EE 240 Circuits I

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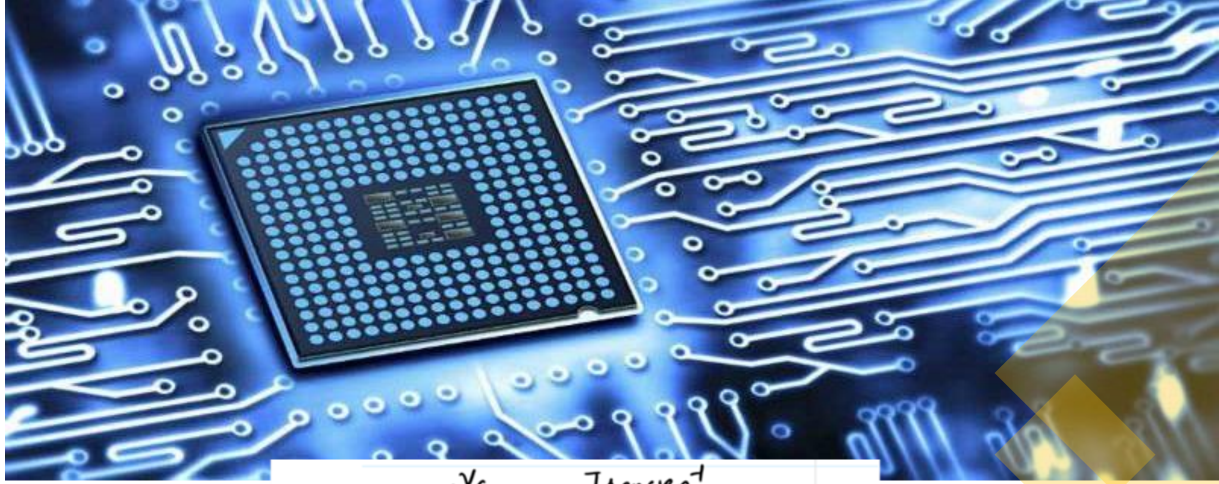
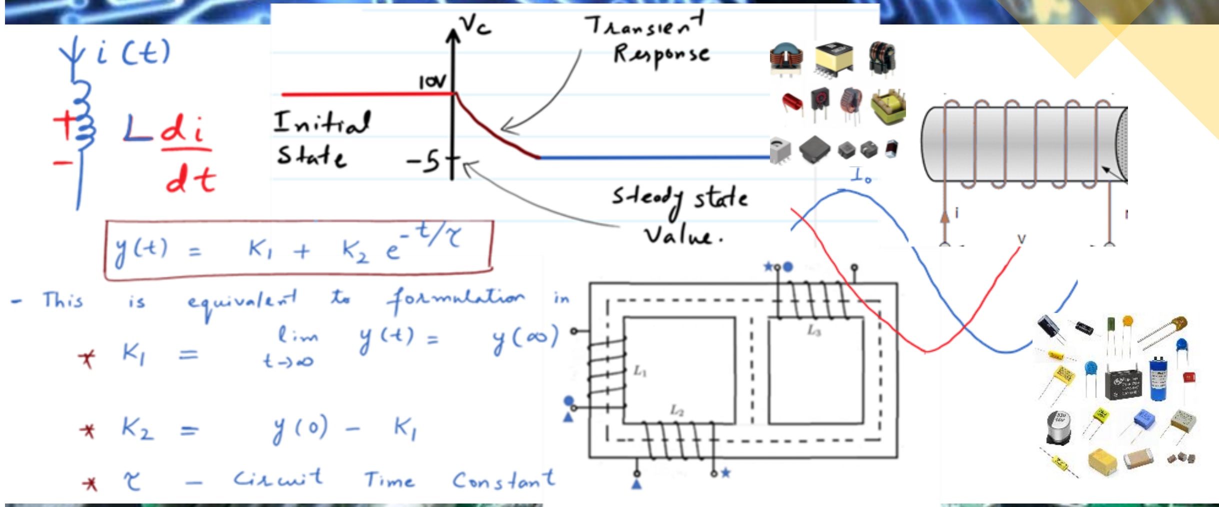
- Recap of Circuit Analysis Techniques
- Thevenin and Norton Equivalent Circuits





Nodal Analysis – Summary

Steps to Carry out Circuit Analysis:

- Determine the number nodes; mark the ground node and assign node voltages to other nodes.
- For an N-node circuit, there are N-1 node voltages, and we need (N-1) linearly independent equations to determine these node voltages.
- If voltage source is connected between a node and the ground, the voltage at the node is equal to the voltage of the source.
- If voltage source (dependent or independent) is connected between two nodes, we employ the super-node concept to obtain two equations: 1) equation of the super-node using KCL and 2) equation relating the node voltages and the voltage source.
- Formulate the remaining node equations by employing KCL.
- For each dependent source, write a constraint equation, which is one of the necessary linearly independent equations. While writing the constraint equation, express the controlling quantity in terms of nodal voltages.

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Loop Analysis – Summary

Steps to Carry out Circuit Analysis:

- Assign a loop current to analyze each independent loop.
- For an N-loop circuit, there are N loop currents, and we need N linearly independent equations to determine the loop currents.
- If current source is connected in a branch carrying only one loop current, the current is equal to the current of the source.
- If current source (dependent or independent) is shared between two loops, we employ the super-loop concept to obtain two equations: 1) equation of the super-loop using KVL and 2) Equation relating the currents in the two loops and the current source.
- Formulate the remaining loop equations by employing KVL.
- For each dependent source, write a constraint equation, which is one of the necessary linearly independent equations. While writing the constraint equation, express the controlling quantity in terms of loop currents.

Superposition Summary

Steps to Carry out Circuit Analysis:

- Isolate Each Source:
  - Consider one independent source at a time while turning off all other independent sources.
  - For Voltage Sources: Replace them with a short circuit (a wire).
  - For Current Sources: Replace them with an open circuit (a break).

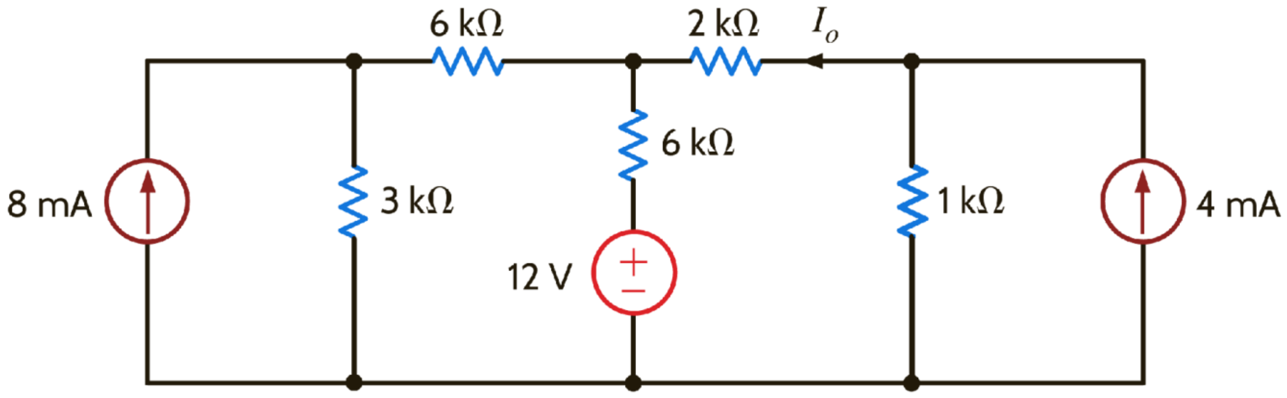
- Analyze the Circuit: Solve the circuit for the desired response (voltage or current) caused by the single active source.

- Repeat for All Sources: Repeat the process for each independent source.

- Sum the Responses: Add the responses from each independent source to obtain the total response in the circuit.

Source Transformation:

Reinforcement Example: Find I_o using Source Transformation:



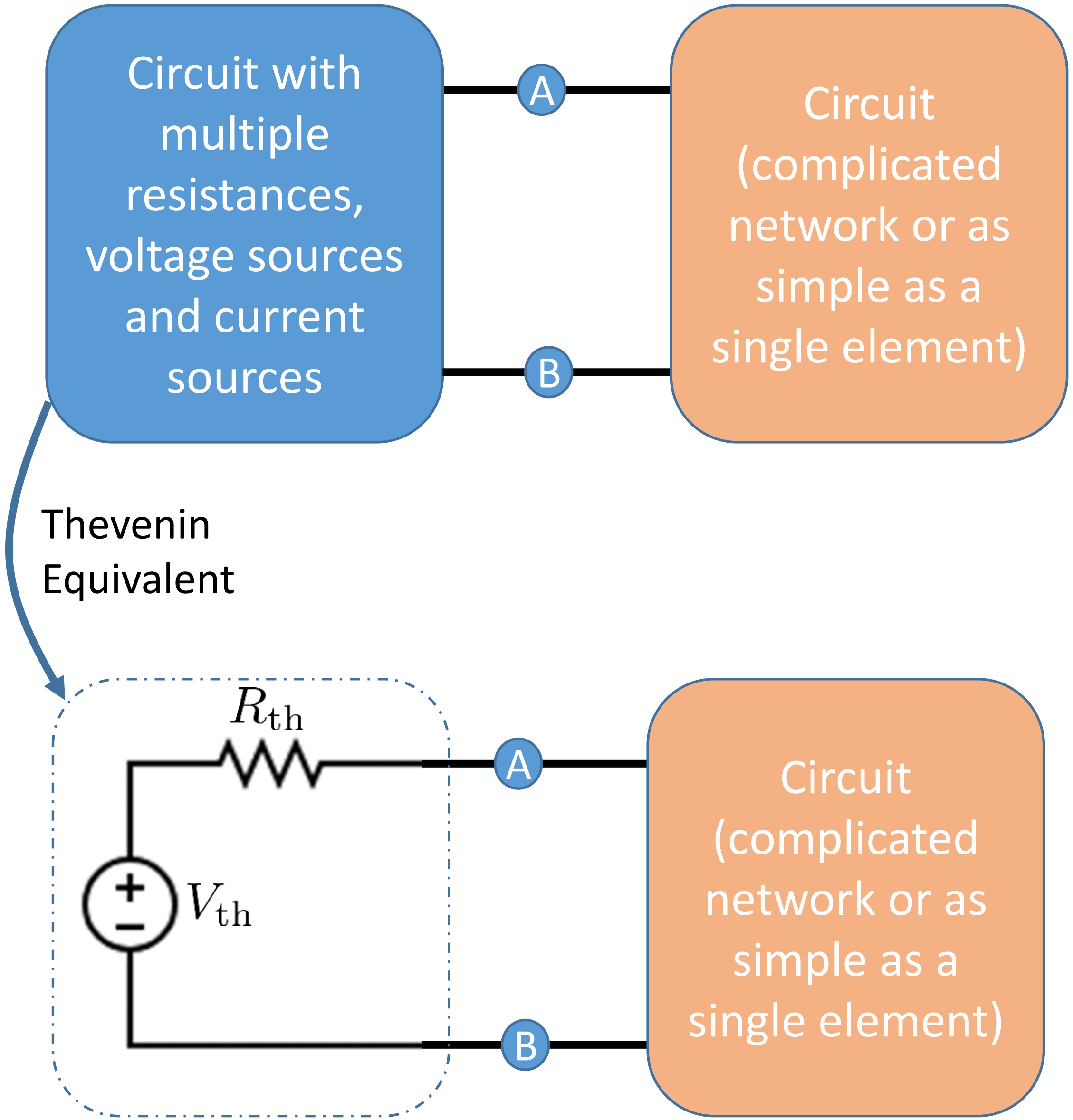
Thevenin's Theorem

Overview

Thevenin theorem is used to change a complicated circuit into a simple equivalent circuit consisting of a single voltage source, referred to as Thevenin voltage V_{th} in series with a single resistance, referred to as Thevenin Resistance R_{th}.

Thevenin Theorem Statement:

Any circuit containing only resistances, voltage sources, and current sources be replaced at the terminals A-B by an equivalent combination of a voltage source V_{th} in a series connection with a resistance R_{th}.



Thevenin's Theorem

How to Obtain Thevenin Equivalent

Key Idea: Use the concept of equivalence: same current and voltage characteristics across terminals A-B.

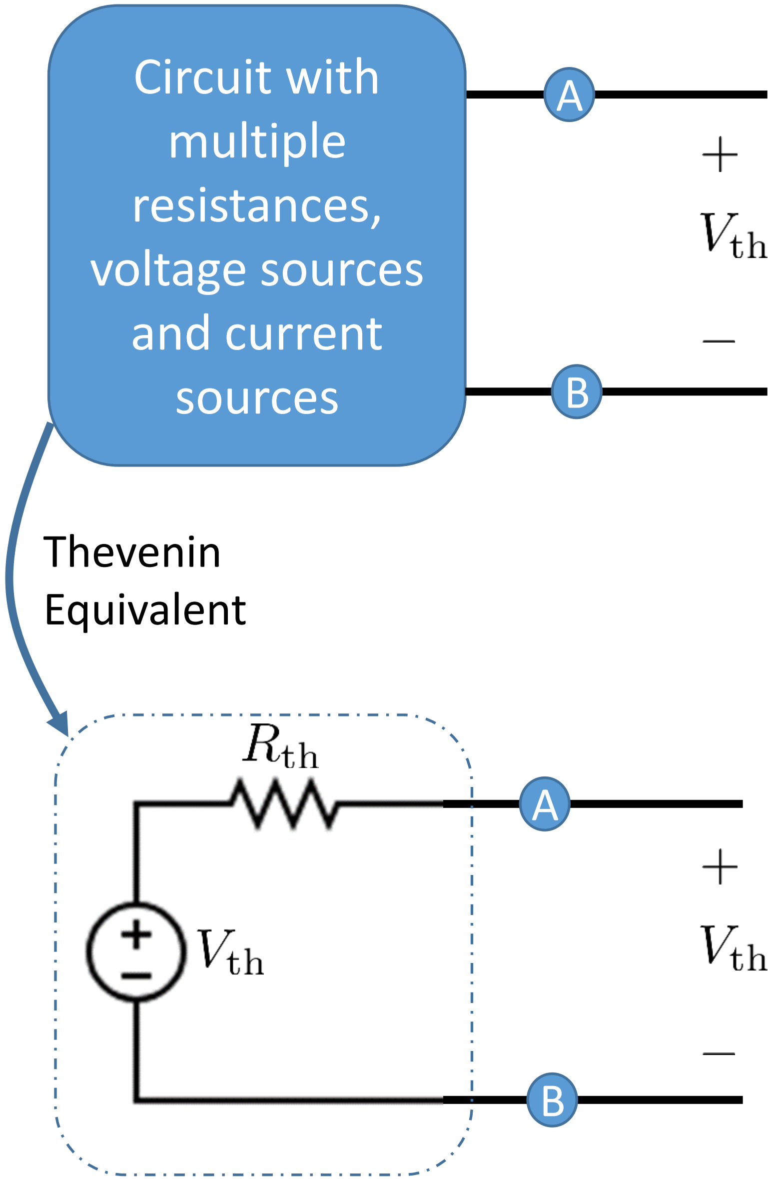
Only keep the circuit for which we want to find the equivalent circuit; disconnect the rest of the circuit.

Determine V_{th}:

As no current is flowing from A to B, V_{th} is simply a voltage across terminals A-B.

We can determine V_{th} by analysisng the circuit inside blue box and determine the voltage across terminals A-B.

We will learn different methods to obtain Thevenin Resistance.



Thevenin's Theorem

How to Obtain Thevenin Equivalent

Determine R_{th}:

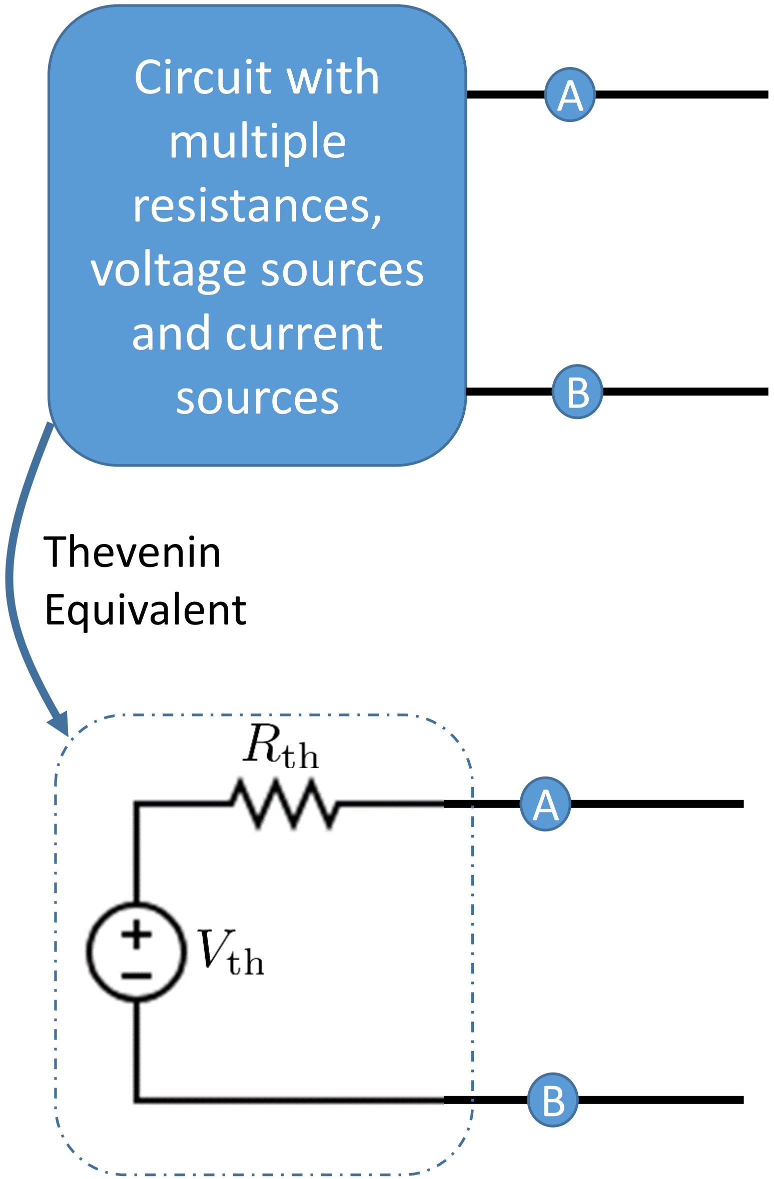
Method 1:

- Switch off the independent sources and determine the equivalent resistance across terminals A and B.

Method 2:

- Short-circuit terminals A and B and determine the current flowing from A to B, referred to as I_{SC}.
- Using this current, we can determine R_{th} as

$$R_{\text{th}} = \frac{V_{\text{th}}}{I_{\text{sc}}}$$



Thevenin's Theorem

How to Obtain Thevenin Equivalent

Determine R_{th}:

Method 3:

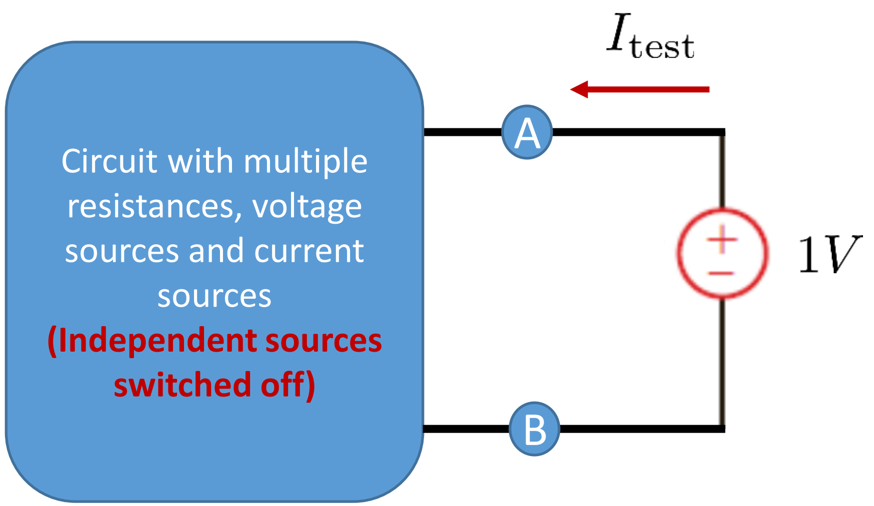
- Switch off the independent sources
- Connect a test source across terminals A-B

- **If 1V (known) voltage test-source is connected:**

Determine the current I_{test} supplied by voltage source.

We can determine R_{th} as follows:

$$R_{\text{th}} = \frac{1}{I_{\text{test}}}.$$

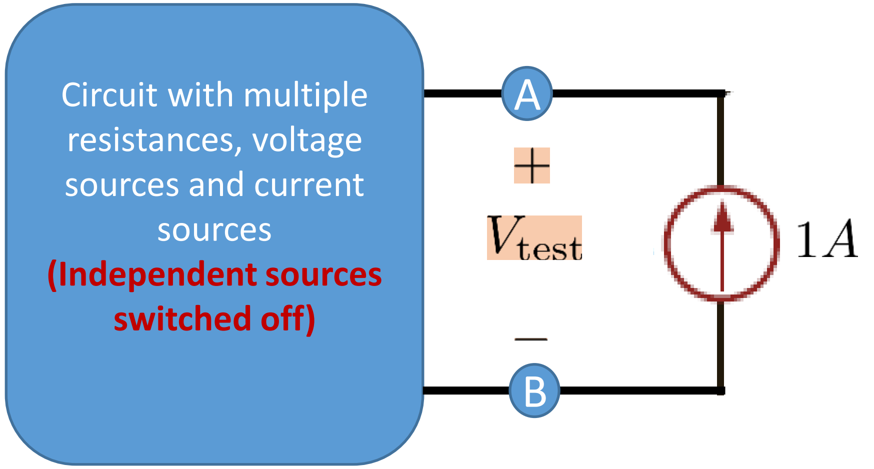


- **If 1A (known) current test-source is connected:**

Determine the voltage V_{test} developed across the current source.

We can determine R_{th} as follows:

$$R_{\text{th}} = \frac{V_{\text{test}}}{1}.$$



Thevenin's Theorem

How to Obtain Thevenin Equivalent

Determine R_{th}:

Which method to use?

Independent sources	Dependent sources	Method – can be used	Justification
✓	✗	Methods 1, 2 and 3	
✓	✓	Methods 2 and 3	Method 1 cannot be used due to the presence of dependent sources
✗	✓	Method 3	No independent source driving V_{th} or I_{SC}
✗	✗	Methods 1 and 3	No independent source driving V_{th} or I_{SC}

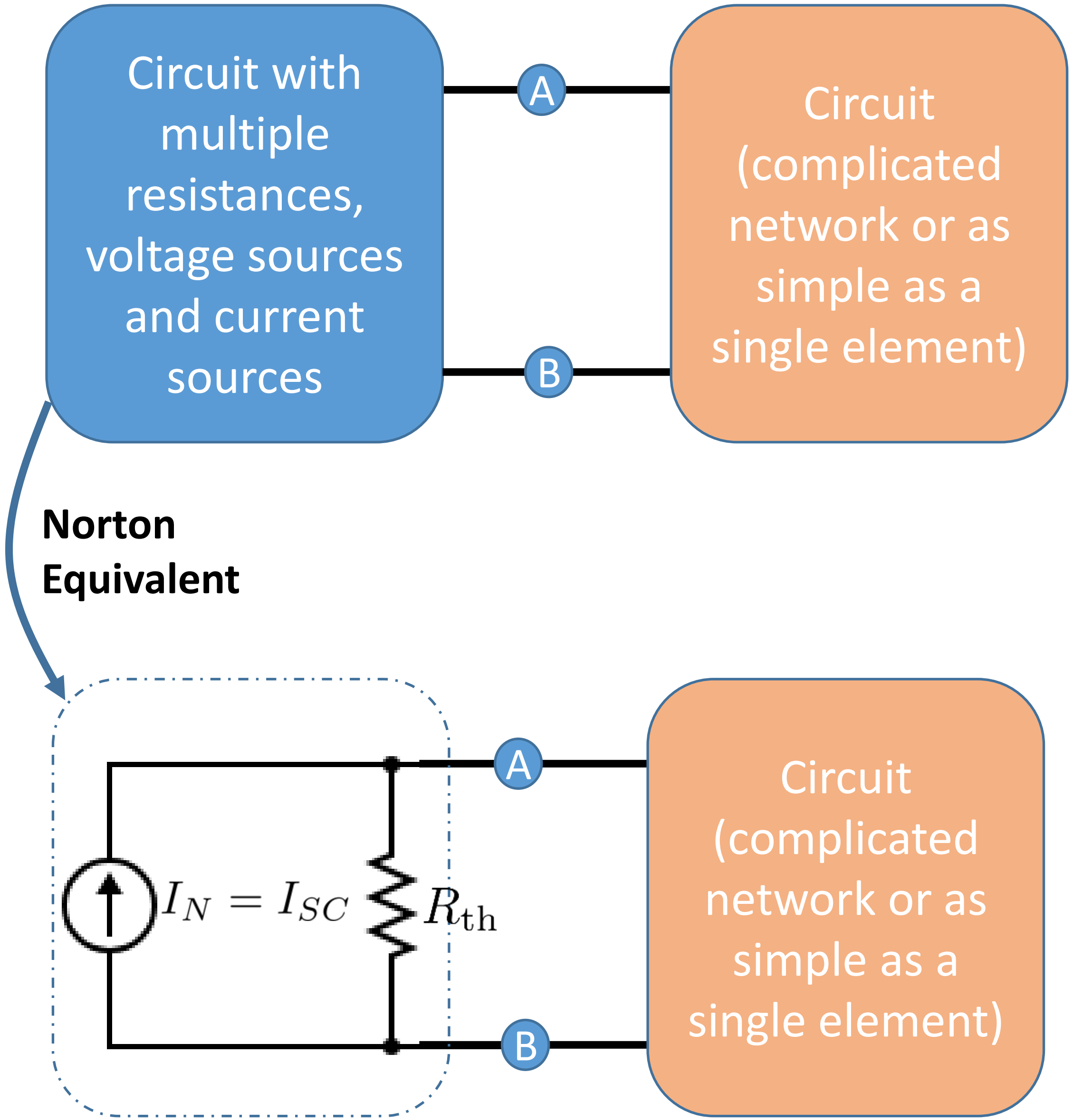
Note: Equivalent circuit does not have the voltage source if there is no independent source in the circuit.

Norton's Theorem

Norton theorem is used to change a complicated circuit into a simple equivalent circuit consisting of a single current source, referred to as Norton current voltage I_N in parallel with a single resistance (the same as Thevenin Resistance R_{th} , explained below).

- The value of the Norton current is one that flows from terminal A to B when the two terminals are shorted together. This is in fact I_{sc}, that is short-circuit current.

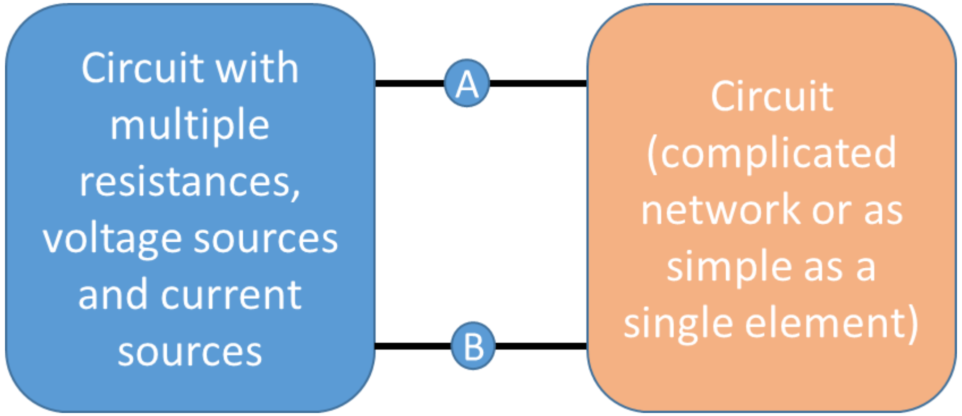
- The resistance represents the resistance looking back into the terminals when source is switched off. This is in fact Thevenin Resistance.



Connection with the Thevenin's Theorem

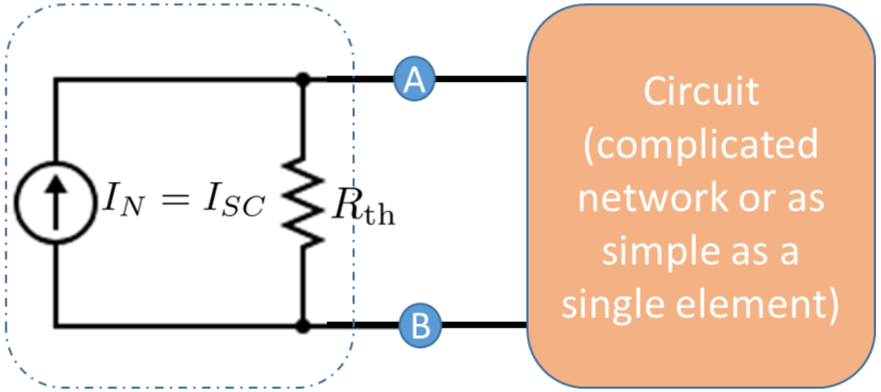
The source transformation of Thevenin's equivalent yields Norton's equivalent and vice versa.

Thevenin's and Norton's Equivalent

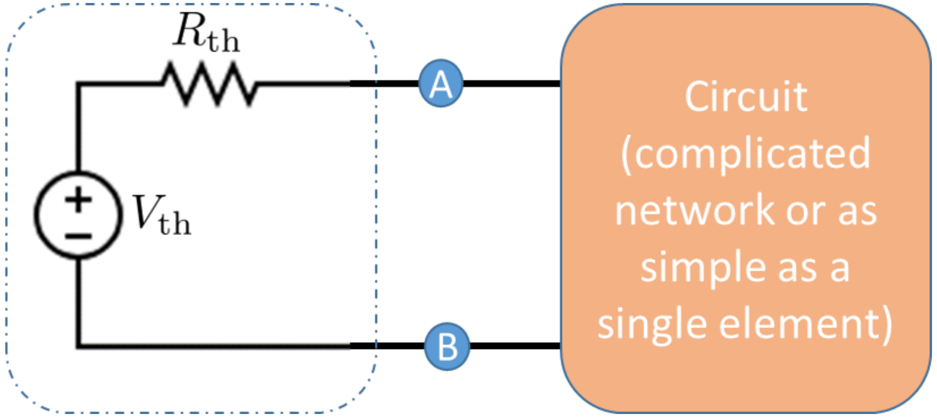


Summary:

Norton Equivalent



Thevenin Equivalent



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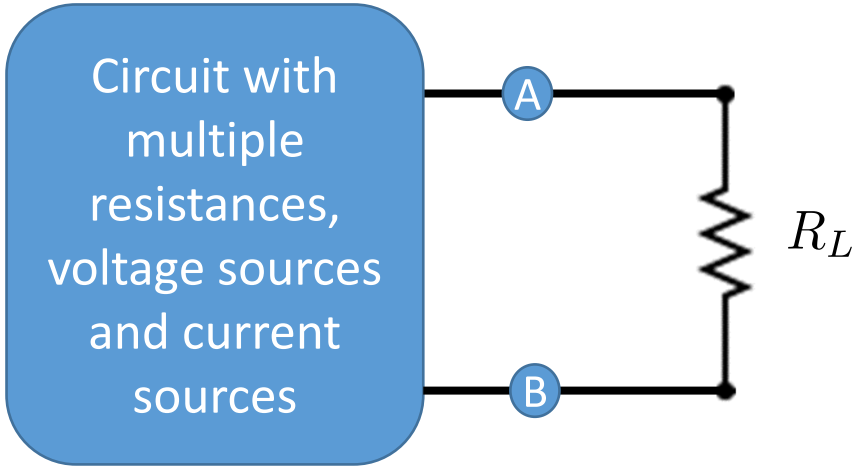
Maximum Power Transfer Theorem

Problem:

Consider a scenario where we have a complex electrical network delivering power to a load resistor R_L.



Question:

Determine the value of R_L

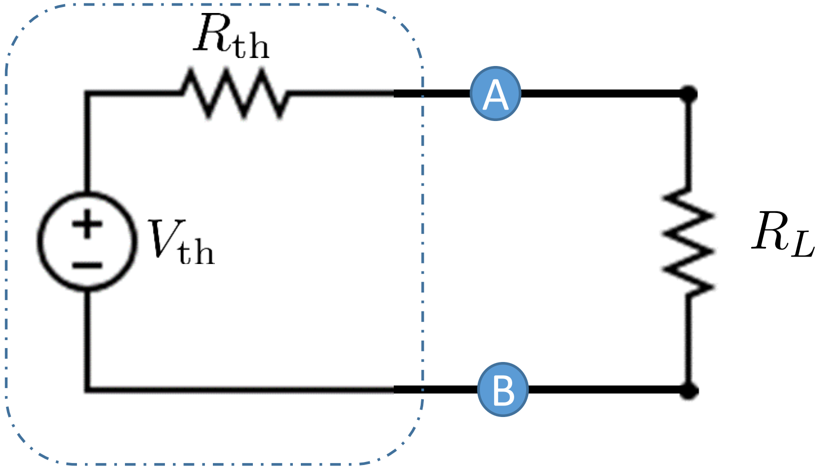
such that

the maximum power is transferred to the load.

Maximum Power Transfer Theorem

Solution:

To address this question, we replace the original complex circuit with its *Thevenin equivalent*, that is,



Formulating the Power Equation

The voltage across the load resistor R_L is given by:

$$V_L = \frac{V_{th} R_L}{R_{th} + R_L}$$

The power delivered to the load R_L is the square of the voltage across R_L, divided by R_L:

$$P_L = \frac{V_L^2}{R_L}$$

Substituting the expression for V_L:

$$P_L = \frac{\left(\frac{V_{th} R_L}{R_{th} + R_L}\right)^2}{R_L}$$

Simplifying this:

$$P_L = \frac{V_{th}^2 R_L}{(R_{th} + R_L)^2}$$

This is the general expression for the power delivered to the load resistor R_L.

Maximum Power Transfer Theorem

Solution:

To find the value of R_L that maximizes the power, we take the derivative of P_L with respect to R_L and set it equal to zero:

$$\frac{dP_L}{dR_L} = 0$$

Differentiating the power equation:

$$\frac{d}{dR_L}\left(\frac{V_{th}^2 R_L}{(R_{th} + R_L)^2}\right) = 0$$

$$\frac{V_{th}^2\left((R_{th} + R_L)^2 - 2R_L(R_{th} + R_L)\right)}{(R_{th} + R_L)^4} = 0$$

Simplifying the numerator:

$$(R_{th} + R_L)^2 - 2R_L(R_{th} + R_L) = R_{th}^2 - R_L^2$$

For this expression to be zero, we find:

$$R_{th} = R_L$$

Thus, the maximum power is transferred when $R_L = R_{th}$.

Maximum Power Delivered to the Load

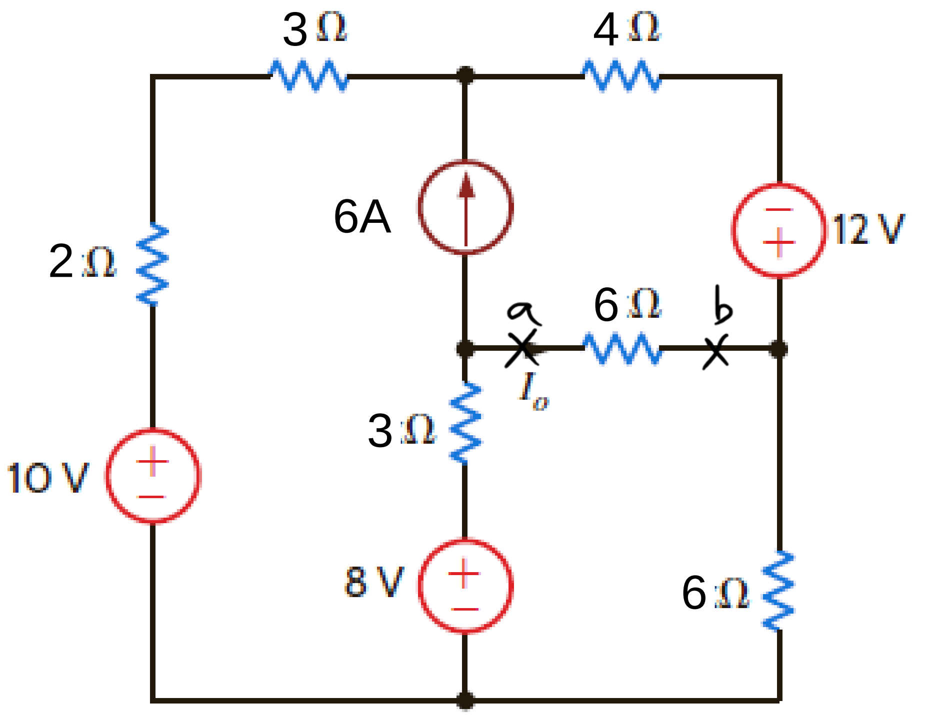
Substitute $R_L = R_{th}$ to get the maximum power:

$$P_{\max} = \frac{V_{th}^2}{4R_{th}}$$

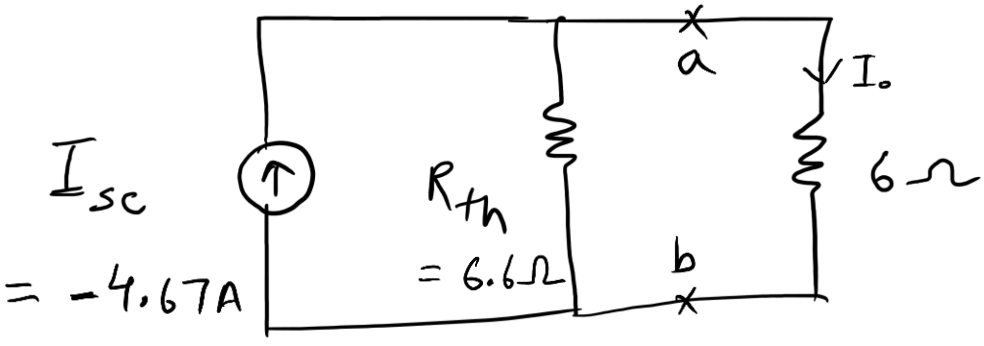
Thevenin's and Norton's Theorems

Problems – In class

Example 3: Find I_o using Thevenin's or Norton's theorem



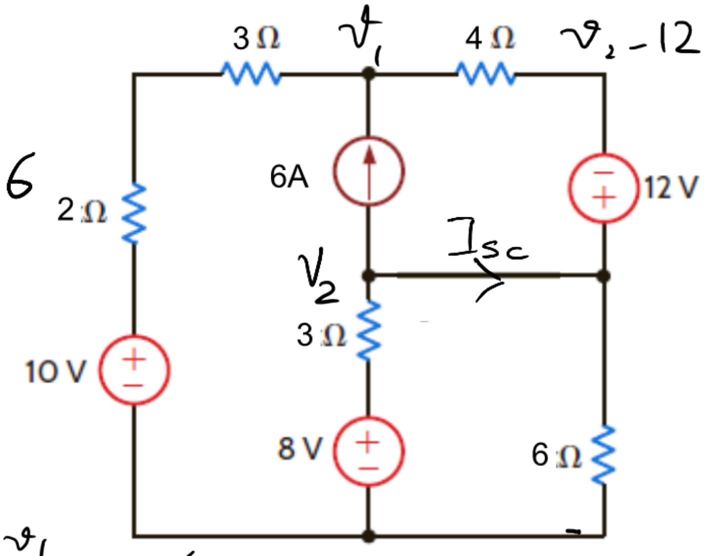
Equivalent: (computed on next page)



$$I_o = \frac{6.6}{12.6} \times (-4.67) = -2.44A$$

Thevenin's and Norton's Theorems

Problems – In class



$$\frac{v_1 - 10}{5} + \frac{v_1 - (v_2 - 12)}{4} = 6$$

$$\boxed{9v_1 - 5v_2 = 100}$$

$$\frac{v_2 - 8}{3} + \frac{v_2}{6} + \frac{v_2 - 12 - v_1}{4} = -6$$

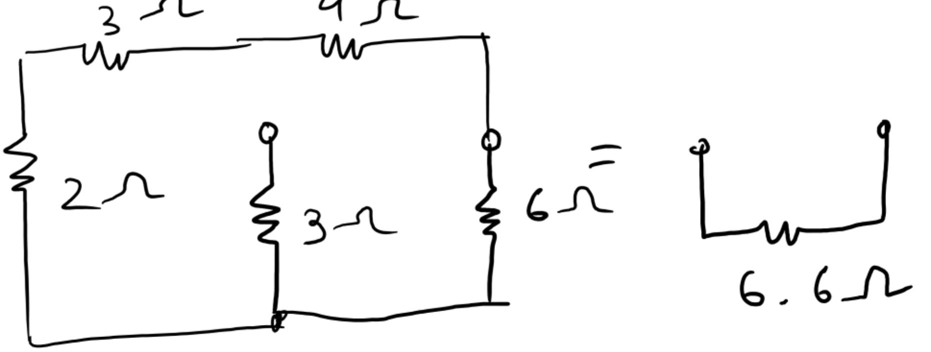
$$\Rightarrow \frac{4v_2 - 24 + 2v_2 + 3v_2 - 36 - 3v_1}{12} = -6$$

$$\boxed{-3v_1 + 9v_2 = -4} \Rightarrow -9v_1 + 27v_2 = -12$$

$$\Rightarrow 22v_2 = 88 \Rightarrow \boxed{v_2 = 4V}$$

$$\frac{v_2 - 8}{3} + 6 + I_{sc} = 0 \Rightarrow \boxed{I_{sc} = -4.667\,A}$$

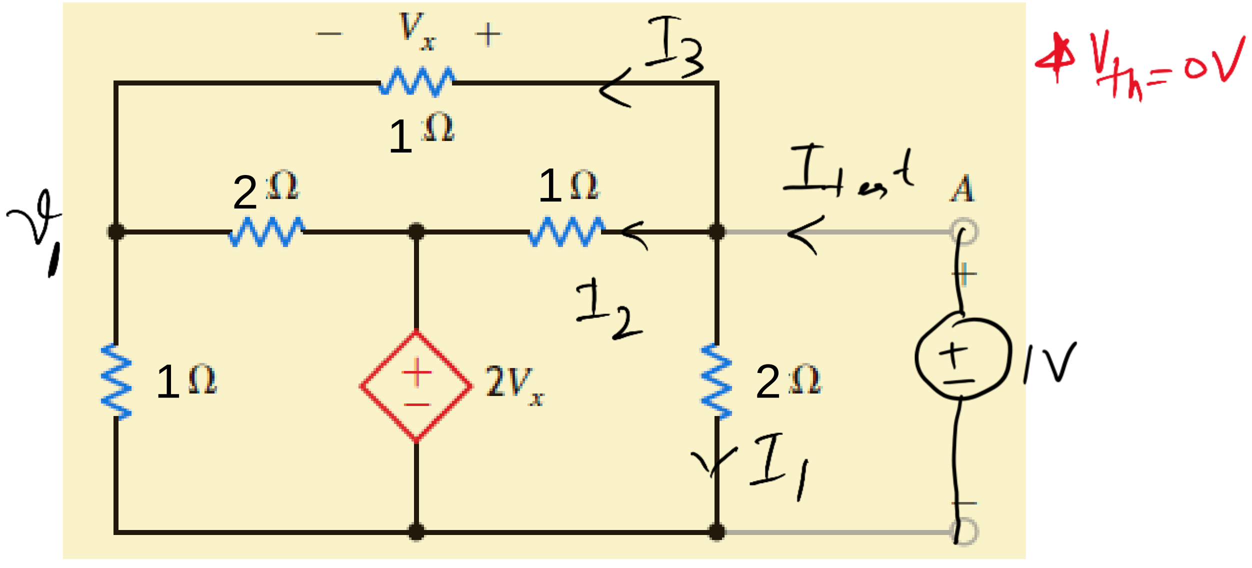
R_{th}:



Thevenin's and Norton's Theorems

Problems – In class

Example 4: Find the Thevenin equivalent circuit for the following circuit with respect to the terminals AB (Irwin – Example 5.8)



We have

$$v_x + v_1 = 1V \Rightarrow v_1 = 1 - v_x$$

KCL ;

$$\frac{v_1}{1} + \frac{v_1 - 2v_x}{2} + \frac{v_1 - 1}{} = 0$$

$$\Rightarrow 1 - v_x + \frac{1 - 3v_x}{2} - v_x = 0$$

$$\Rightarrow 3 = 7v_x \Rightarrow \boxed{v_x = \frac{3}{7} V}$$

$$I_1 = \frac{1}{2} A, \quad I_2 = \frac{1 - 2(3/7)}{1} = 1/7 A$$

$$I_3 = 3/7 A$$

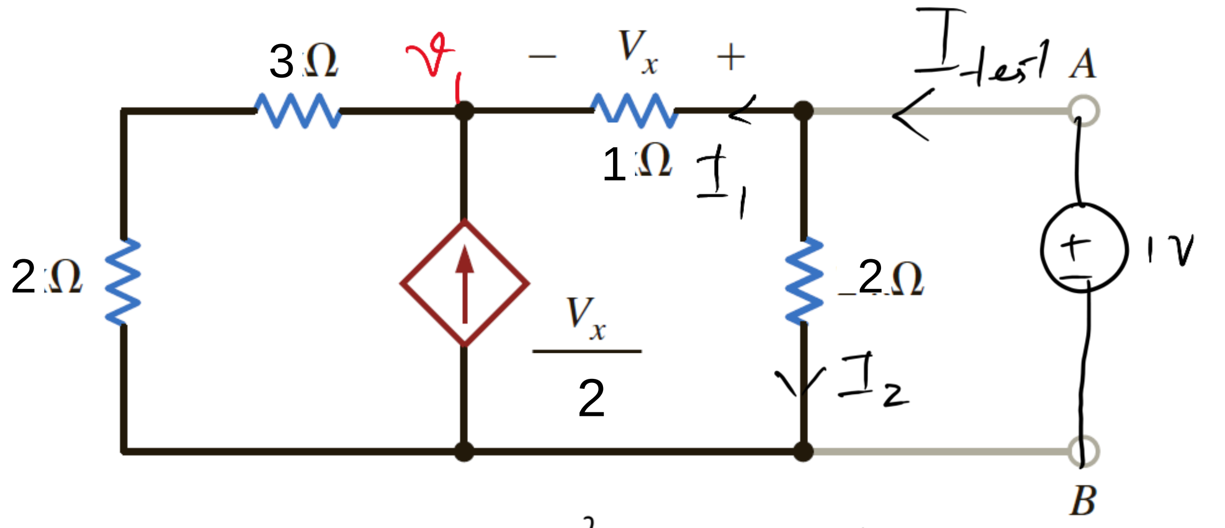
$$I_{test} = I_1 + I_2 + I_3 = \boxed{\frac{15}{14} A}$$

$$\Rightarrow R_{th} = \frac{1}{I_{test}} = \frac{14}{15} \Omega$$

Thevenin’s and Norton's Theorems

Problems – In class

Example 5: Find the Thevenin equivalent circuit for the following circuit with respect to the terminals AB (Irwin – E 5.13)



$$\frac{v_1}{5} + \frac{v_1 - 1}{1} = \frac{v_x}{2}$$

$$\left\{ v_x + v_1 = 1 \Rightarrow v_x = 1 - v_1 \right.$$

$$\Rightarrow \boxed{v_1 = \frac{15}{17}\,V}$$

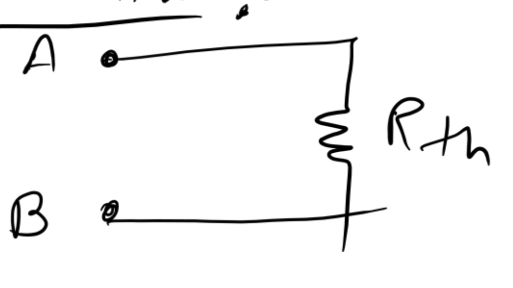
$$I_{test} = I_1 + I_2 = \frac{1}{2} + \frac{2}{17} = \frac{21}{34}\,A$$

$$I_1 = \frac{1 - v_1}{1} = \frac{2}{17}\,A$$

$$I_2 = \frac{1}{2}$$

$$\Rightarrow R_{th} = \frac{1}{I_{test}} = \frac{34}{21}\,\Omega$$

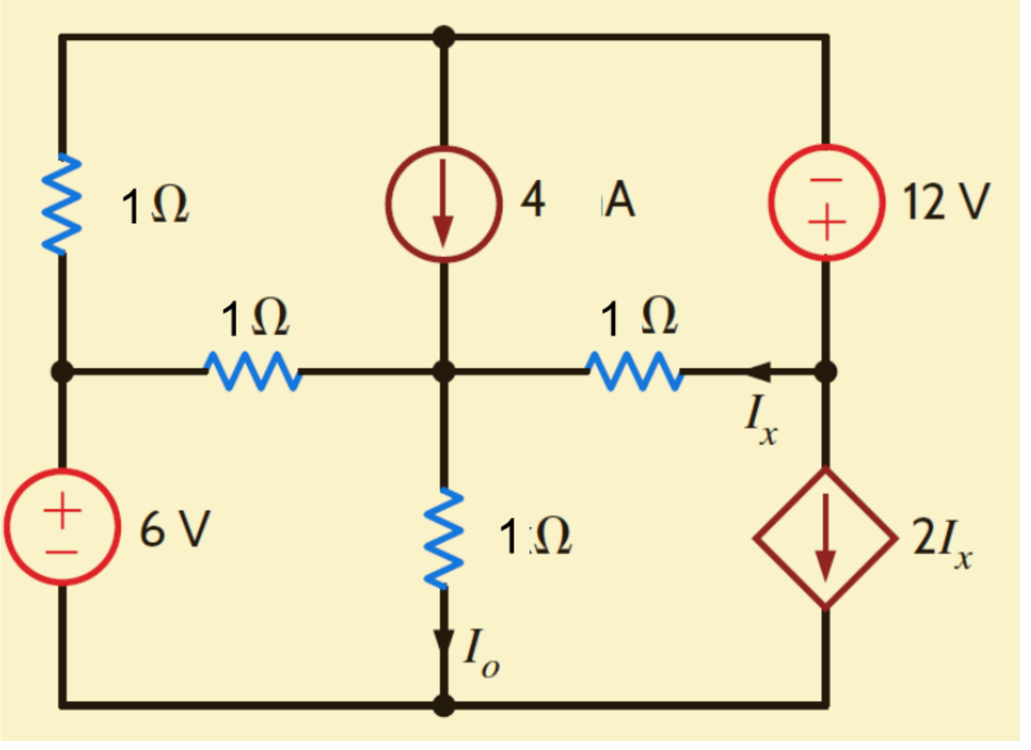
Equivalent :-



Thevenin’s and Norton's Theorems

Problems – In class

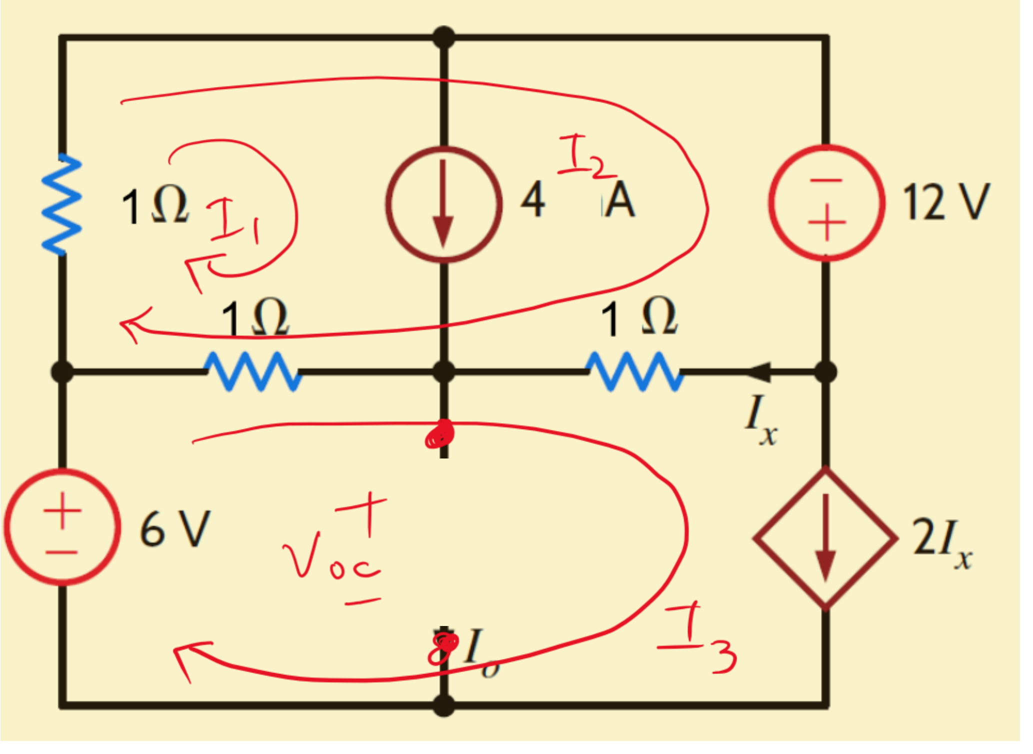
Example 6: Find I_o using Thevenin’s theorem or Norton’s theorem



Thevenin's and Norton's Theorems

Problems – In class

Determine V_{oc} first:



$I_1 = 4A, \quad I_3 = 2I_x$

Loop 2

$$-12 + 1(I_2 - I_3) + 1(I_1 + I_2 - I_3) + (I_1 + I_2)1 = 0$$

$I_x = I_2 - I_3 \Rightarrow$ $\boxed{3I_x = I_2}$

Solving

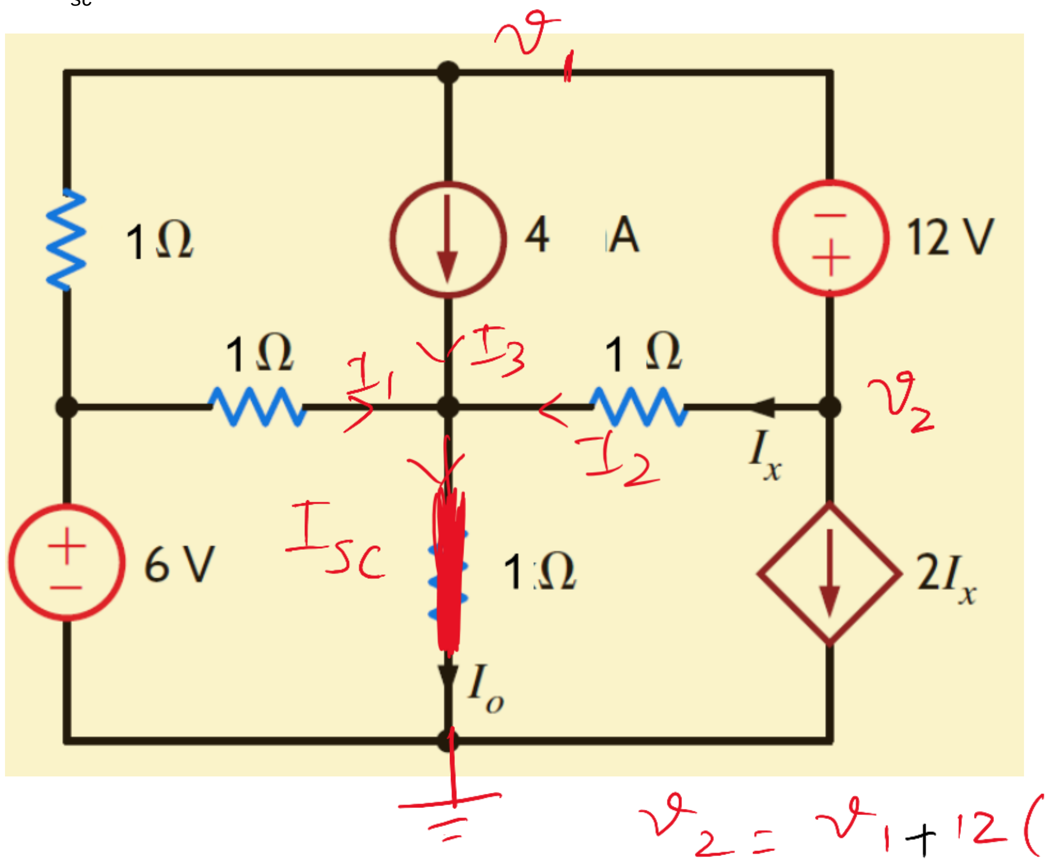
$I_3 = \frac{8}{5}A, \quad I_2 = \frac{12}{5}A, \quad I_x = \frac{4}{5}A$

$$V_{oc} = 6 + (I_1 + I_2 - I_3)1 = \frac{54}{5} V$$

Thevenin's and Norton's Theorems

Problems – In class

Determine I_{sc}



$v_2 = v_1 + 12$ (Super Node)

$$\frac{v_1 - 6}{1} + 4 + \frac{v_2}{1} + 2I_x = 0 \qquad I_x = \frac{v_2}{1}$$

$$\Rightarrow v_1 - 6 + 4 + 3v_1 + 36 = 0$$

$$\Rightarrow v_1 = -\frac{17}{2}\ V, \quad v_2 = 7/2$$

$$I_{sc} = \frac{6}{1} + \frac{7}{2} + 4 = \frac{27}{2}\ A$$

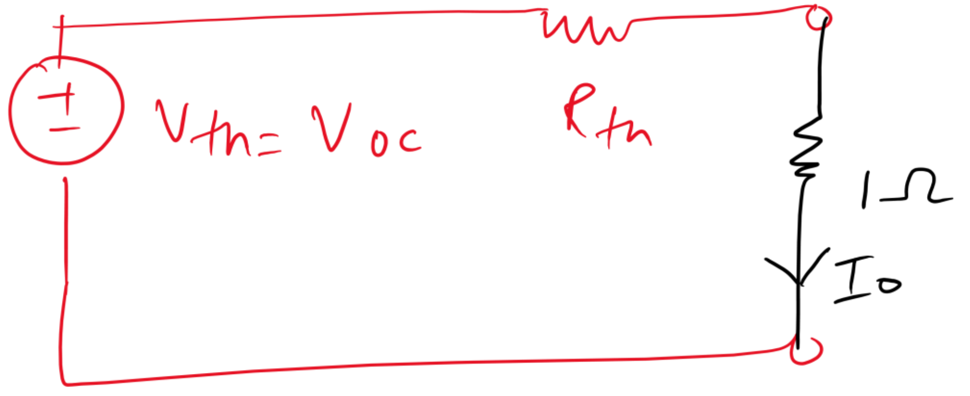
$$\Rightarrow R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{54/5}{27/2} = \frac{4}{5}\ \Omega$$

Thevenin’s and Norton's Theorems

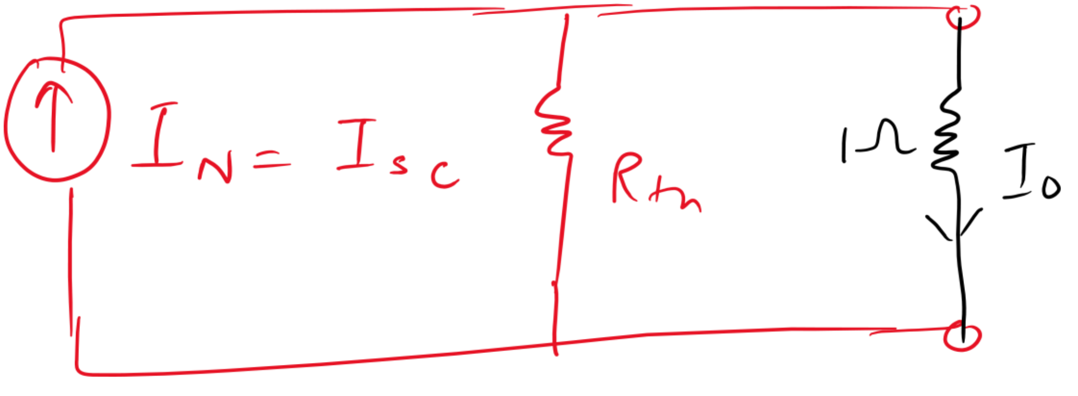
Problems – In class

Using equivalent circuits to determine I_o

Thevenin Equivalent



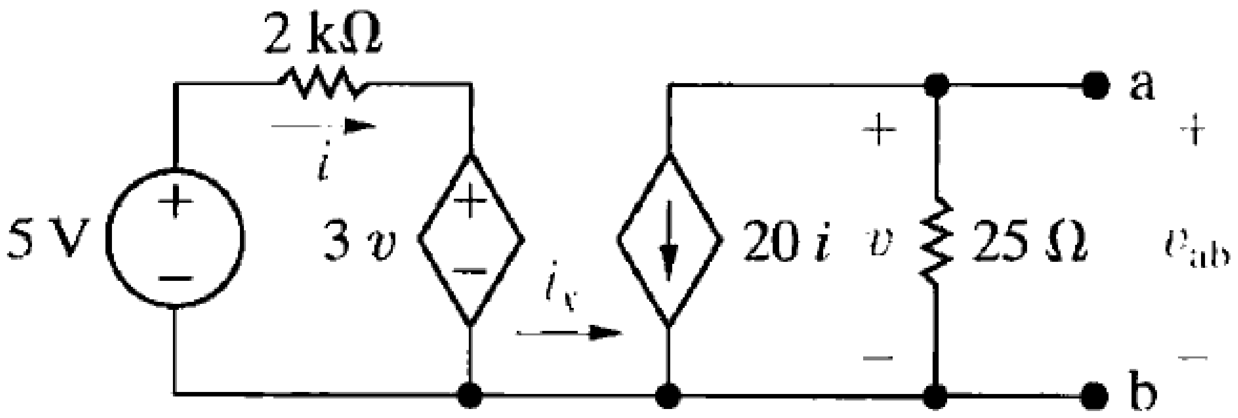
Norton Equivalent



$$I_o = \frac{54/5}{1+\frac{4}{5}} = \boxed{6\ A}$$

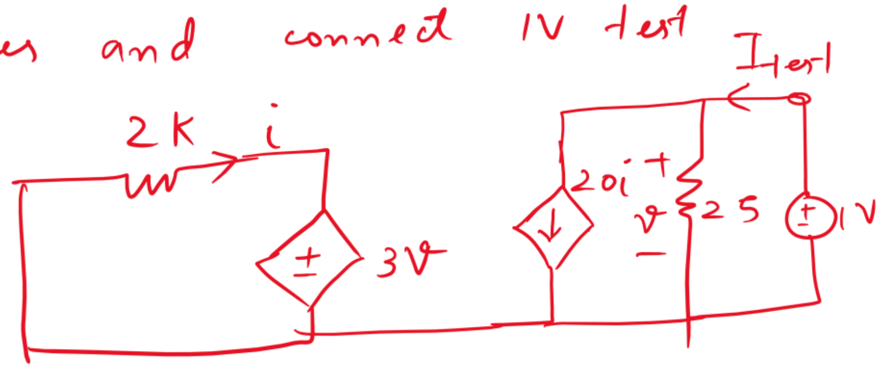
Problems – In class

Example 7: Find the Thevenin equivalent circuit for the following circuit with respect to the terminals a,b



First find R_{th} :-

switch off sources and connect 1V test source



$v = 1V$

$3v = 3V$

$i = -3/2K$

$\Rightarrow \quad I_{test} = \frac{1}{25} - (20)(i) = 0.04 - 0.03 = 0.01A$

$\Rightarrow \boxed{R_{th} = 100\Omega} \leftarrow \frac{1}{I_{sc}}$

V_{th} :- $\quad V_{ab} = v = v_{th}$

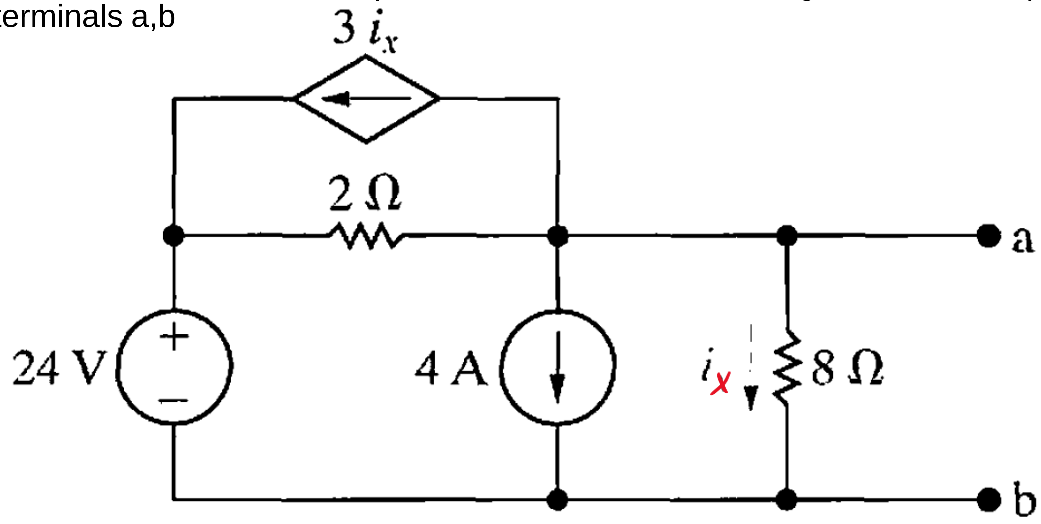
$V_{th} = -20i \times 25$ — ①

$i = \frac{25 - v_{th}}{2K}$ — ②

① + ② $\Rightarrow \boxed{V_{th} = -5V}$

Problems – In class

Example 8: Find the Thevenin equivalent circuit for the following circuit with respect to the terminals a,b



* Circuit contains both dependent and Independent sources; we can use either of the following techniques

1) Determine V_{ab} and I_{sc}
2) Determine V_{ab}; Determine R_{th} by switching off independent sources and applying test current (or voltage) source at a–b.

Let's apply 2)

V_{ab} : Using KCL;

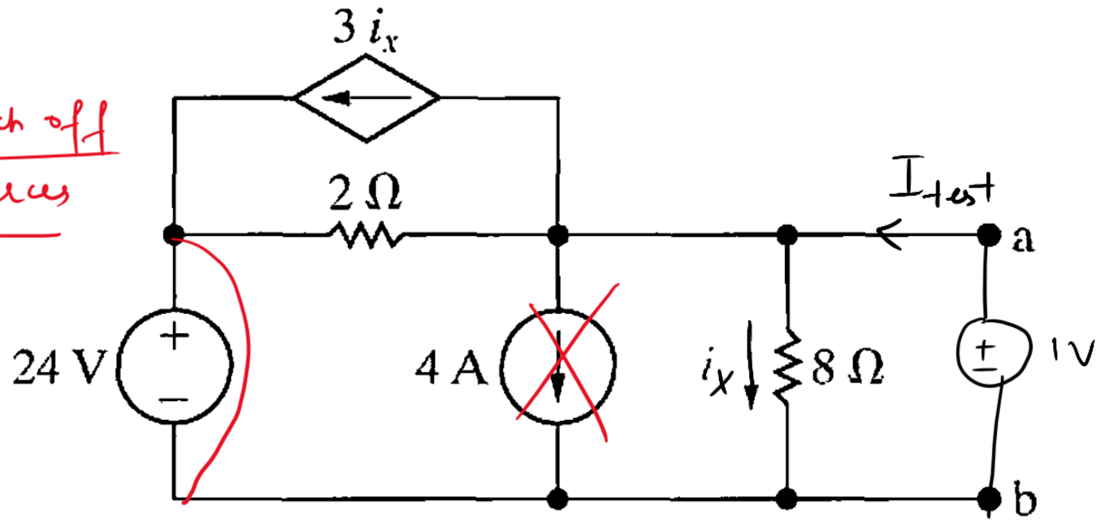
$$\frac{V_{ab}}{8} + \frac{V_{ab}-24}{2} + 3i_x + 4 = 0$$

where $i_x = \dfrac{V_{ab}}{8} \Rightarrow \boxed{v_{ab} = 8 i_x}$

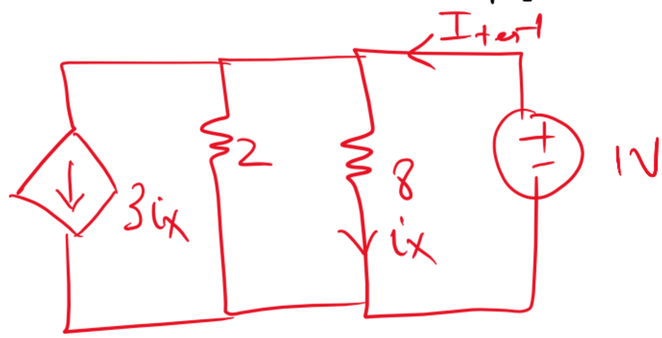
Solving $\boxed{v_{ab} = 8V}$

R_{th} : Apply 1V voltage source :

Switch off Sources



$i_x = \frac{1}{8}$ A



$$I_{test} = \frac{1}{8} + \frac{1}{2} + \frac{3}{8}$$

$$= 8/8 = 1A$$

$$\Rightarrow \quad R_{th} = \frac{1V}{I_{test}} = \boxed{1\,\Omega}$$