A Not-for-Profit University

School of Science and Engineering

EE 310 Signals and Systems

PROBLEM SET 8 – SOLUTIONS

Key Concepts of Sampling, Zero-Order Hold, and the z-Transform

This problem set covers the core ideas from ideal sampling, Nyquist rate, signal reconstruction, zero-order hold, processing continuous-time signals using discrete-time systems, and the z-transform. The aim is to build both computational skill and conceptual understanding.

Ideal Sampling and Nyquist Rate

A continuous-time signal $x_c(t)$ is sampled every T_s seconds to form the discrete-time signal

$$x[n] = x_c(nT_s)$$

where the sampling frequency is

$$f_s = \frac{1}{T_s}$$

If the highest analog frequency present in the signal is $f_{\max}$, then to avoid aliasing we require

$$f_s \geq 2f_{\max}$$

The quantity $2f_{\max}$ is called the **Nyquist rate**, and the corresponding maximum sampling interval is called the **Nyquist interval**.

Ideal Reconstruction

If sampling is done above the Nyquist rate and the signal is bandlimited, then the original continuous-time signal can be perfectly reconstructed using an ideal low-pass filter, or equivalently by sinc interpolation:

$$x_c(t) = \sum_{n=-\infty}^{\infty} x[n]\ \mathrm{sinc}\left(\frac{t - nT_s}{T_s}\right)$$

Zero-Order Hold

A zero-order hold (ZOH) converts a discrete-time sequence into a continuous-time piecewise-constant waveform by holding each sample value constant over one sampling interval. Its impulse response is

$$h_0(t) = u(t) - u(t - T_s)$$

and its Fourier transform is

$$H_0(j\omega) = \frac{1 - e^{-j\omega T_s}}{j\omega} = T_s\, e^{-j\omega T_s/2}\ \mathrm{sinc}\left(\frac{\omega T_s}{2\pi}\right)$$

The magnitude response is $|H_0(j\omega)| = T_s\left|\mathrm{sinc}\left(\frac{\omega T_s}{2\pi}\right)\right|$, which introduces a frequency-dependent distortion (droop) that must be compensated if faithful reconstruction is required.

z-Transform

The bilateral z-transform of a sequence $x[n]$ is

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

The region of convergence (ROC) is the set of values of z for which this sum converges.

Some standard pairs are:

$$a^n u[n] \overset{\mathcal{Z}}{\longleftrightarrow} \frac{1}{1 - az^{-1}}, \qquad |z| > |a|$$

$$-a^n u[-n-1] \overset{\mathcal{Z}}{\longleftrightarrow} \frac{1}{1 - az^{-1}}, \qquad |z| < |a|$$

The ROC is essential because the same algebraic expression can correspond to different time-domain signals depending on the ROC.

Problem 1

A continuous-time signal is

$$x_c(t) = \cos(200\pi t) + 2\cos(600\pi t)$$

(a) Find the frequencies present in $x_c(t)$ in Hz and determine the Nyquist rate.

(b) State whether aliasing occurs if the signal is sampled at $f_s = 1000$ Hz and at $f_s = 500$ Hz.

(c) For $f_s = 1000$ Hz, find the discrete-time sequence $x[n]$.

Solution: Part (a): Frequencies and Nyquist rate

For a cosine of the form $\cos(2\pi f t)$, the frequency is f Hz.

Comparing:

$$\cos(200\pi t) = \cos(2\pi \cdot 100\, t)$$

so the first component has frequency

$$f_1 = 100 \text{ Hz}$$

Also,

$$2\cos(600\pi t) = 2\cos(2\pi \cdot 300\, t)$$

so the second component has frequency

$$f_2 = 300 \text{ Hz}$$

The highest frequency is

$$f_{\max} = 300 \text{ Hz}$$

Therefore the Nyquist rate is

$$f_N = 2f_{\max} = 600 \text{ Hz}$$

Part (b): Aliasing check

If $f_s = 1000$ Hz, then

$$1000 > 600$$

so there is **no aliasing**.

If $f_s = 500$ Hz, then

$$500 < 600$$

so **aliasing does occur**.

Part (c): Discrete-time sequence for $f_s = 1000$ Hz

The sampling period is

$$T_s = \frac{1}{1000} = 0.001 \text{ s}$$

Then

$$x[n] = x_c(nT_s) = \cos(200\pi n T_s) + 2\cos(600\pi n T_s)$$

Substitute $T_s = 0.001$:

$$x[n] = \cos(200\pi \cdot 0.001\, n) + 2\cos(600\pi \cdot 0.001\, n)$$

So

$$\boxed{x[n] = \cos(0.2\pi n) + 2\cos(0.6\pi n)}$$

Problem 2

A continuous-time signal

$$x_c(t) = \cos(2\pi \cdot 700\, t)$$

is sampled at

$$f_s = 800 \text{ Hz}$$

Find the aliased frequency that appears after sampling, and write the sampled sequence $x[n]$.

Solution: The original analog frequency is

$$f_0 = 700 \text{ Hz}$$

The Nyquist frequency is

$$\frac{f_s}{2} = 400 \text{ Hz}$$

Since

$$700 > 400$$

aliasing occurs.

To find the aliased frequency, reflect around f_s:

$$f_a = |f_0 - f_s| = |700 - 800| = 100 \text{ Hz}$$

So the sampled signal appears as a 100 Hz cosine.

Now find the discrete-time sequence:

$$x[n] = x_c(nT_s) = \cos\left(2\pi \cdot 700 \cdot \frac{n}{800}\right)$$

$$x[n] = \cos\left(2\pi \cdot \frac{7}{8} n\right)$$

Thus

$$x[n] = \cos\left(\frac{7\pi}{4} n\right)$$

Because discrete-time frequency is periodic modulo 2π,

$$\frac{7\pi}{4} \equiv -\frac{\pi}{4} \pmod{2\pi}$$

and cosine is even, so

$$\cos\left(\frac{7\pi}{4} n\right) = \cos\left(\frac{\pi}{4} n\right)$$

Therefore,

$$\boxed{x[n] = \cos\left(\frac{\pi}{4} n\right)}$$

This corresponds to the aliased analog frequency

$$\boxed{100 \text{ Hz}}$$

Problem 3

A bandlimited continuous-time signal has highest frequency component equal to 2 kHz. It is sampled at 5 kHz.

(a) Can the signal be perfectly reconstructed in theory?

(b) Write the ideal interpolation formula for reconstruction.

(c) Briefly explain why sinc functions appear in ideal reconstruction.

Solution: Part (a): Perfect reconstruction

The highest frequency is

$$f_{\max} = 2 \text{ kHz}$$

The Nyquist rate is

$$2f_{\max} = 4 \text{ kHz}$$

Since the sampling frequency is

$$f_s = 5 \text{ kHz} > 4 \text{ kHz}$$

perfect reconstruction is theoretically possible.

Part (b): Ideal interpolation formula

The sampling period is

$$T_s = \frac{1}{f_s} = \frac{1}{5000}$$

The reconstruction formula is

$$\boxed{x_c(t) = \sum_{n=-\infty}^{\infty} x[n]\ \mathrm{sinc}\left(\frac{t - nT_s}{T_s}\right)}$$

Part (c): Why sinc appears

Ideal reconstruction in frequency domain uses an ideal low-pass filter that keeps the baseband copy of the spectrum and removes the repeated shifted copies caused by sampling.

The inverse Fourier transform of an ideal rectangular low-pass filter is a sinc function. Therefore each sample contributes a shifted sinc pulse, and the full signal is reconstructed by summing all of them.

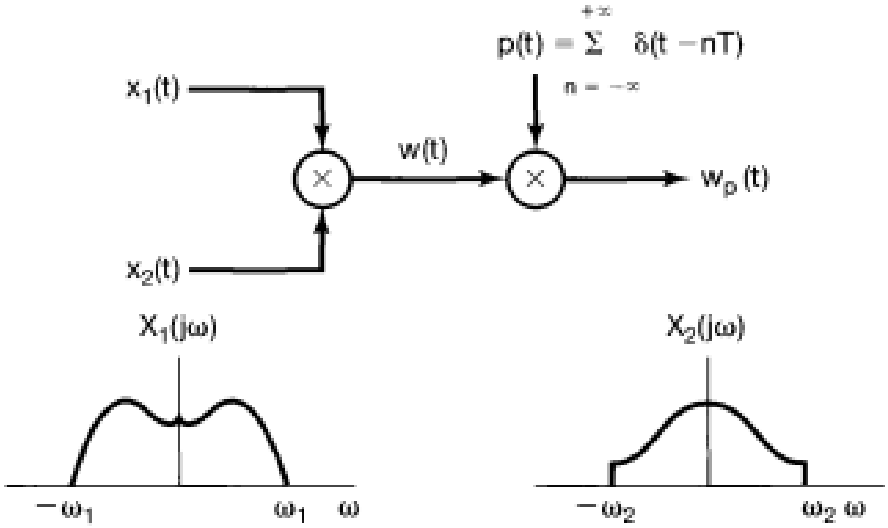
So sinc appears because:

$$\text{ideal low-pass in frequency} \Longleftrightarrow \text{sinc in time}$$

Problem 4

In the system shown below, two functions of time, $x_1(t)$ and $x_2(t)$, are multiplied together, and the product $w(t)$ is sampled by a periodic impulse train. $x_1(t)$ is band limited to ω_1, and $x_2(t)$ is band limited to ω_2; that is,

$$X_1(j\omega) = 0, \quad |\omega| \geq \omega_1, \qquad X_2(j\omega) = 0, \quad |\omega| \geq \omega_2.$$



Determine the **maximum** sampling interval T such that $w(t)$ is recoverable from $w_p(t)$ through the use of an ideal lowpass filter.

Hint: Multiplication in time = convolution in frequency. Two signals walk into a convolution... they leave wider. Bandwidth adds.

Solution: Multiplication in time corresponds to convolution in frequency:

$$W(j\omega) = \frac{1}{2\pi} X_1(j\omega) * X_2(j\omega).$$

Convolving a spectrum supported on $[-\omega_1, \omega_1]$ with one supported on $[-\omega_2, \omega_2]$ gives a result supported on $[-(\omega_1 + \omega_2),\ \omega_1 + \omega_2]$.

Hence the highest frequency in $w(t)$ is

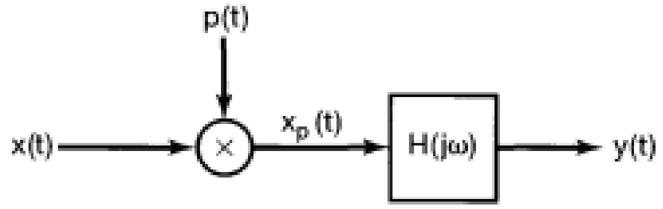
$$\omega_{\max} = \omega_1 + \omega_2.$$

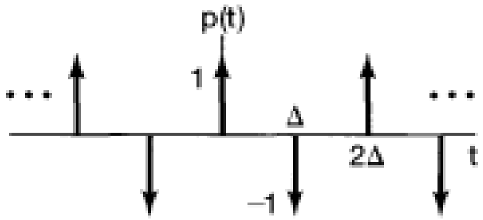
The Nyquist sampling rate is $\omega_s = 2(\omega_1 + \omega_2)$, so the maximum sampling interval is

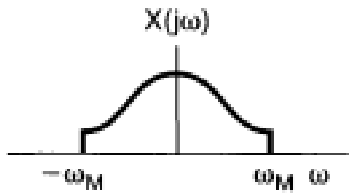
$$\boxed{T_{\max} = \frac{2\pi}{\omega_s} = \frac{\pi}{\omega_1 + \omega_2}.}$$

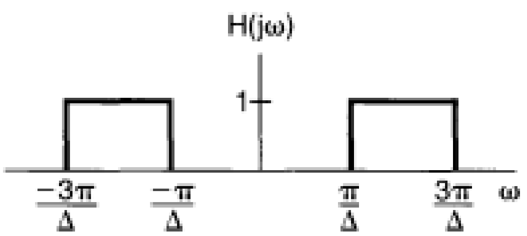
Problem 5

Shown below is a system in which the sampling signal $p(t)$ is an impulse train with alternating sign. The Fourier transform of the input signal is as indicated in the figure.









(a) For $\Delta < \pi/(2\omega_M)$, sketch the Fourier transform of $x_p(t)$ and $y(t)$.

(b) For $\Delta < \pi/(2\omega_M)$, determine a system that will recover $x(t)$ from $x_p(t)$.

(c) For $\Delta < \pi/(2\omega_M)$, determine a system that will recover $x(t)$ from $y(t)$.

(d) What is the **maximum** value of Δ in relation to ω_M for which $x(t)$ can be recovered from either $x_p(t)$ or $y(t)$?

Hint: $+1, -1, +1, -1, \ldots = e^{j\pi n}$. *Your spectrum didn't disappear — it rage quit DC and moved to* $\pm\pi/\Delta$. *Bring it back.*

Solution: Setup.

The alternating-sign impulse train can be written as

$$p(t) = \sum_{n=-\infty}^{\infty} (-1)^n \,\delta(t - n\Delta) = e^{j\pi t/\Delta} \sum_{n=-\infty}^{\infty} \delta(t - n\Delta).$$

Its Fourier transform is

$$P(j\omega) = \frac{2\pi}{\Delta} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{(2k+1)\pi}{\Delta}\right),$$

i.e. impulses at *odd* multiples of π/Δ only (none at $\omega = 0$).

Consequently,

$$X_p(j\omega) = \frac{1}{\Delta} \sum_{k=-\infty}^{\infty} X\left(j\left(\omega - \frac{(2k+1)\pi}{\Delta}\right)\right).$$

Part (a): Spectra of $x_p(t)$ and $y(t)$

For $\Delta < \pi/(2\omega_M)$ the sampling rate $1/\Delta$ exceeds ω_M/π, so the shifted replicas do not overlap. The spectrum $X_p(j\omega)$ consists of copies of $X(j\omega)$ centred at $\pm\pi/\Delta$, $\pm 3\pi/\Delta, \ldots$—none at $\omega = 0$.

The filter $H(j\omega)$ shown in the figure is a bandpass filter passing $\pi/\Delta - \omega_M \le |\omega| \le \pi/\Delta + \omega_M$. Its output $y(t)$ contains the single replica centred at $\pm\pi/\Delta$, scaled by the filter gain.

Part (b): Recovering $x(t)$ from $x_p(t)$

Multiply $x_p(t)$ by $e^{j\pi t/\Delta}$ (demodulate), then apply an ideal lowpass filter with cutoff ω_M and gain Δ:

$$\boxed{r(t) = x_p(t) \cdot e^{j\pi t/\Delta}, \quad \text{followed by ideal LPF: } |\omega| \le \omega_M, \text{ gain } \Delta.}$$

Part (c): Recovering $x(t)$ from $y(t)$

$y(t)$ already contains the $k = 0$ replica centred at π/Δ. Multiply by $e^{-j\pi t/\Delta}$ and apply the same lowpass filter:

$$\boxed{r(t) = y(t) \cdot e^{-j\pi t/\Delta}, \quad \text{followed by ideal LPF: } |\omega| \le \omega_M, \text{ gain } 1.}$$

Part (d): Maximum Δ

To avoid overlap between adjacent replicas (centred at π/Δ and $3\pi/\Delta$), we need

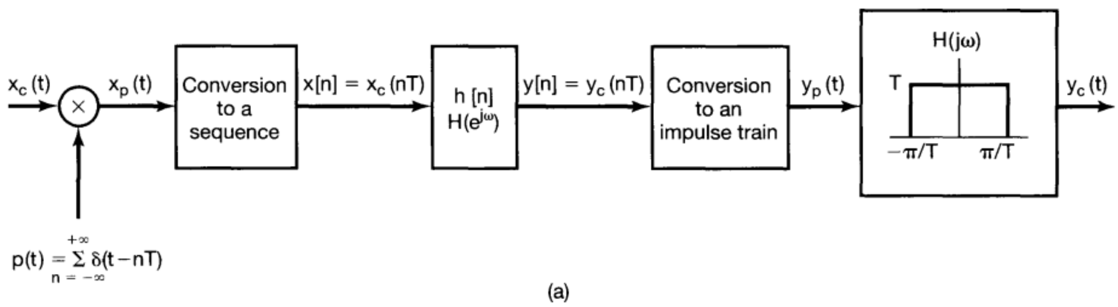
$$\frac{\pi}{\Delta} - \omega_M \ge \frac{\pi}{\Delta} - \frac{2\pi}{\Delta} + \omega_M \implies \frac{2\pi}{\Delta} \ge 2\omega_M \implies \Delta \le \frac{\pi}{\omega_M}.$$

However, the LPF used in parts (b) and (c) must also not overlap with the $k = -1$ replica, giving the tighter condition

$$\boxed{\Delta_{\max} = \frac{\pi}{2\omega_M}.}$$

Problem 6

The figure below shows the overall system for filtering a continuous-time signal using a discrete-time filter. If $X_c(j\omega)$ and $H(e^{j\omega})$ are as shown, with $1/T = 20$ kHz, sketch $X_p(j\omega)$, $X(e^{j\omega})$, $Y(e^{j\omega})$, $Y_p(j\omega)$, and $Y_c(j\omega)$.



(a)

Solution: We use $\omega = \Omega T$ and $\Omega_s = 2\pi/T = 2\pi \times 20{,}000$ rad/s.

From the figures: $X_c(j\Omega)$ is a smooth (triangular-like) spectrum with $|X_c(j\Omega)| = 1$ at $\Omega = 0$ and zero for $|\Omega| \geq \pi \times 10^4$ rad/s. $H(e^{j\omega})$ is a bandpass filter passing $\pi/4 \leq |\omega| \leq \pi$ with gain 1.

Step 1 — $X_p(j\Omega)$

Ideal C/D produces

$$X_p(j\Omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c(j(\Omega - k\Omega_s)) .$$

Since X_c is bandlimited to $\pi \times 10^4 = \Omega_s/2$, sampling at exactly the Nyquist rate means the copies just touch but do not overlap. $X_p(j\Omega)$ is a periodic repetition of $X_c(j\Omega)/T$ with period Ω_s.

Step 2 — $X(e^{j\omega})$

Substituting $\omega = \Omega T$ maps the baseband copy ($|\Omega| \leq \Omega_s/2$) to $|\omega| \leq \pi$:

$$X(e^{j\omega}) = \frac{1}{T} X_c\left(\frac{j\omega}{T}\right), \qquad |\omega| \leq \pi.$$

$X(e^{j\omega})$ has the same triangular shape as X_c but stretched to fill $[-\pi, \pi]$, with peak value $1/T$.

Step 3 — $Y(e^{j\omega})$

The discrete-time filter passes $\pi/4 \leq |\omega| \leq \pi$ and rejects $|\omega| < \pi/4$:

$$Y(e^{j\omega}) = H(e^{j\omega}) \cdot X(e^{j\omega}).$$

So $Y(e^{j\omega})$ retains only the portion of $X(e^{j\omega})$ for $\pi/4 \leq |\omega| \leq \pi$; the low-frequency part is zeroed out.

Step 4 — $Y_p(j\Omega)$

The D/C conversion maps $\omega \to \Omega T$, giving $Y_p(j\Omega)$ as a periodic repetition of $T \cdot Y(e^{j\Omega T})$.

Step 5 — $Y_c(j\Omega)$

The ideal reconstruction LPF with cutoff $\Omega_s/2$ and gain T selects the baseband copy and restores amplitude:

$$\boxed{Y_c(j\Omega) = X_c(j\Omega) \cdot H\left(e^{j\Omega T}\right),}$$

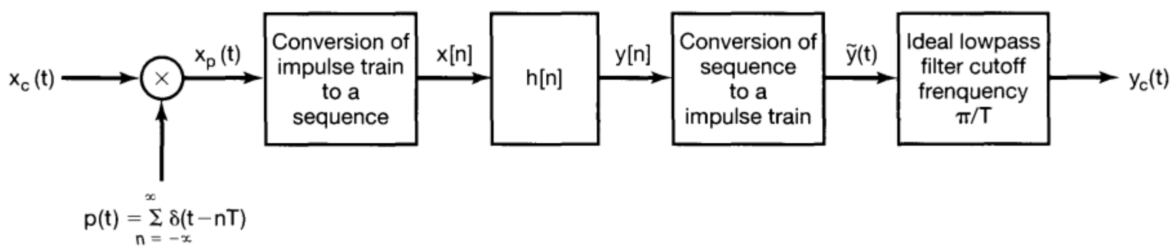
which is $X_c(j\Omega)$ with its low-frequency content ($|\Omega| < \pi \times 5 \times 10^3$) removed. The overall system acts as a highpass filter on the continuous-time signal, passing frequencies in the upper half of the band.

Problem 7

Shown below is a system that processes continuous-time signals using a digital filter $h[n]$ that is linear and causal with difference equation

$$y[n] = \tfrac{1}{2}\, y[n-1] + x[n].$$

For input signals that are band limited such that $X_c(j\omega) = 0$ for $|\omega| > \pi/T$, the system in the figure is equivalent to a continuous-time LTI system.



Determine the frequency response $H_c(j\omega)$ of the equivalent overall system with input $x_c(t)$ and output $y_c(t)$.

Solution: Step 1: Discrete-time frequency response $H(e^{j\omega})$

Take the DTFT of $y[n] = \frac{1}{2}y[n-1] + x[n]$:

$$Y(e^{j\omega}) = \tfrac{1}{2}\, e^{-j\omega}Y(e^{j\omega}) + X(e^{j\omega}).$$

Solving,

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1}{1 - \frac{1}{2}e^{-j\omega}}.$$

Step 2: Equivalent continuous-time response $H_c(j\Omega)$

For a band-limited input, the C/D–H–D/C–LPF cascade is equivalent to a continuous-time LTI system. The equivalent frequency response is obtained by the substitution $\omega = \Omega T$:

$$\boxed{H_c(j\Omega) = H(e^{j\omega})\big|_{\omega=\Omega T} = \frac{1}{1 - \frac{1}{2}\, e^{-j\Omega T}}, \qquad |\Omega| \le \frac{\pi}{T}.}$$

Step 3: Interpretation

Evaluate the magnitude response at a few key frequencies:

$$\Omega = 0 \quad \Rightarrow \quad |H_c(0)| = \frac{1}{1 - 1/2} = 2,$$

$$\Omega = \frac{\pi}{T} \quad \Rightarrow \quad |H_c(j\pi/T)| = \frac{1}{|1 + 1/2|} = \frac{2}{3}.$$

The gain is largest at $\Omega = 0$ and decreases toward the Nyquist frequency, so the equivalent continuous-time system is a **lowpass filter**.

Problem 8

A discrete-time LTI system has transfer function

$$H(z) = \frac{1}{1 - 0.5z^{-1}}$$

Find the impulse response $h[n]$ and state whether the system is stable.

Solution: For a causal right-sided sequence, we use the standard z-transform pair

$$a^n u[n] \overset{\mathcal{Z}}{\longleftrightarrow} \frac{1}{1 - az^{-1}}, \qquad |z| > |a|$$

Comparing

$$H(z) = \frac{1}{1 - 0.5z^{-1}}$$

with the standard form, we identify

$$a = 0.5$$

Therefore the impulse response is

$$\boxed{h[n] = (0.5)^n u[n]}$$

Now check stability.

A discrete-time LTI system is BIBO stable if its impulse response is absolutely summable:

$$\sum_{n=-\infty}^{\infty} |h[n]| < \infty$$

Here,

$$\sum_{n=0}^{\infty} |(0.5)^n| = \sum_{n=0}^{\infty} (0.5)^n = \frac{1}{1 - 0.5} = 2$$

This is finite, so the system is stable.

Thus,

$$\boxed{\text{The system is stable}}$$

Problem 9

Find the bilateral z-transform and ROC of

$$x[n] = a^n u[n]$$

Also explain what changes if we instead use the unilateral z-transform.

Solution: The bilateral z-transform is

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

Since

$$x[n] = a^n u[n]$$

we know that $x[n] = 0$ for $n < 0$, so the sum becomes

$$X(z) = \sum_{n=0}^{\infty} a^n z^{-n}$$

This is a geometric series:

$$X(z) = \sum_{n=0}^{\infty} (az^{-1})^n$$

Using

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}, \qquad |r| < 1$$

we get

$$X(z) = \frac{1}{1 - az^{-1}}$$

The convergence condition is

$$|az^{-1}| < 1$$

which gives

$$|a| < |z|$$

So the ROC is

$$\boxed{|z| > |a|}$$

Hence,

$$\boxed{x[n] = a^n u[n] \overset{\mathcal{Z}}{\longleftrightarrow} \frac{1}{1 - az^{-1}}, \qquad |z| > |a|}$$

For the unilateral z-transform, the sum is defined only from $n = 0$ to ∞:

$$X^+(z) = \sum_{n=0}^{\infty} x[n] z^{-n}$$

Since this sequence is already zero for $n < 0$, the unilateral and bilateral z-transforms give the same algebraic expression here:

$$\boxed{X^+(z) = \frac{1}{1 - az^{-1}}}$$

The main difference is conceptual: unilateral z-transform is especially useful when solving difference equations with initial conditions.

Problem 10

Find the z-transform and ROC of

$$x[n] = -a^n u[-n-1]$$

Solution: First note that $u[-n-1] = 1$ for

$$n \leq -1$$

and zero otherwise. So

$$x[n] = -a^n \quad \text{for } n \leq -1$$

Now compute the bilateral z-transform:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} = \sum_{n=-\infty}^{-1} (-a^n)z^{-n}$$

So

$$X(z) = -\sum_{n=-\infty}^{-1} a^n z^{-n}$$

Let $m = -n$, so that when $n = -1, -2, \dots$, we get $m = 1, 2, \dots$. Then

$$a^n z^{-n} = a^{-m} z^m = \left(\frac{z}{a}\right)^m$$

Thus

$$X(z) = -\sum_{m=1}^{\infty} \left(\frac{z}{a}\right)^m$$

This is a geometric series with ratio

$$r = \frac{z}{a}$$

Using

$$\sum_{m=1}^{\infty} r^m = \frac{r}{1-r}, \qquad |r| < 1$$

we get

$$X(z) = -\frac{z/a}{1 - z/a}$$

Simplify:

$$X(z) = -\frac{z}{a-z} = \frac{z}{z-a}$$

Dividing numerator and denominator by z:

$$\boxed{X(z) = \frac{1}{1 - az^{-1}}}$$

The ROC comes from

$$\left|\frac{z}{a}\right| < 1$$

so

$$\boxed{|z| < |a|}$$

Therefore,

$$\boxed{-a^n u[-n-1] \overset{\mathcal{Z}}{\longleftrightarrow} \frac{1}{1 - az^{-1}}, \qquad |z| < |a|}$$

This shows that the same algebraic expression can correspond to a different time-domain signal if the ROC changes.

Problem 11

Find the inverse z-transform of

$$X(z) = \frac{1}{1 - \frac{1}{2}z^{-1}}$$

for each of the following ROCs:

(a) $|z| > \frac{1}{2}$

(b) $|z| < \frac{1}{2}$

Hint: Same formula, different ROC = different personality. One is causal and chill. The other lives in the past.

Solution: We use the known z-transform pair

$$a^n u[n] \overset{\mathcal{Z}}{\longleftrightarrow} \frac{1}{1 - az^{-1}}, \qquad |z| > |a|$$

and

$$-a^n u[-n-1] \overset{\mathcal{Z}}{\longleftrightarrow} \frac{1}{1 - az^{-1}}, \qquad |z| < |a|$$

Here,

$$a = \frac{1}{2}$$

Part (a): ROC $|z| > \frac{1}{2}$

This corresponds to the right-sided sequence:

$$\boxed{x[n] = \left(\frac{1}{2}\right)^n u[n]}$$

Part (b): ROC $|z| < \frac{1}{2}$

This corresponds to the left-sided sequence:

$$\boxed{x[n] = -\left(\frac{1}{2}\right)^n u[-n-1]}$$

So the inverse z-transform depends on the ROC. The same algebraic expression does not uniquely determine the time-domain sequence unless the ROC is also specified.

Problem 12

A system has transfer function

$$H(z) = \frac{1}{(1 - 0.2z^{-1})(1 - 0.8z^{-1})}$$

Determine the poles, the ROC for a causal system, and whether the causal system is stable.

Solution: First find the poles.

The denominator is zero when

$$(1 - 0.2z^{-1})(1 - 0.8z^{-1}) = 0$$

So the poles are at

$$z = 0.2, \qquad z = 0.8$$

Hence,

$$\boxed{\text{Poles at } z = 0.2 \text{ and } z = 0.8}$$

For a **causal** system, the ROC lies outside the outermost pole. Since the outermost pole has magnitude 0.8, the ROC is

$$\boxed{|z| > 0.8}$$

Now check stability.

A discrete-time LTI system is stable if the ROC includes the unit circle:

$$|z| = 1$$

Since

$$|z| > 0.8$$

does include the unit circle, the causal system is stable.

Therefore,

$$\boxed{\text{The causal system is stable}}$$

So the final answers are:

$$\boxed{\text{Poles: } 0.2,\ 0.8}$$

$$\boxed{\text{Causal ROC: } |z| > 0.8}$$

$$\boxed{\text{Causal system is stable}}$$

— End of Problem Set —