Name: _____

Campus ID: _____

Total Marks: 10
Time Duration: 15 minutes

Comparing Apples to Oranges: We will be implementing PCA to bridge the gap between fundamentally different quantities. The input features are speed (km/hr), cost (PKR) and beauty (on a scale of 1-10, with 10 being best). The data for the problem is given as follows:

car $= [120, 90, 6]$ #speed, cost, beauty
bicycle $= [15, 30, 3]$
cow $= [3, 60, 9]$

Question 1 (1 mark)

Arrange the data in the form of an input feature matrix.

> **Solution:**
> $$X = \begin{pmatrix} 120 & 90 & 6 \\ 15 & 30 & 3 \\ 3 & 60 & 9 \end{pmatrix}$$

Question 2 (3 marks)

Standardize the data using (only) the difference from feature vectors mean values i.e $s_i = x_i - \bar{x}$.

> **Solution:**
> $$\bar{x} = [46, 60, 6]$$
>
> $$S = \begin{pmatrix} 74 & 30 & 0 \\ -31 & -30 & -3 \\ -43 & 0 & 3 \end{pmatrix}$$

Question 3 (2 marks)

Find the covariance matrix for the standardized data of the previous part, i.e., $\sum = \frac{1}{n} S S^T$.

> **Solution:**
> $$\sum = \begin{pmatrix} 2762 & 1050 & -12 \\ 1050 & 600 & 30 \\ -12 & 30 & 6 \end{pmatrix}$$

Question 4 (3 marks)

Given that the Eigenvalues and corresponding Eigen vectors for the covariance matrix are as follows

$$\lambda = [3188, 180, 0]$$

Eigenvectors

$$\mathbf{V} = \begin{pmatrix} 0.93 & 0.37 & 0.068 \\ 0.38 & -0.91 & -0.17 \\ 0 & -0.18 & 0.98 \end{pmatrix}$$

- Project the input feature matrix onto the direction covering the maximum variance ($z = V_j^T X$ where for jth PC).

Solution:
$$\mathbf{z} = \begin{pmatrix} 0.93 & 0.38 & 0 \end{pmatrix} \begin{pmatrix} 120 & 90 & 6 \\ 15 & 30 & 3 \\ 3 & 60 & 9 \end{pmatrix} = \begin{pmatrix} 117 & 95 & 7 \end{pmatrix}$$

Question 5 (1 mark)

Plot the projected data on a scatter plot, and based on the plotted values, comment on the insights from this new feature representation for our beautiful cows (Hint: observe the distance between the projected values).

Solution: car and bicycle are more relatable to each other in terms of the new feature space and compared to cows. Moral: speed matters more than money or beauty (maybe)