LAHORE UNIVERSITY OF MANAGEMENT SCIENCES
Department of Electrical Engineering

EE310 Signals and Systems
Quiz 4 Solutions

Name: ____________________

Campus ID: ____________________

Total Marks: 10
Time Duration: 15 minutes

Question 1 (4 marks)

Consider two continuous-time periodic signals, $x(t)$ and $y(t)$, that share the same fundamental period T. Let their respective Fourier series representations be: $x(t) \overset{FS}{\longleftrightarrow} a_k$ and $y(t) \overset{FS}{\longleftrightarrow} b_k$. The multiplication property states that the Fourier series coefficients c_k of the product signal $z(t) = x(t)y(t)$ are given by the discrete convolution of a_k and b_k:

$$c_k = \sum_{m=-\infty}^{\infty} a_m b_{k-m}$$

Prove the multiplication property stated above. ***Hint:*** *To start your proof, express both $x(t)$ and $y(t)$ using the Fourier series synthesis equation:*

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

Make sure to use different dummy summation indices (for example, use m for $x(t)$ and n for $y(t)$) before multiplying the two series together.

Solution:

Proof:

Let $x(t)$ and $y(t)$ be periodic with fundamental frequency ω_0. Their Fourier series representations are:

$$x(t) = \sum_{m=-\infty}^{\infty} a_m e^{jm\omega_0 t}$$

$$y(t) = \sum_{n=-\infty}^{\infty} b_n e^{jn\omega_0 t}$$

Then:

$$z(t) = x(t)y(t) = \left(\sum_{m=-\infty}^{\infty} a_m e^{jm\omega_0 t}\right)\left(\sum_{n=-\infty}^{\infty} b_n e^{jn\omega_0 t}\right)$$

Multiplying the sums:

$$z(t) = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} a_m b_n e^{j(m+n)\omega_0 t}$$

Let $k = m + n$. Then $n = k - m$, and as m, n range over all integers, k also ranges over all integers:

$$z(t) = \sum_{k=-\infty}^{\infty}\left[\sum_{m=-\infty}^{\infty} a_m b_{k-m}\right] e^{jk\omega_0 t}$$

This is in the form of a Fourier series:

$$z(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

Comparing, we get:

$$c_k = \sum_{m=-\infty}^{\infty} a_m b_{k-m}$$

which is the discrete-time convolution of a_k and b_k.

Question 2 (6 marks)

Consider the following continuous-time signals with fundamental period $T = 6$:

$$x(t) = e^{j\,\frac{2\pi}{6}t}, \quad y(t) = \cos\left(\frac{4\pi}{6}t\right)$$

(a) [3 marks] Determine the Fourier series coefficients of $x(t)$ and $y(t)$.

(b) [1 mark] Sketch the Fourier series coefficients of $x(t)$ and $y(t)$.

(c) [2 marks] Using the multiplication property (Question 1) or otherwise, find the Fourier series coefficients of $z(t) = x(t) \cdot y(t)$.

Solution:

The fundamental frequency of both signals is:

$$\omega_0 = \frac{2\pi}{T} = \frac{\pi}{3} \text{ rad/s}$$

By directly comparing $x(t)$ with the Fourier series synthesis equation given by

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t},$$

we see that there is only one term, corresponding to $k = 1$, that is,

$$a_k = \begin{cases} 1, & \text{for } k = 1 \\ 0, & \text{otherwise} \end{cases}$$

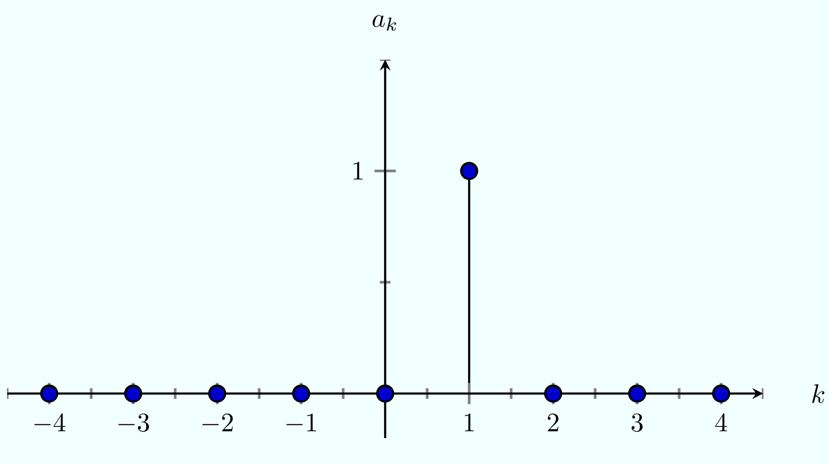
Using Euler's identity, we can express $y(t)$ as a sum of complex exponentials:

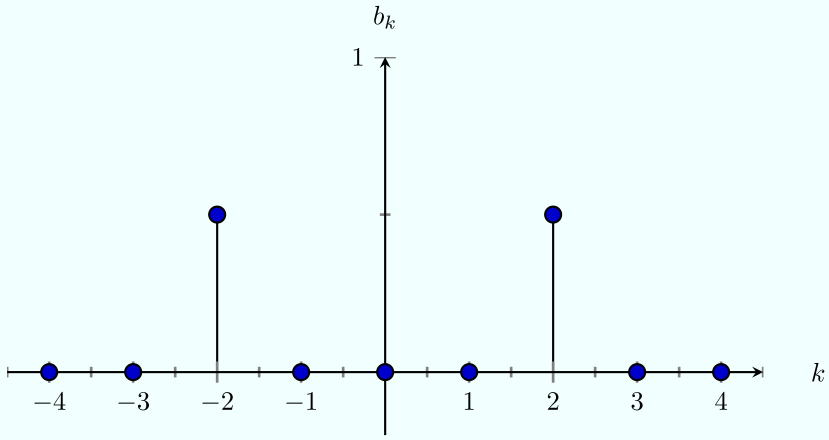
$$y(t) = \frac{1}{2}e^{j(2)\omega_0 t} + \frac{1}{2}e^{-j(2)\omega_0 t}$$

By directly comparing this with the synthesis equation, we identify the non-zero terms at $k = 2$ and $k = -2$. Let's denote the coefficients for $y(t)$ as b_k:

$$b_k = \begin{cases} \frac{1}{2}, & \text{for } k = 2 \text{ and } k = -2 \\ 0, & \text{otherwise} \end{cases}$$

Below are the sketches for the discrete coefficients a_k and b_k.





We know that multiplication in the time domain corresponds to discrete convolution in the frequency domain:

$$c_k = \sum_{m=-\infty}^{\infty} a_m b_{k-m}$$

Since a_m is zero everywhere except at $m = 1$, the infinite sum collapses to just a single term:

$$c_k = a_1 b_{k-1} = (1) b_{k-1} = b_{k-1}$$

This tells us that the coefficients c_k are simply the sequence b_k shifted to the right by $k = 1$. Therefore, the non-zero Fourier series coefficients for $z(t)$ are:

$$c_3 = \frac{1}{2}, \quad c_{-1} = \frac{1}{2}$$