EE 240 Circuits I

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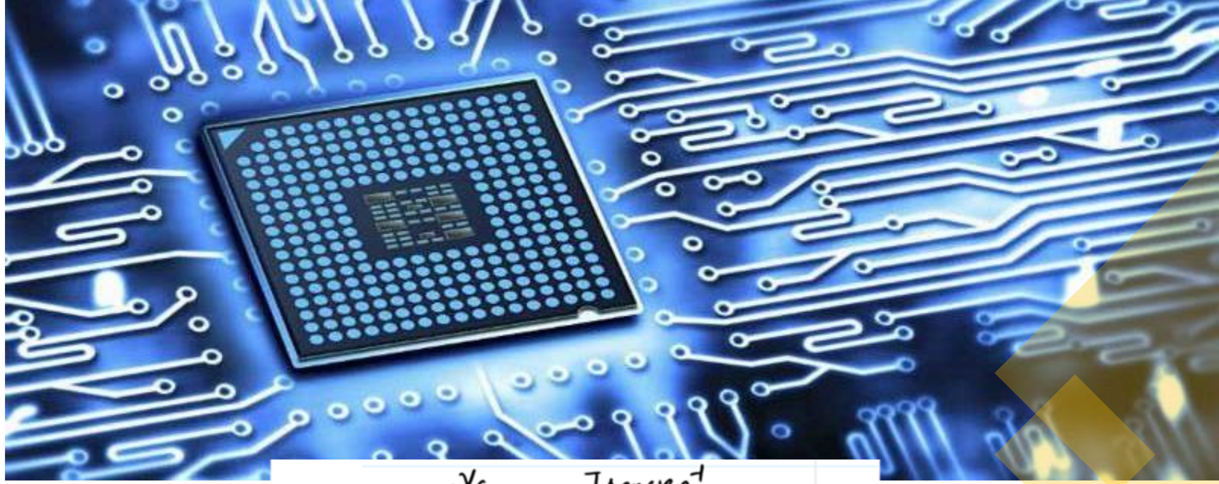
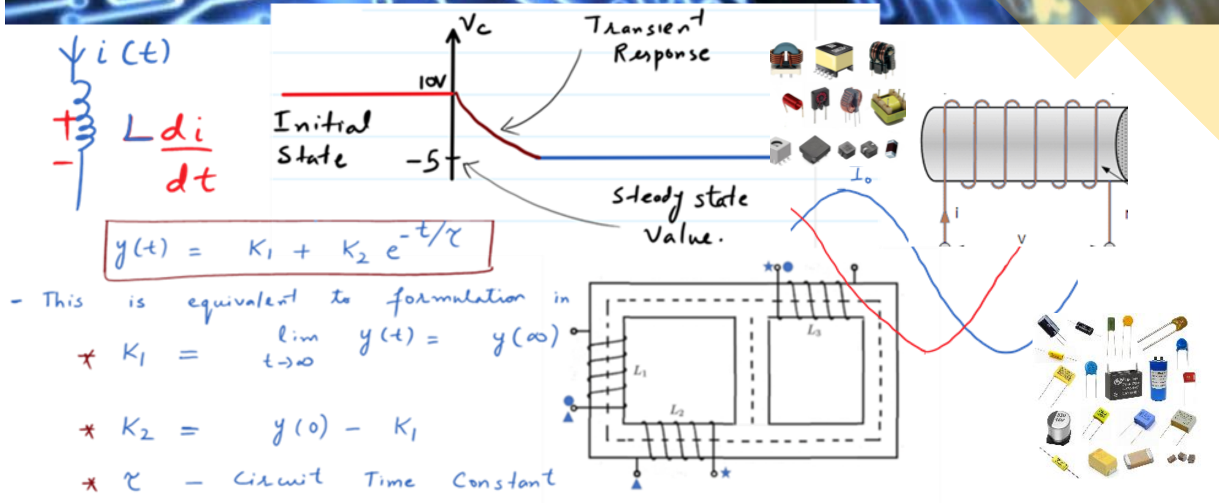
- Loop Analysis
- Super Loop
- Additional Analysis Techniques



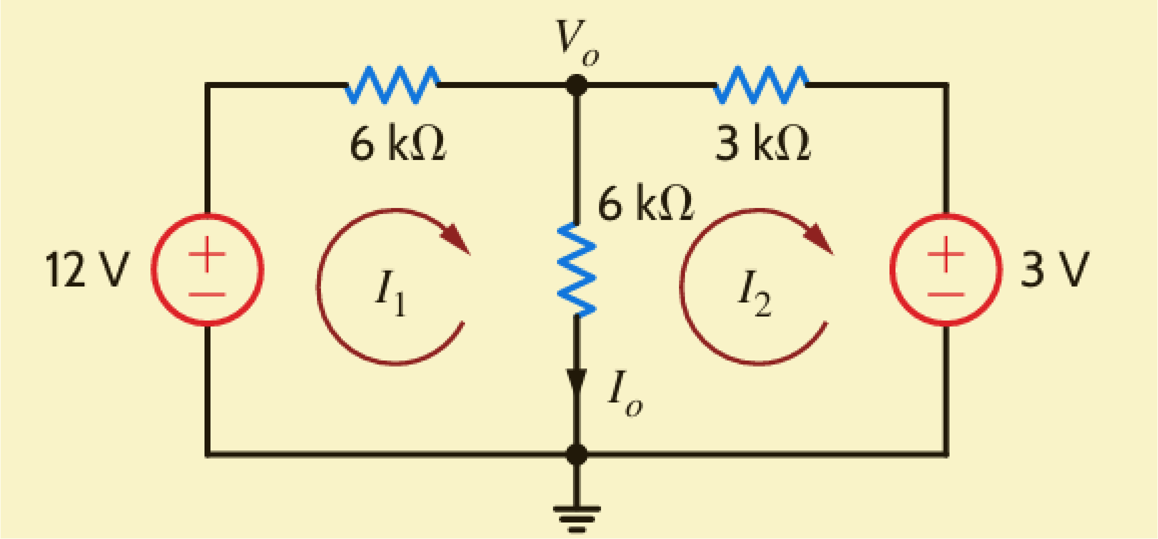


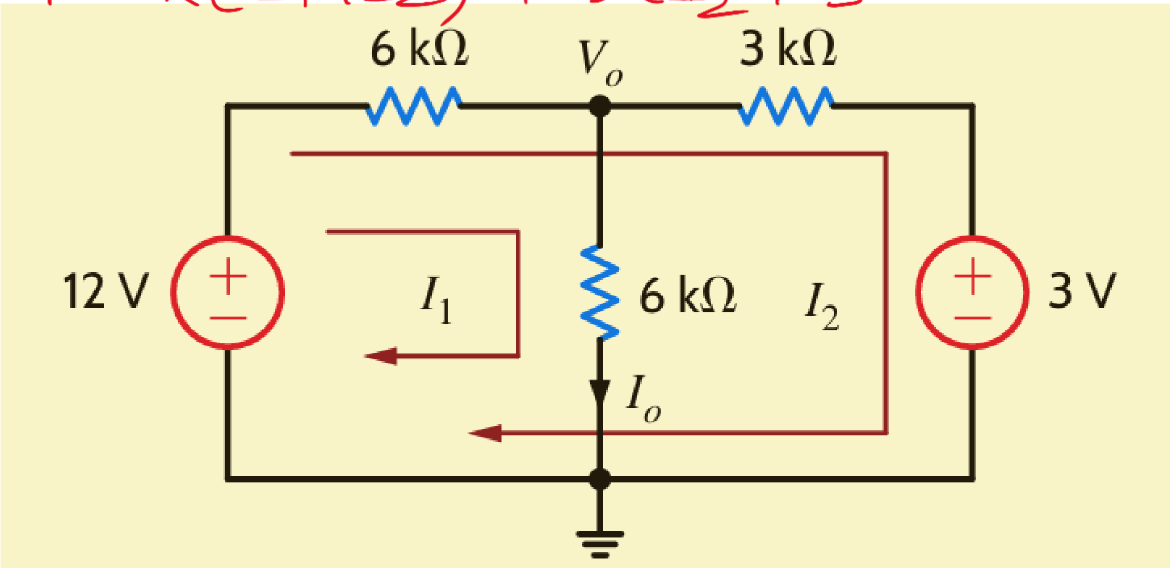
Loop Analysis using Kirchhoff's Voltage Law

Circuits Involving Independent Voltage Sources:

$* \; -12 + 6K(I_1 + I_2) + 6k(I_1) = 0$

$* \; -12 + 6K(I_1 + I_2) + 3KI_2 + 3 = 0$

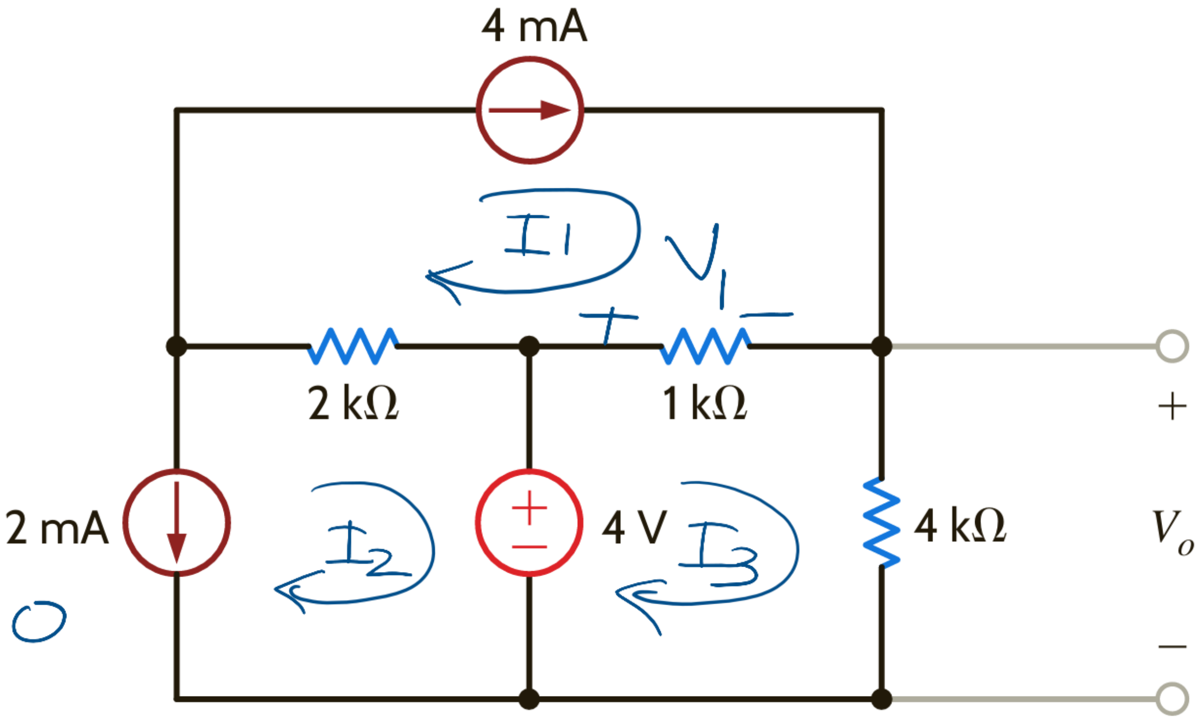




Independent loop: if loop traverses at-least one element that is not part of any other loop

Loop Analysis using Kirchhoff's Voltage Law

Circuits Involving Independent Current Sources:



* $I_1 = 4mA$
* $I_2 = -2mA$
* $-4 + 1K(I_3 - I_1) + 4K\,I_3 = 0$

$I_3 = 1.6mA$

$V_o = (1.6)(4) = 6.4V$

$V_1 = 1K(I_3 - I_1) = -2.4V$

$V_o + V_1 = 4V$

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Loop Analysis using Kirchhoff's Voltage Law

Circuits Involving Independent Current Sources and Super-Loop:

Choice of loop currents

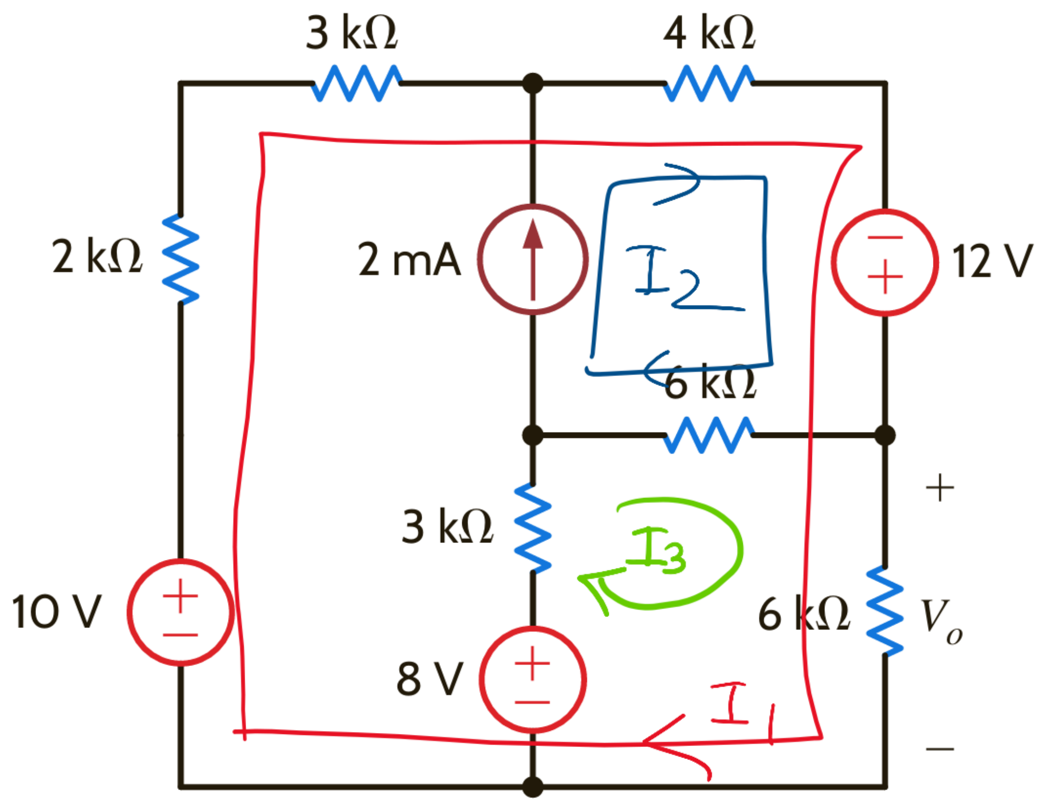
* $I_2 = 2mA$

* $3KI_1 + 4K(I_1 + I_2) - 12$

$+ 6K(I_1 + I_3) - 10$

$+ 2KI_1 = 0$

* $6K(I_3 - I_2) + 6K(I_3 + I_1) - 8 + 3KI_3 = 0$



Loop Analysis using Kirchhoff's Voltage Law

Circuits Involving Independent Current Sources and Super-Loop:

Now let's learn and apply the concept of Super-Loop:

$$5kI_1 + V_1 + 3k(I_1 - I_3) + 8 - 10 = 0$$

$$4kI_2 + 6k(I_2 - I_3) - V_1 - 12 = 0$$

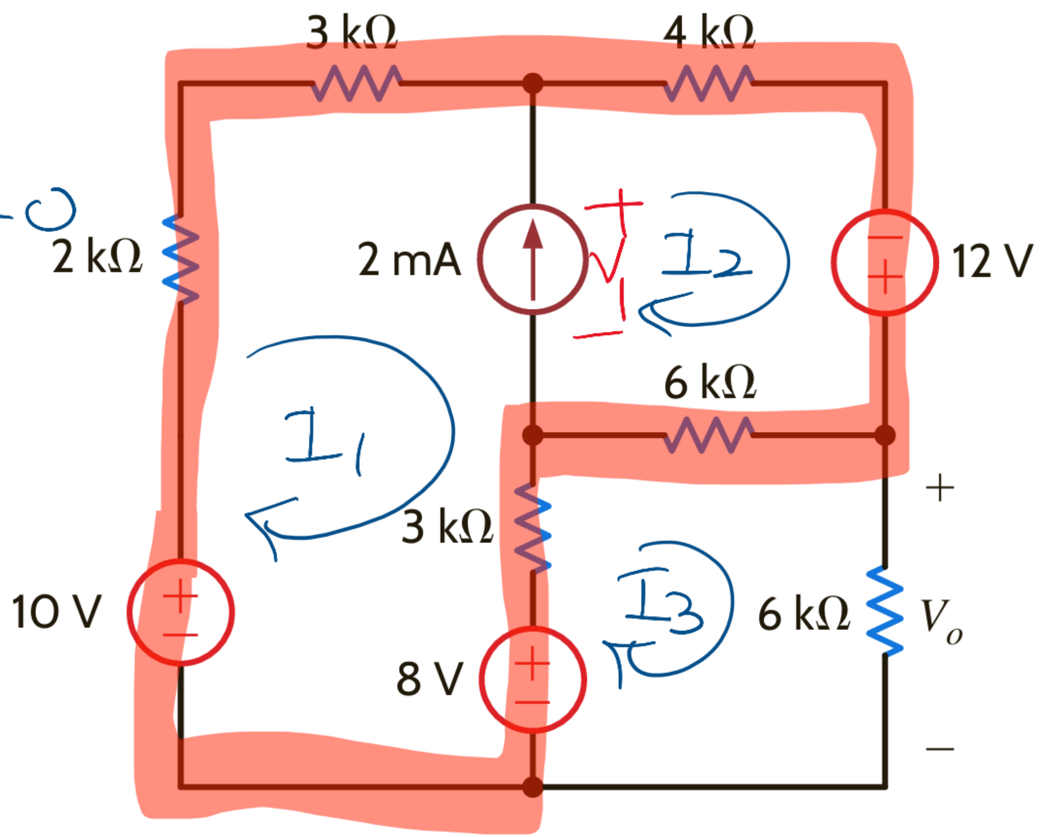
01

$$5kI_1 + 4kI_2 - 12 + 6k(I_2 - I_3) + 3k(I_1 - I_3) + 8 - 10 = 0$$

02

$$* \quad 6k(I_3 - I_2) + 6kI_3 - 8 + 3k(I_3 - I_1) = 0$$

03 $I_2 - I_1 = 2m$



KCL, KVL – Number of Equations

Connection with graphical representation

We analyse the electric circuit by forming network equations governed by KCL and KVL.

We should learn to use these laws **intelligently** to formulate **minimum** number of **independent** equations describing the network **completely.**

KCL – Nodal Analysis
KVL – Loop Analysis

To determine the number of equations obtained by using KCL or KVL, we use the graphical representation of the circuit.

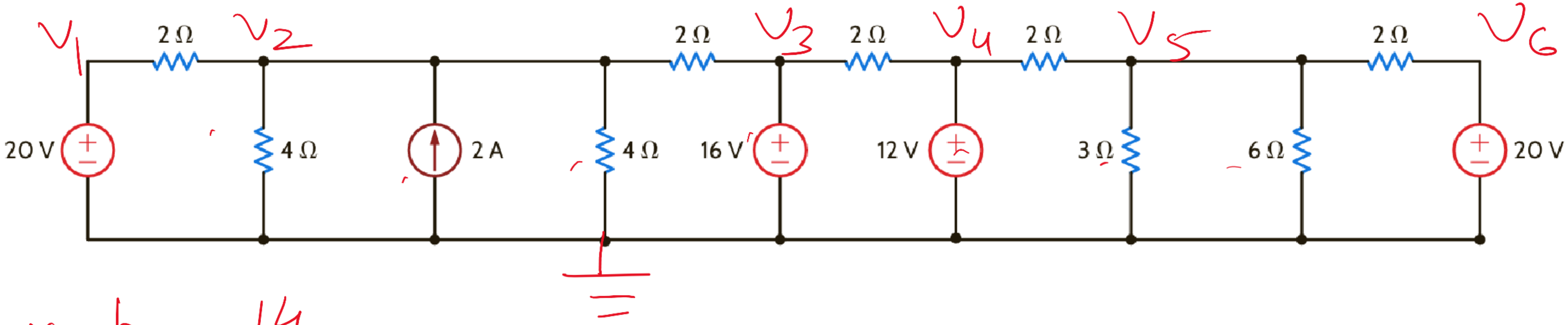
For a circuit with **n** nodes and **b** branches (or edges)
- KCL: number of equations = **n-1**
- KVL: number of equations = **b-(n-1)**

For a circuit with **p** parts
- KCL: number of equations = **n-1+(p-1)**
- KVL: number of equations = **b-(n-1)+(p-1)**

Recall **b-(n-1)** represents the number of chords to be removed from the graph of the circuit to obtain its tree.

Nodal or Loop Analysis

Circuits Involving Independent Current Sources and Super-Loop:



* $b = 14$
* $n = 7$

Nodal; 6

Loop; 8

* v_1, v_3, v_4, v_6 are known
* only 2 equations are needed to find v_2 and v_5

Equivalent Networks

Concept:

Two networks are equivalent at a pair of terminals if the current and voltage have a same relationship at these terminals.

We had been dealing with this concept in the course. For example,

- Resistors/Capacitors/Inductors in Series or Parallel
- Current sources in parallel
- Voltage sources in series

Now we are going to learn the following techniques to obtain an equivalent network to facilitate the analysis of the circuits.

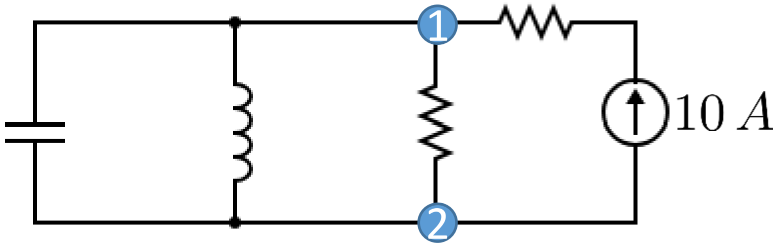
- Removing Extra Element
- Source Transformation
- Moving Sources

Equivalent Networks

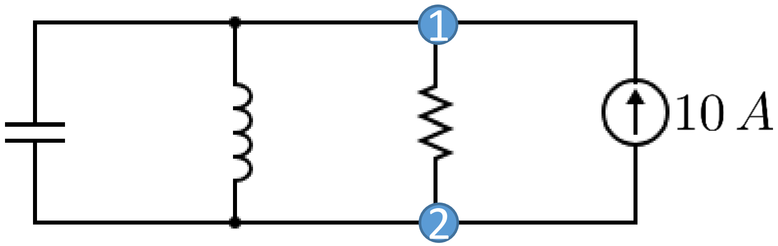
Removing Extra Element

Idea: As far circuit analysis is concerned, we can remove a circuit element if it is appearing in parallel to the voltage source or in series to the current source.

Element in series to the current source:

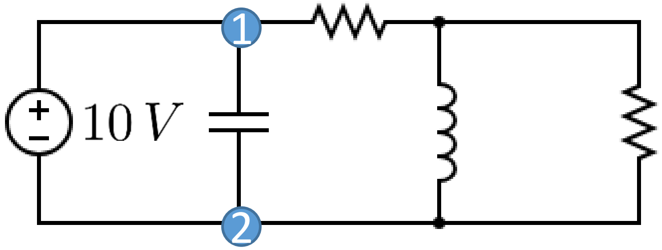


Equivalent to

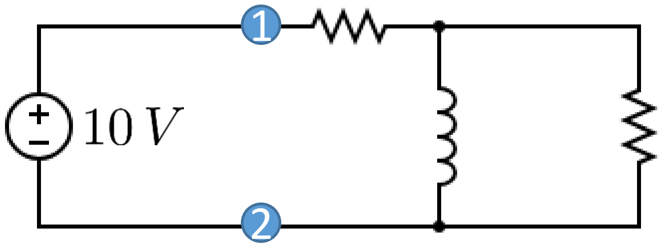


10A is being fed to node 1 and 10A is being drawn from the node 2.

Element in parallel to the voltage source:



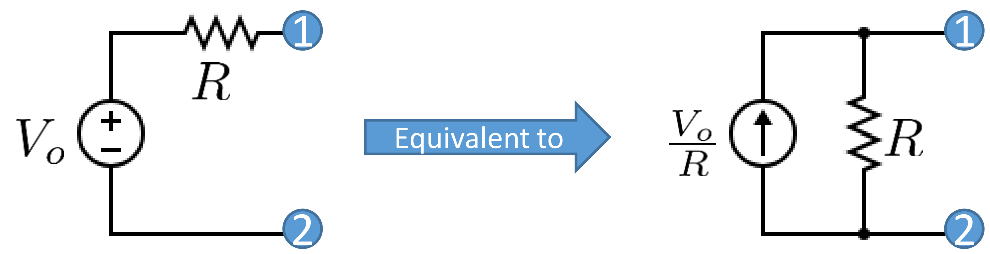
Equivalent to

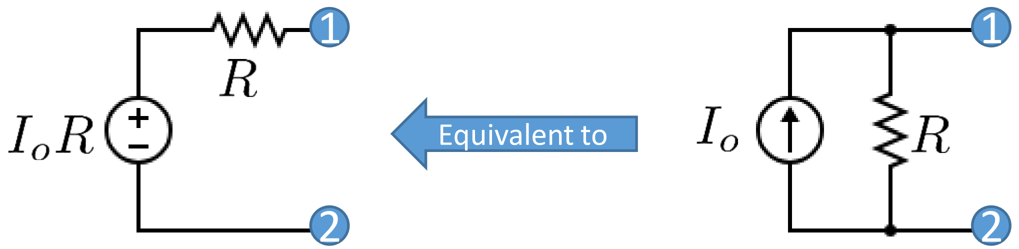


10V is the voltage across the nodes (terminals) 1 and 2.

Equivalent Networks

Source Transformation:





Show Equivalence:

Using KVL

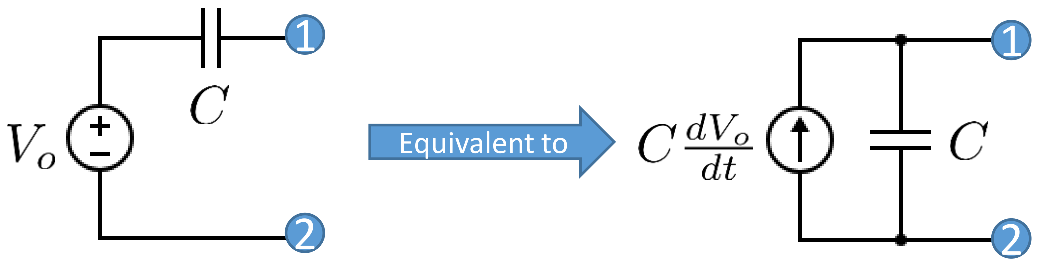
$$iR + v(t) - V_0 = 0$$

Using KCL

$$i + \frac{v(t)}{R} - \frac{V_0}{R} = 0$$

Idea: Voltage source and element in series can be transformed as current source with element in parallel.

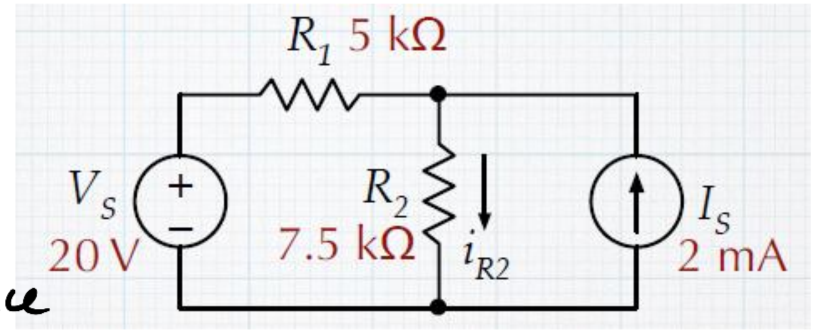
Similarly:



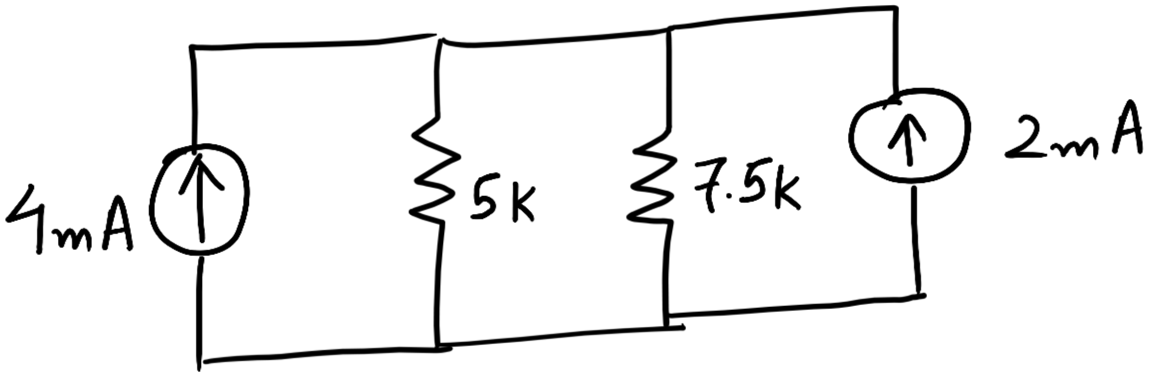
Source Transformation

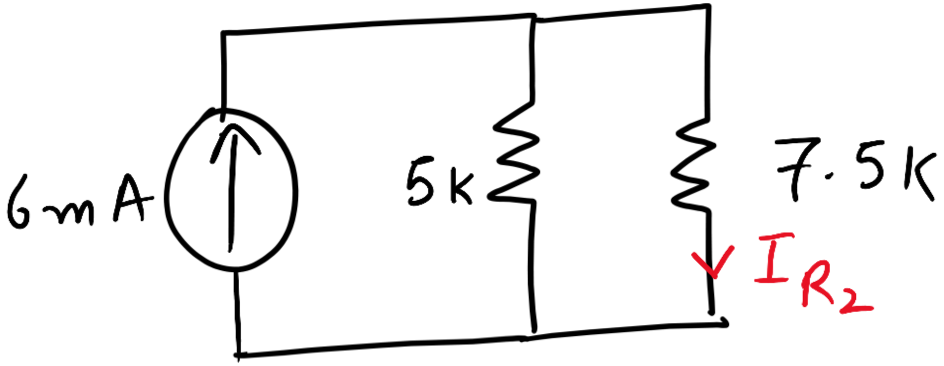
Example

Determine I_{R_2} using source transformation.



Source transformation of voltage source





Applying current division rule;

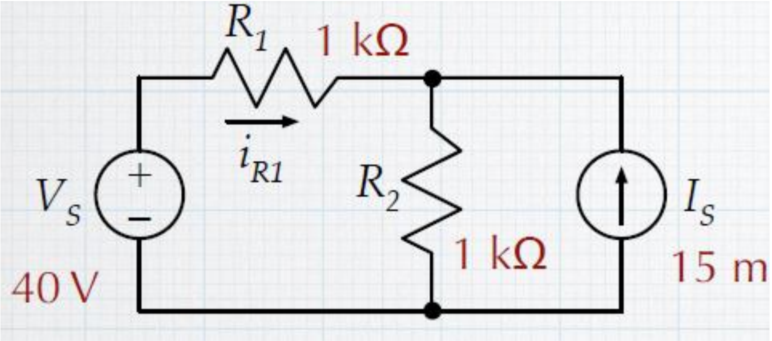
$$I_{R_2} = \frac{5k}{7.5k + 5k} \; 6m$$

$$= \underline{2.4mA}.$$

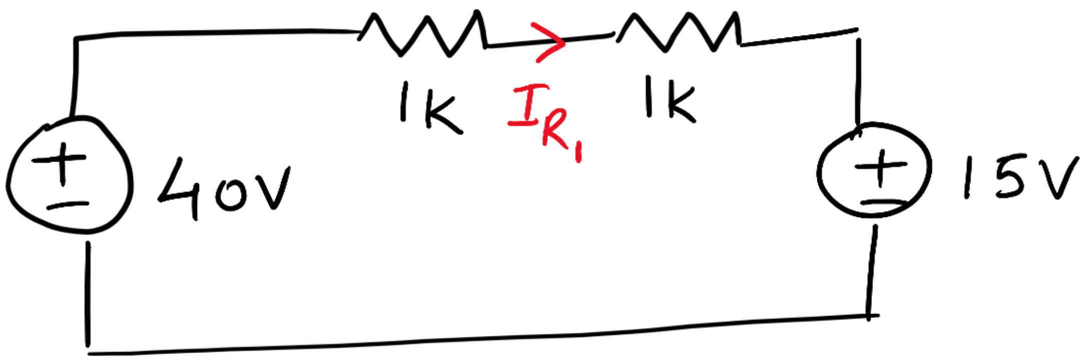
Equivalent Networks

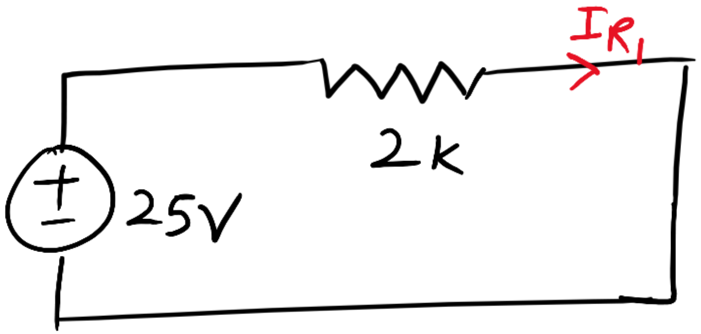
Example

Determine I_{R_1} using source transformation.



Source Transformation of current source





$$I_{R_1} = \frac{25}{2k} = \underline{12.5\,mA}.$$

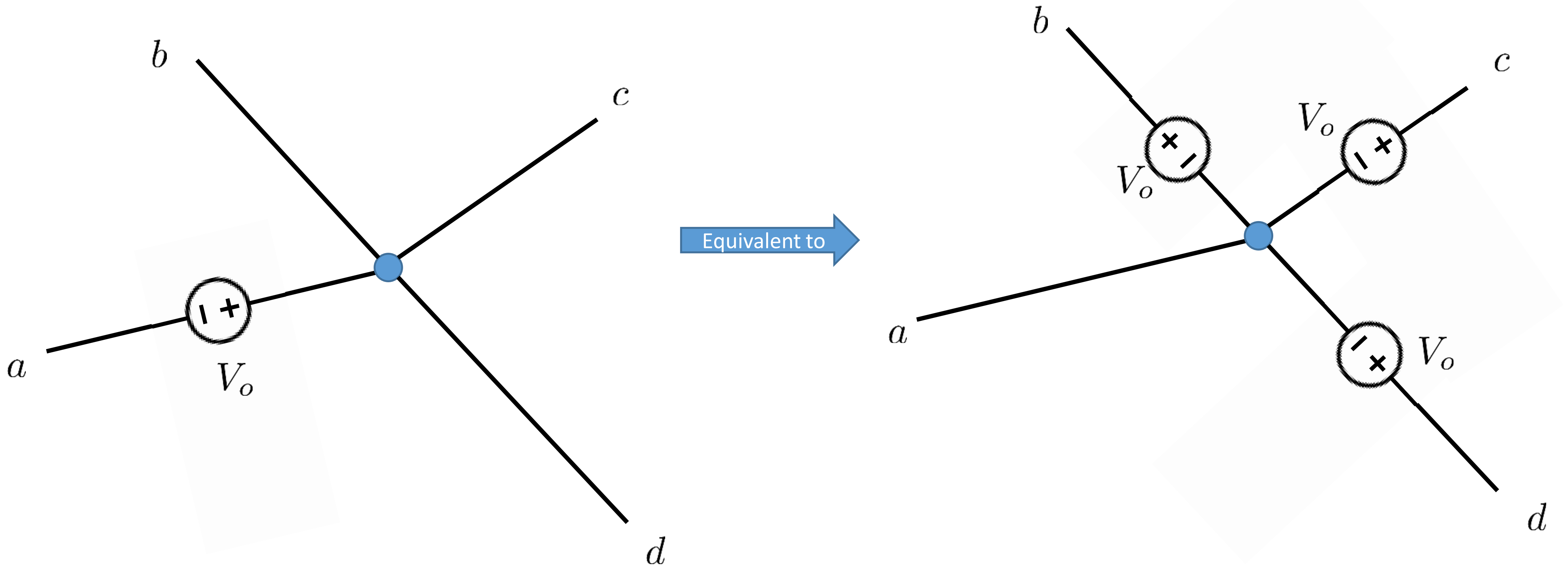
Caution: Do not apply source transformation using an element for which we are finding current or voltage.

Equivalent Networks

Moving Sources

Voltage Source: Move (Push) through a node

Consider a node of the circuit with 4 branches connected to the node.



Idea: *When voltage source is pushed from one branch through the node, it appears in every other branch connected to the node.*

$$V_{ba} = V_{ca} = V_{da} = V_o$$

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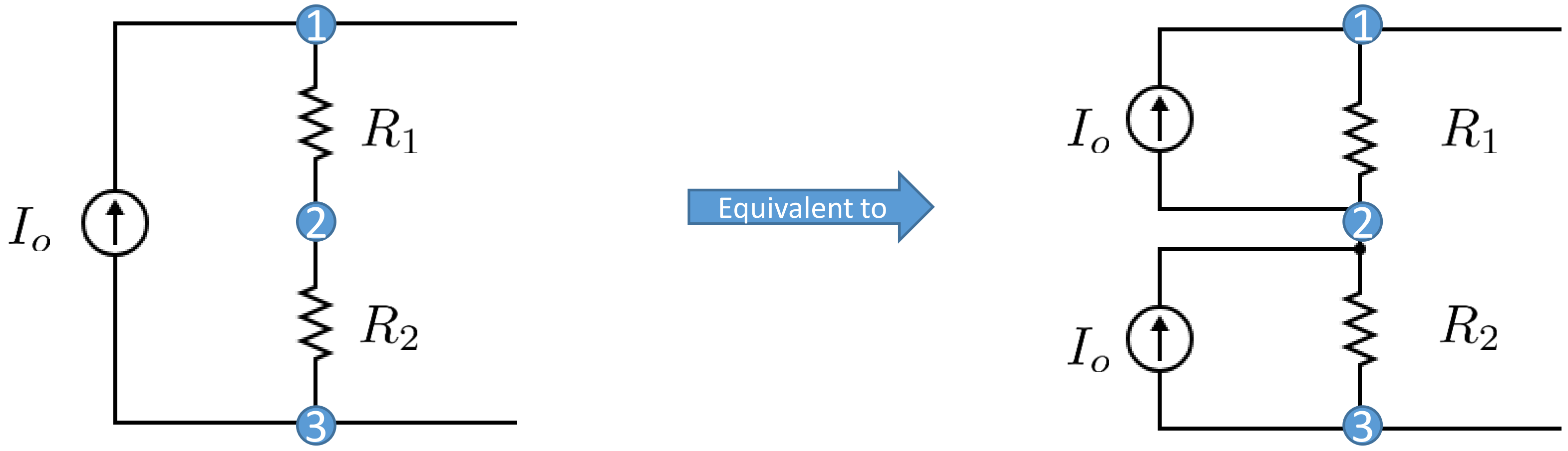
Equivalent Networks

Moving Sources

Current Source: Move around a node



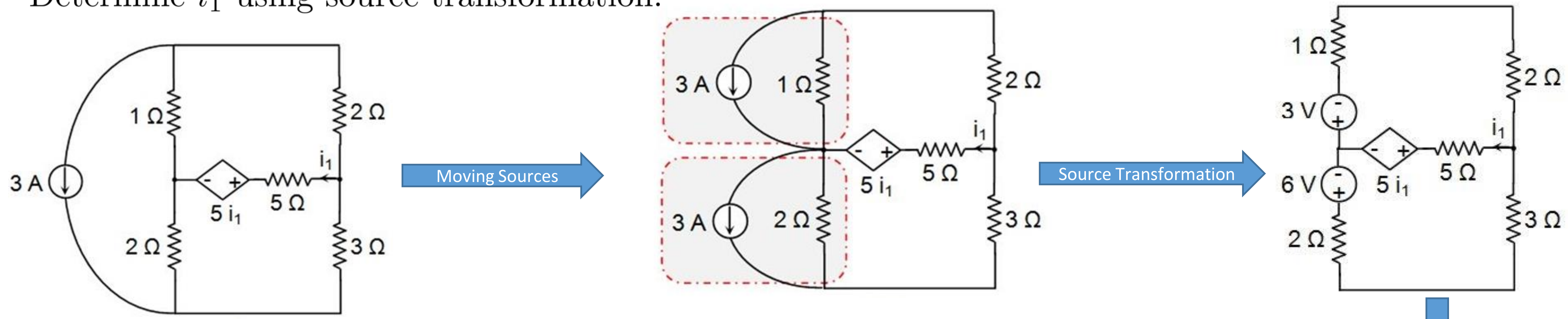
Current being fed (or drawn) to Node 1 (from Node 3) stays the same.

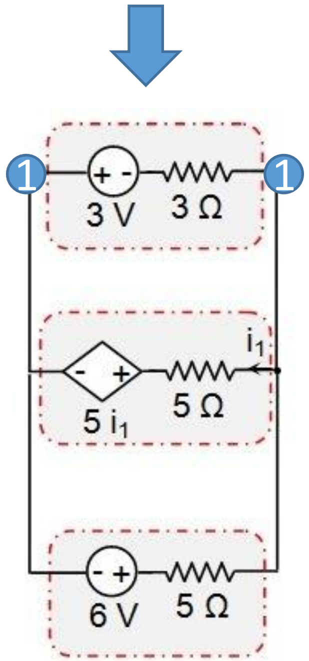
Idea: *Drawing the amount of current and feeding the same amount of current from the node does not change the circuit.*

Equivalent Networks

Example (illustrate moving sources + source transformation)

Determine i_1 using source transformation.





The circuit has 2 nodes. Considering node 2 as ground, we can apply KCL to write the equation in terms of the voltage v_1 at node 1:

$$\frac{v_1+3}{3} + \frac{v_1-5i}{5} + \frac{v_1-6}{5} = 0$$

For controlled source, we can relate i_1 with v_1 as

$$\frac{v_1 - 5i_1}{5} = i_1 \Rightarrow v_1 = 10i_i$$

Substituting this in the network equation yields $i_1 = \frac{3}{95}$ A

Superposition Theorem (Principle)

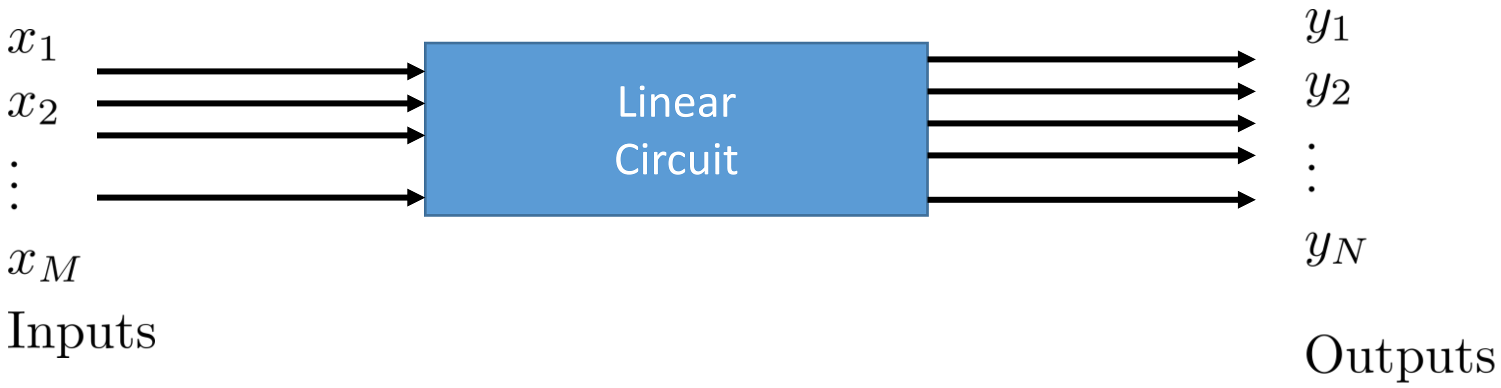
Idea:

*In a linear system (circuit), the branch current (or node voltage) due to multiple **independent** current or voltage sources is equal to the algebraic sum of branch current (or node voltage) due to each/single **independent** current or voltage source.*

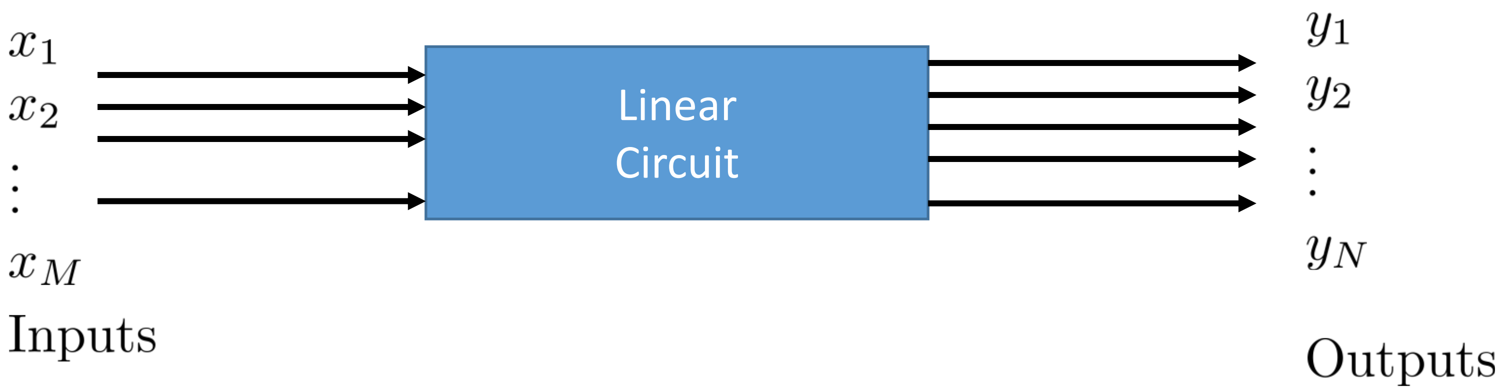
Linear circuit: resistors, capacitors, inductors, independent/dependent sources, transformers.

Superposition simply follows from the definition of linearity.

A circuit can be considered as a system with inputs as independent sources and outputs representing current through any branch or voltage across any element.



Superposition Theorem (Principle)



Each output $y_k(t)$ is a linear combination of inputs and/or derivatives of inputs.

Switch off all the inputs except one input, say x_ℓ, and observe the k-th output $y_{k,\ell}$, that is the output y_k when only input x_ℓ is active.

By superposition theorem,

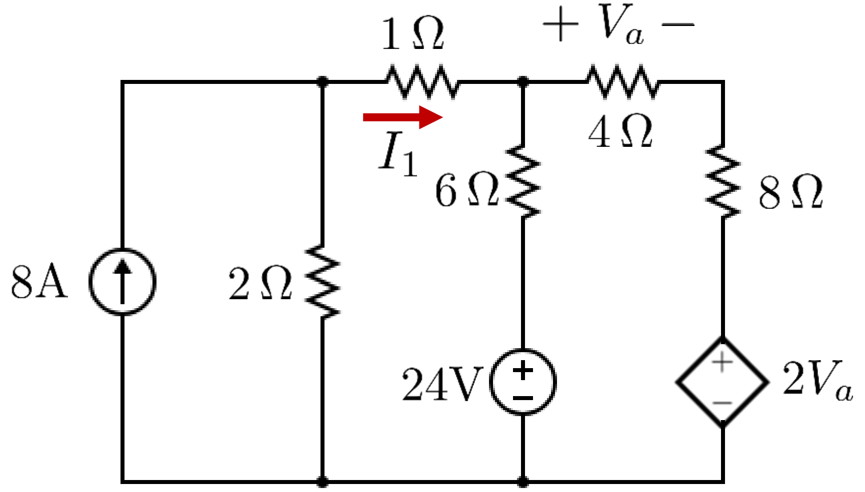
$$y_k = \sum_{\ell=1}^{M} y_{k,\ell}$$

You only need to know the concept of superposition and obviously how to apply it.

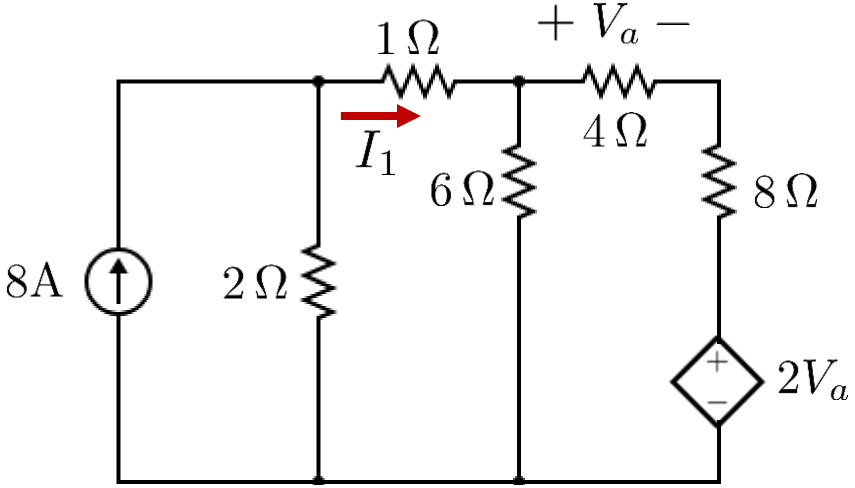
__Caution:__ Dependent sources stay in the circuit as they are.

Superposition Theorem (Principle)

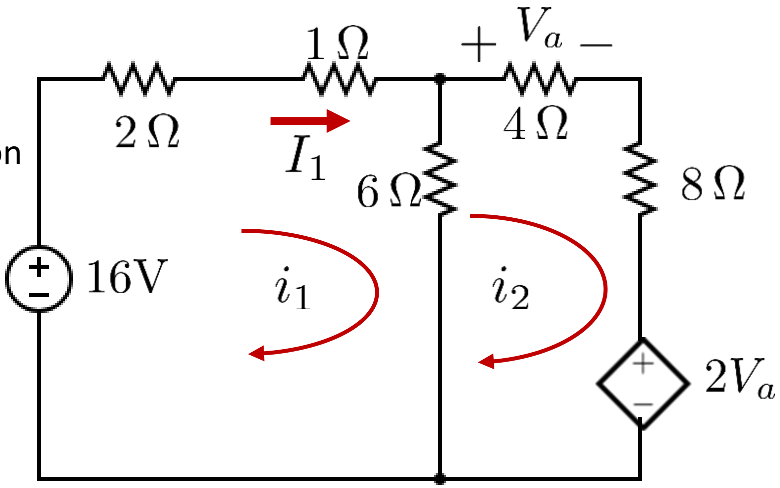
Example Determine I_1 using superposition principle.



Keep only 8A source.
Switch off 24V source (replace it with short-circuit).



Source Transformation



Loop 1 Equation:

$$9i_1 - 6i_2 = 16$$

Loop 2 Equation:

$$-6i_1 + 18i_2 = -2V_a$$

$$6i_1 = 26i_2$$

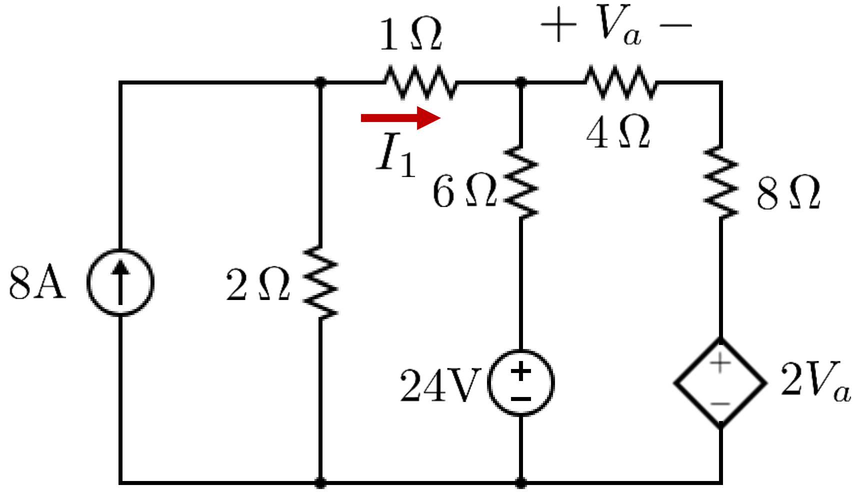
Controlled source equation:

$$V_a = 4i_2$$

$$i_1 = \frac{208}{99}\ \text{A} \qquad i_2 = \frac{16}{33}\ \text{A}$$

$$I_{1,8A} = i_1 = \frac{208}{99}\ \text{A}$$

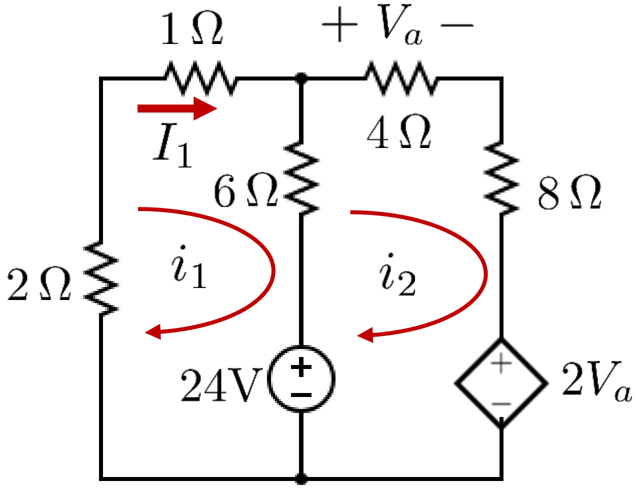
Superposition Theorem (Principle)



Example

Determine I_1 using superposition principle.

Keep only 24V source.
Switch off 8A source (replace it with open-circuit).



Loop 1 Equation:

$$9i_1 - 6i_2 = -24$$

Loop 2 Equation:

$$-6i_1 + 18i_2 = -2V_a + 24$$

$$-6i_1 + 26i_2 = 24$$

Controlled source equation:

$$V_a = 4i_2$$

$$i_1 = -\frac{80}{33}\text{ A} \qquad i_2 = \frac{12}{33}\text{ A} \qquad I_{1,24V} = i_1 = -\frac{80}{33}\text{ A}$$

$$I_1 = I_{1,24V} + I_{1,8A} = -\frac{80}{33} + \frac{208}{99} = -\frac{32}{99}\text{A}$$