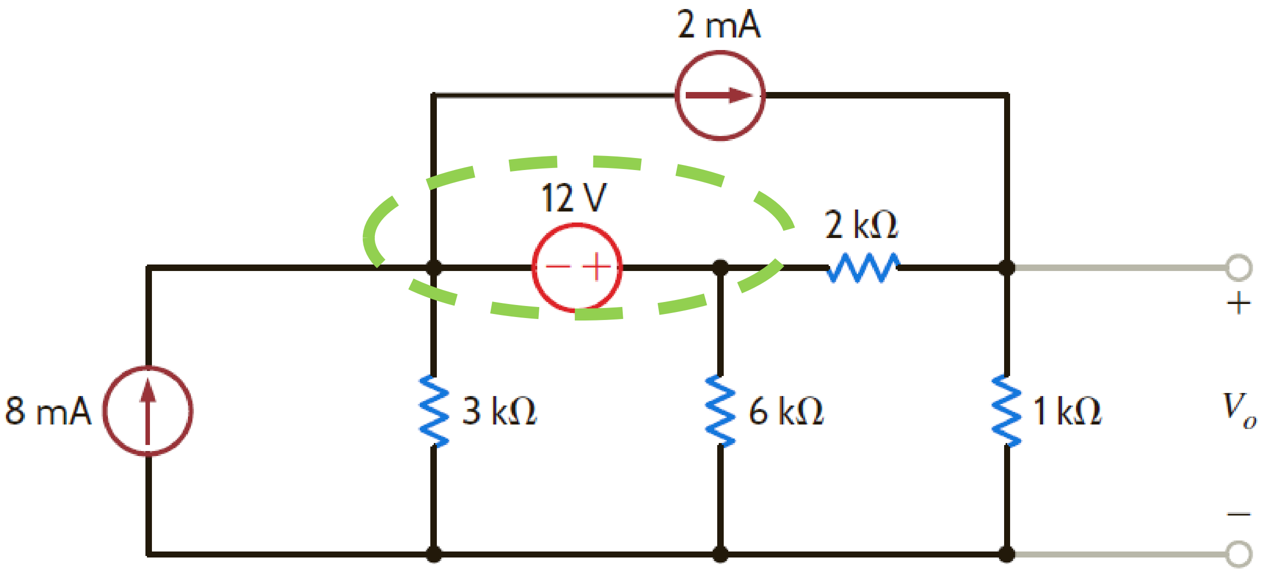
$$\frac{v_1}{3k} + \frac{v_2 - v_3}{2k} + \frac{v_2}{6k} - 8 + 2 = 0$$

Node 3 Equation:

$$\frac{v_3 - v_2}{2k} + \frac{v_3}{1k} - 2 = 0$$

Equation relating voltages at super-node:

$$v_2 - v_1 = 12$$

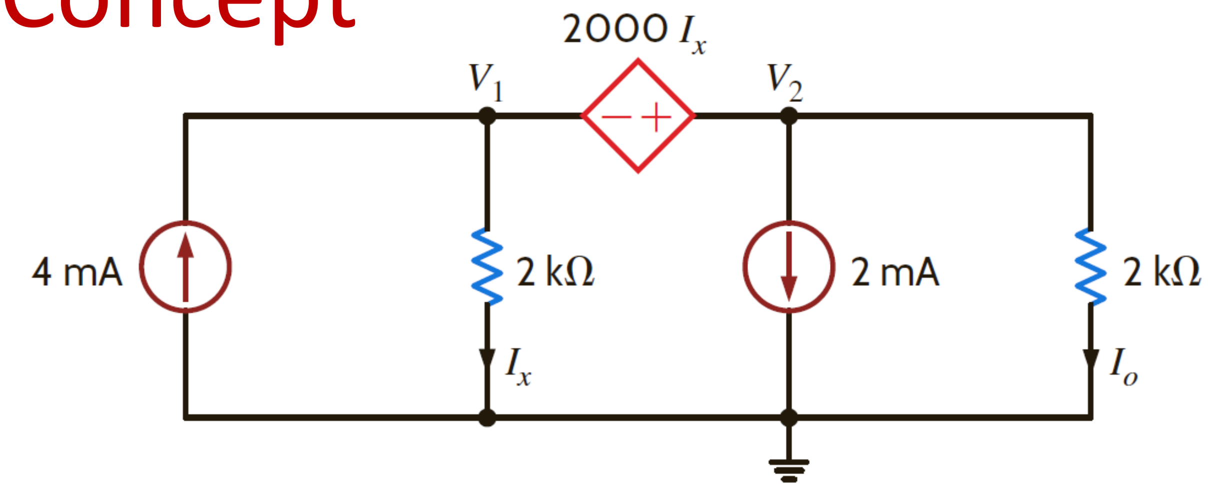


By solving these equations, we obtain

$$v_3 = V_o = 5.6V$$

Nodal Analysis – Super-node Concept

Reinforcement Example: Determine the current I_o.



Super-node Equation:

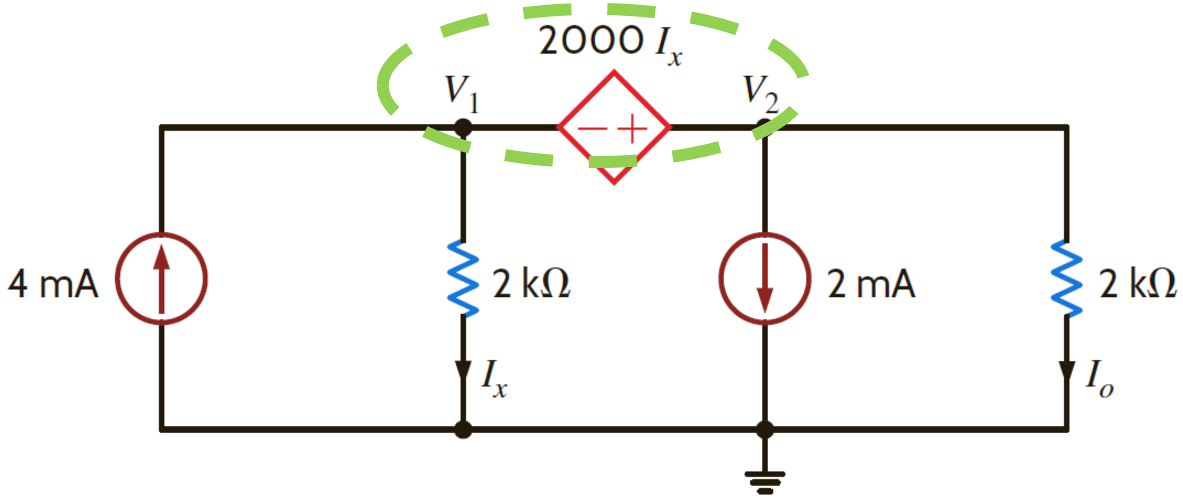
$$\frac{V_1}{2k} + \frac{V_2}{2k} - 4 + 2 = 0$$

Controlled Source Equation:

$$I_x = \frac{V_1}{2k}$$

Equation relating voltages at super-node:

$$V_2 - V_1 = 2000 I_x$$



By solving these equations, we obtain

$$I_o = \frac{V_2}{2k} = \frac{4}{3}\text{mA}$$

LUMS
A Not-for-Profit University

Nodal Analysis using Kirchhoff's Current Law

Summary:

STEP 1.
Identify all nodes in the circuit. Pick one node as the *reference node* (also called ground), and assign a voltage variable to each remaining node (these are called *nonreference nodes*). If there are N nodes, you will need to find $N-1$ node voltages.

STEP 2.
For each voltage source (either independent or dependent), write an equation describing the voltage source. Each voltage source will give one equation to help solve the circuit. If you have a dependent voltage source (one controlled by a circuit variable), express its value using the node voltages.

Nodal Analysis using Kirchhoff's Current Law

Summary:

STEP 3.

Use Kirchhoff's Current Law (KCL) to write the remaining equations. This law states that the total current entering a node equals the total current leaving that node. Apply KCL to each nonreference node that isn't directly connected to a voltage source. If a voltage source connects two nonreference nodes, treat them as a single "supernode" and apply KCL to this supernode as a whole. For dependent current sources, express their controlling variables in terms of the node voltages.

Loop (Mesh) Analysis using Kirchhoff's Voltage Law

No. of Independent Loops:

A network with **b** branches, **n** nodes, and **l** independent loops will satisfy the fundamental theorem of network topology:

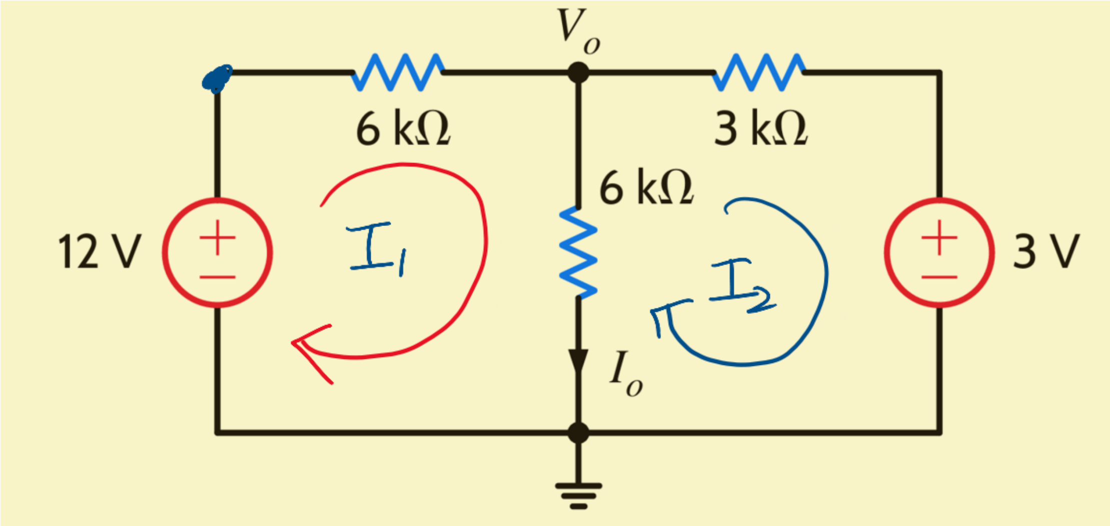
number of nodes + number of independent loops = number of branches + 1

$$n + l = b + 1$$

$$l = b - n + 1$$

Loop Analysis using Kirchhoff's Voltage Law

Circuits Involving Independent Voltage Sources:



* $I_1 6K + (I_1 - I_2) 6K - 12 = 0$

* $3 + \underbrace{(I_2 - I_1) 6K}_{V_o} + I_2 3K = 0$

In matrix form

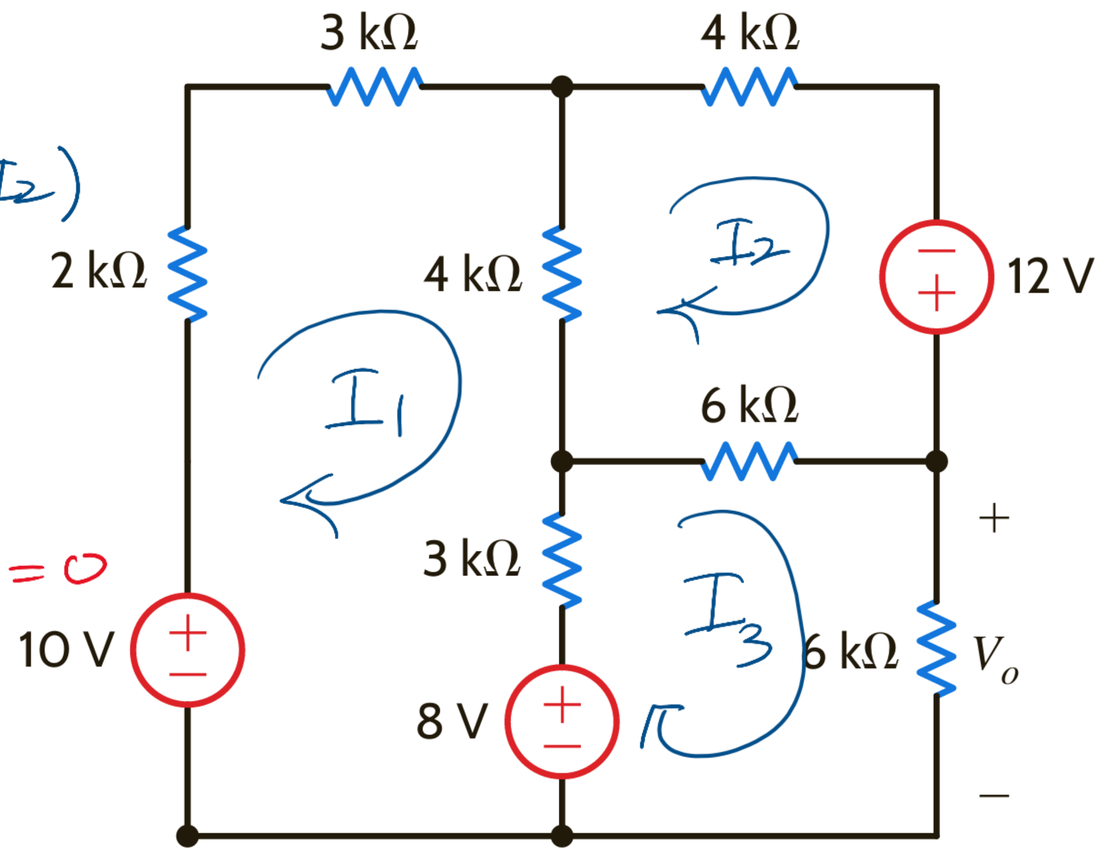
$$\begin{bmatrix} 12K & -6K \\ -6K & 3K \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -3 \end{bmatrix}$$

$I_o = I_1 - I_2$

$V_o = 6K(I_o)$

Loop Analysis using Kirchhoff's Voltage Law

Circuits Involving Independent Voltage Sources:



$$-10 + (2k)(I_1) + (3k)I_1 + (4k)(I_1 - I_2) + 3k(I_1 - I_3) + 8 = 0$$

$$4kI_2 - 12 + 6k(I_2 - I_3) + 4k(I_2 - I_1) = 0$$

$$3k(I_3 - I_1) + 6k(I_3 - I_2) + 6k(I_3) - 8 = 0$$

Loop Analysis using Kirchhoff's Voltage Law

Circuits Involving Independent Voltage Sources:

$$12kI_1 - 4kI_2 - 3kI_3 = 2$$

$$-4kI_1 + 14kI_2 - 6kI_3 = 12$$

$$-3kI_1 - 6kI_2 + 15kI_3 = 8$$

$$\begin{bmatrix} 12k & -4k & -3k \\ -4k & 14k & -6k \\ -3k & -6k & 15k \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 12 \\ 8 \end{bmatrix}$$

$$\underline{R}\ \underline{I} = \underline{V}$$