LAHORE UNIVERSITY OF MANAGEMENT SCIENCES
Department of Electrical Engineering

EE310 Signals and Systems
Quiz 6 Solutions

Name: ______________________

Campus ID: ______________________

Total Marks: 10
Time Duration: 20 minutes

Question 1 (5 marks)

(2+2+1)

Let $m(t)$ be a real message signal whose Fourier transform $M(j\omega)$ is bandlimited to $|\omega| \le W$. The DSB-SC modulated signal is

$$s(t) = m(t)\cos(\omega_c t + \theta), \qquad \omega_c \gg W,$$

where θ is an unknown constant phase offset.

1. Derive $S(j\omega)$ in terms of $M(j\omega)$, ω_c, and θ.
2. A coherent demodulator multiplies $s(t)$ by the phase-mismatched local carrier $2\cos(\omega_c t)$ and passes the result through an ideal low-pass filter with cutoff ω_c and unit gain. Show that the recovered signal is $m(t)\cos\theta$, and identify all values of $\theta \in [0, 2\pi)$ for which the demodulator *completely suppresses* the output.

Solution:

(a) Derivation of $S(j\omega)$:

Given:

$$s(t) = m(t)\cos(\omega_c t + \theta)$$

Using Euler identity:

$$\cos(\omega_c t + \theta) = \frac{1}{2}\left(e^{j(\omega_c t+\theta)} + e^{-j(\omega_c t+\theta)}\right)$$

$$s(t) = \frac{1}{2}m(t)\left(e^{j\theta}e^{j\omega_c t} + e^{-j\theta}e^{-j\omega_c t}\right)$$

Using frequency shift property:

$$m(t)e^{j\omega_c t} \leftrightarrow M(j(\omega - \omega_c))$$
$$m(t)e^{-j\omega_c t} \leftrightarrow M(j(\omega + \omega_c))$$

Thus,

$$S(j\omega) = \frac{1}{2}\left[e^{j\theta}M(j(\omega - \omega_c)) + e^{-j\theta}M(j(\omega + \omega_c))\right]$$

(b) Demodulation:

Multiply:

$$y(t) = 2s(t)\cos(\omega_c t)$$

$$= 2m(t)\cos(\omega_c t + \theta)\cos(\omega_c t)$$

Using identity:

$$2\cos A\cos B = \cos(A - B) + \cos(A + B)$$

$$y(t) = m(t)\left[\cos(\theta) + \cos(2\omega_c t + \theta)\right]$$

After ideal LPF (removes high-frequency term):

$$y_{out}(t) = m(t)\cos\theta$$

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Condition for complete suppression:

$$m(t)\cos\theta = 0 \Rightarrow \cos\theta = 0$$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

Question 2 (5 marks)

(1+1+2+1) Define the signal

$$p(t) = \frac{\sin(3t)}{\pi t} \cdot \frac{\sin(2t)}{\pi t}$$

Using the convolution theorem, find and sketch $P(j\omega)$.

Solution:

Given:

$$p(t) = \frac{\sin(3t)}{\pi t} \cdot \frac{\sin(2t)}{\pi t}$$

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Step 1: Fourier transforms

$$\frac{\sin(Wt)}{\pi t} \longleftrightarrow \begin{cases} 1, & |\omega| \le W \\ 0, & \text{otherwise} \end{cases}$$

So,

$$X_1(j\omega) = \begin{cases} 1, & |\omega| \le 3 \\ 0, & \text{otherwise} \end{cases} \quad X_2(j\omega) = \begin{cases} 1, & |\omega| \le 2 \\ 0, & \text{otherwise} \end{cases}$$

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Step 2: Convolution

$$P(j\omega) = \frac{1}{2\pi}\left(X_1 * X_2\right)$$

The convolution of two rectangular functions gives a trapezoid.

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Step 3: Final expression

$$P(j\omega) = \begin{cases} \frac{1}{2\pi}(5 - |\omega|), & 1 \le |\omega| \le 5 \\ \frac{2}{\pi}, & |\omega| \le 1 \\ 0, & |\omega| > 5 \end{cases}$$

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Sketch:

- Trapezoid - Base from -5 to 5 - Flat top from -1 to 1 - Peak value $= \frac{2}{\pi}$